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ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

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I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

II. Formulation

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where $\vec{a}$ is some unit vector, yields the value $+1$ then, according to quantum mechanics, measurement of $\vec{\sigma}_2 \cdot \vec{a}$ must yield the value -1 and vice versa. Now we make the hypothesis [2], and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of $\vec{\sigma}_2$, by previously measuring the same component of $\vec{\sigma}_1$, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does *not* determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state.

Let this more complete specification be effected by means of parameters λ. It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, we write as if λ were a single continuous parameter. The result A of measuring $\vec{\sigma}_1 \cdot \vec{a}$ is then determined by $\vec{a}$ and λ, and the result B of measuring $\vec{\sigma}_2 \cdot \vec{b}$ in the same instance is determined by $\vec{b}$ and λ, and

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$$A(\vec{a}, \lambda) = \pm 1, \ B(\vec{b}, \lambda) = \pm 1. \tag{1}$$

The vital assumption [2] is that the result B for particle 2 does not depend on the setting $\vec{a}$, of the magnet for particle 1, nor A on $\vec{b}$.

If $\rho(\lambda)$ is the probability distribution of λ then the expectation value of the product of the two components $\vec{\sigma}_1 . \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$ is

$$P(\vec{a}, \vec{b}) = \int d\lambda \, \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) \tag{2}$$

This should equal the quantum mechanical expectation value, which for the singlet state is

$$< \vec{\sigma}_1 \cdot \vec{a} \ \vec{\sigma}_2 \cdot \vec{b} > = - \vec{a} \cdot \vec{b} . \tag{3}$$

But it will be shown that this is not possible.

Some might prefer a formulation in which the hidden variables fall into two sets, with A dependent on one and B on the other; this possibility is contained in the above, since λ stands for any number of variables and the dependences thereon of A and B are unrestricted. In a complete physical theory of the type envisaged by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can then be thought of as initial values of these variables at some suitable instant.

III. Illustration

The proof of the main result is quite simple. Before giving it, however, a number of illustrations may serve to put it in perspective.

Firstly, there is no difficulty in giving a hidden variable account of spin measurements on a single particle. Suppose we have a spin half particle in a pure spin state with polarization denoted by a unit vector $\vec{p}$. Let the hidden variable be (for example) a unit vector $\vec{\lambda}$ with uniform probability distribution over the hemisphere $\vec{\lambda} \cdot \vec{p} > 0$. Specify that the result of measurement of a component $\vec{\sigma} \cdot \vec{a}$ is

$$\text{sign } \vec{\lambda} \cdot \vec{a}' \ , \tag{4}$$

where $\vec{a}'$ is a unit vector depending on $\vec{a}$ and $\vec{p}$ in a way to be specified, and the sign function is $+1$ or -1 according to the sign of its argument. Actually this leaves the result undetermined when $\lambda \cdot a' = 0$, but as the probability of this is zero we will not make special prescriptions for it. Averaging over $\vec{\lambda}$ the expectation value is

$$< \vec{\sigma} \cdot \vec{a} > = 1 - 2\theta'/\pi \ , \tag{5}$$

where θ' is the angle between $\vec{a}'$ and $\vec{p}$. Suppose then that $\vec{a}'$ is obtained from $\vec{a}$ by rotation towards $\vec{p}$ until

$$1 - \frac{2\theta'}{\pi} = \cos\theta \tag{6}$$

where θ is the angle between $\vec{a}$ and $\vec{p}$. Then we have the desired result

$$< \vec{\sigma} \cdot \vec{a} > = \cos\theta \tag{7}$$

So in this simple case there is no difficulty in the view that the result of every measurement is determined by the value of an extra variable, and that the statistical features of quantum mechanics arise because the value of this variable is unknown in individual instances.

Secondly, there is no difficulty in reproducing, in the form (2), the only features of (3) commonly used in verbal discussions of this problem:

$$\left.\begin{aligned} P(\vec{a}, \vec{a}) &= -P(\vec{a}, -\vec{a}) = -1 \\ P(\vec{a}, \vec{b}) &= 0 \text{ if } \vec{a} \cdot \vec{b} = 0 \end{aligned}\right\} \tag{8}$$

For example, let λ now be unit vector $\vec{\lambda}$, with uniform probability distribution over all directions, and take

$$\left.\begin{aligned} A(\vec{a}, \vec{\lambda}) &= \text{sign } \vec{a} \cdot \vec{\lambda} \\ B(a, b) &= -\text{sign } \vec{b} \cdot \vec{\lambda} \end{aligned}\right\} \tag{9}$$

This gives

$$P(\vec{a}, \vec{b}) = -1 + \frac{2}{\pi}\theta , \tag{10}$$

where θ is the angle between a and b, and (10) has the properties (8). For comparison, consider the result of a modified theory [6] in which the pure singlet state is replaced in the course of time by an isotropic mixture of product states; this gives the correlation function

$$-\frac{1}{3}\vec{a} \cdot \vec{b} \tag{11}$$

It is probably less easy, experimentally, to distinguish (10) from (3), than (11) from (3).

Unlike (3), the function (10) is not stationary at the minimum value -1(at $\theta = 0$). It will be seen that this is characteristic of functions of type (2).

Thirdly, and finally, there is no difficulty in reproducing the quantum mechanical correlation (3) if the results A and B in (2) are allowed to depend on $\vec{b}$ and $\vec{a}$ respectively as well as on $\vec{a}$ and $\vec{b}$. For example, replace $\vec{a}$ in (9) by $\vec{a}'$, obtained from $\vec{a}$ by rotation towards $\vec{b}$ until

$$1 - \frac{2}{\pi}\theta' = \cos\theta ,$$

where θ' is the angle between $\vec{a}'$ and $\vec{b}$. However, for given values of the hidden variables, the results of measurements with one magnet now depend on the setting of the distant magnet, which is just what we would wish to avoid.

IV. Contradiction

The main result will now be proved. Because ρ is a normalized probability distribution,

$$\int d\lambda\, \rho(\lambda) = 1 , \tag{12}$$

and because of the properties (1), P in (2) cannot be less than -1. It can reach -1 at $\vec{a} = \vec{b}$ only if

$$A(\vec{a}, \lambda) = -B(\vec{a}, \lambda) \tag{13}$$

except at a set of points λ of zero probability. Assuming this, (2) can be rewritten

$$P(\vec{a}, \vec{b}) = -\int d\lambda\, \rho(\lambda)\, A(\vec{a}, \lambda)\, A(\vec{b}, \lambda) . \tag{14}$$

It follows that $\vec{c}$ is another unit vector

$$P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) = -\int d\lambda\, \rho(\lambda)\, [A(\vec{a}, \lambda)\, A(\vec{b}, \lambda) - A(\vec{a}, \lambda)\, A(\vec{c}, \lambda)]$$
$$= \int d\lambda\, \rho(\lambda)\, A(\vec{a}, \lambda)\, A(\vec{b}, \lambda)\, [A(\vec{b}, \lambda)\, A(\vec{c}, \lambda) - 1]$$

using (1), whence

$$|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \le \int d\lambda\, \rho(\lambda)\, [1 - A(\vec{b}, \lambda)\, A(\vec{c}, \lambda)]$$

The second term on the right is $P(\vec{b}, \vec{c})$, whence

$$1 + P(\vec{b}, \vec{c}) \ge |P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \tag{15}$$

Unless P is constant, the right hand side is in general of order $|\vec{b} - \vec{c}|$ for small $|\vec{b} - \vec{c}|$. Thus $P(\vec{b}, \vec{c})$ cannot be stationary at the minimum value (-1 at $\vec{b} = \vec{c}$) and cannot equal the quantum mechanical value (3).

Nor can the quantum mechanical correlation (3) be arbitrarily closely approximated by the form (2). The formal proof of this may be set out as follows. We would not worry about failure of the approximation at isolated points, so let us consider instead of (2) and (3) the functions

$$\bar{P}(\vec{a}, \vec{b}) \quad \text{and} \quad \overline{-\vec{a} \cdot \vec{b}}$$

where the bar denotes independent averaging of $P(\vec{a}', \vec{b}')$ and $-\vec{a}' \cdot \vec{b}'$ over vectors $\vec{a}'$ and $\vec{b}'$ within specified small angles of $\vec{a}$ and $\vec{b}$. Suppose that for all $\vec{a}$ and $\vec{b}$ the difference is bounded by ϵ:

$$|\bar{P}(\vec{a}, \vec{b}) + \vec{a} \cdot \vec{b}| \le \epsilon \tag{16}$$

Then it will be shown that ϵ cannot be made arbitrarily small.

Suppose that for all a and b

$$|\overline{\vec{a} \cdot \vec{b}} - \vec{a} \cdot \vec{b}| \le \delta \tag{17}$$

Then from (16)

$$|\bar{P}(\vec{a}, \vec{b}) + \vec{a} \cdot \vec{b}| \le \epsilon + \delta \tag{18}$$

From (2)

$$\bar{P}(\vec{a}, \vec{b}) = \int d\lambda\, \rho(\lambda)\, \bar{A}(\vec{a}, \lambda)\, \bar{B}(\vec{b}, \lambda) \tag{19}$$

where

$$|\bar{A}(\vec{a}, \lambda)| \le 1 \quad \text{and} \quad |\bar{B}(\vec{b}, \lambda)| \le 1 \tag{20}$$

From (18) and (19), with $\vec{a} = \vec{b}$,

$$d\lambda\, \rho(\lambda)\, [\bar{A}(\vec{b}, \lambda)\, \bar{B}(\vec{b}, \lambda) + 1] \le \epsilon + \delta \tag{21}$$

From (19)

$$\bar{P}(\vec{a}, \vec{b}) - \bar{P}(\vec{a}, \vec{c}) = \int d\lambda\, \rho(\lambda)\, [\bar{A}(\vec{a}, \lambda)\, \bar{B}(\vec{b}, \lambda) - \bar{A}(\vec{a}, \lambda)\, \bar{B}(\vec{c}, \lambda)]$$
$$= \int d\lambda\, \rho(\lambda)\, \bar{A}(\vec{a}, \lambda)\, \bar{B}(\vec{b}, \lambda)\, [1 + \bar{A}(\vec{b}, \lambda)\, \bar{B}(\vec{c}, \lambda)]$$
$$- \int d\lambda\, \rho(\lambda)\, \bar{A}(\vec{a}, \lambda)\, \bar{B}(\vec{c}, \lambda)\, [1 + \bar{A}(\vec{b}, \lambda)\, \bar{B}(\vec{b}, \lambda)]$$

Using (20) then

$$|\bar{P}(\vec{a}, \vec{b}) - \bar{P}(\vec{a}, \vec{c})| \leq \int d\lambda \propto(\lambda)\, [1 + \bar{A}(\vec{b}, \lambda)\, \bar{B}(\vec{c}, \lambda)] + \int d\lambda\, \rho(\lambda)\, [1 + \bar{A}(\vec{b}, \lambda)\, \bar{B}(\vec{b}, \lambda)]$$

Then using (19) and 21)

$$|\bar{P}(\vec{a}, \vec{b}) - \bar{P}(\vec{a}, \vec{c})| \leq 1 + \bar{P}(\vec{b}, \vec{c}) + \epsilon + \delta$$

Finally, using (18),

$$|\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b}| - 2(\epsilon + \delta) \leq 1 - \vec{b} \cdot \vec{c} + 2(\epsilon + \delta)$$

or

$$4(\epsilon + \delta) \geq |\vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b}| + \vec{b} \cdot \vec{c} - 1 \tag{22}$$

Take for example $\vec{a} \cdot \vec{c} = 0,\ \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = 1/\sqrt{2}$ Then

$$4(\epsilon + \delta) \geq \sqrt{2} - 1$$

Therefore, for small finite δ, ϵ cannot be arbitrarily small.

Thus, the quantum mechanical expectation value cannot be represented, either accurately or arbitrarily closely, in the form (2).

V. Generalization

The example considered above has the advantage that it requires little imagination to envisage the measurements involved actually being made. In a more formal way, assuming [7] that any Hermitian operator with a complete set of eigenstates is an "observable", the result is easily extended to other systems. If the two systems have state spaces of dimensionality greater than 2 we can always consider two dimensional subspaces and define, in their direct product, operators $\vec{\sigma}_1$ and $\vec{\sigma}_2$ formally analogous to those used above and which are zero for states outside the product subspace. Then for at least one quantum mechanical state, the "singlet" state in the combined subspaces, the statistical predictions of quantum mechanics are incompatible with separable predetermination.

VI. Conclusion

In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant.

Of course, the situation is different if the quantum mechanical predictions are of limited validity. Conceivably they might apply only to experiments in which the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light. In that connection, experiments of the type proposed by Bohm and Aharonov [6], in which the settings are changed during the flight of the particles, are crucial.

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References

1. A. EINSTEIN, N. ROSEN and B. PODOLSKY, *Phys. Rev.* **47**, 777 (1935); see also N. BOHR, *Ibid.* **48**, 696 (1935), W. H. FURRY, *Ibid.* **49**, 393 and 476 (1936), and D. R. INGLIS, *Rev. Mod. Phys.* **33**, 1 (1961).
2. "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former." A. EINSTEIN in *Albert Einstein, Philosopher Scientist,* (Edited by P. A. SCHILP) p. 85, Library of Living Philosophers, Evanston, Illinois (1949).
3. J. VON NEUMANN, *Mathematishe Grundlagen der Quanten-mechanik.* Verlag Julius-Springer, Berlin (1932), [English translation: Princeton University Press (1955)]; J. M. JAUCH and C. PIRON, *Helv. Phys. Acta* **36**, 827 (1963).
4. J. S. BELL, to be published.
5. D. BOHM, *Phys. Rev.* **85**, 166 and 180 (1952).
6. D. BOHM and Y. AHARONOV, *Phys. Rev.* **108**, 1070 (1957).
7. P. A. M. DIRAC, *The Principles of Quantum Mechanics* (3rd Ed.) p. 37. The Clarendon Press, Oxford (1947).

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Is the moon there when nobody looks? Reality and the quantum theory

Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments have now shown that what bothered Einstein is not a debatable point but the observed behaviour of the real world.

N. David Mermin

[David Mermin is director of the Laboratory of Atomic and Solid State Physics at Cornell University. A solid-state theorist, he has recently come up with some quasithoughts about quasicrystals. He is known to PHYSICS TODAY readers as the person who made “boojum” an internationally accepted scientific term. With N.W.Ashcroft, he is about to start updating the world’s funniest solid-state physics text.
He says he *is* bothered by Bell’s theorem, but may have rocks in his head anyway.]

Quantum mechanics is magic[1]

In May 1935, Albert Einstein, Boris Podolsky and Nathan Rosen published[2] an argument that quantum mechanics fails to provide a complete description of physical reality. Today, 50 years later, the EPR paper and the theoretical and experimental work it inspired remain remarkable for the vivid illustration they provide of one of the most bizarre aspects of the world revealed to us by the quantum theory.

Einstein’s talent for saying memorable things did him a disservice when he declared “God does not play dice.” for it has been held ever since the basis for his opposition to quantum mechanics was the claim that a fundamental understanding of the world can only be statistical.

But the EPR paper, his most powerful attack on the quantum theory, focuses on quite a different aspect: the doctrine that physical properties have in general no objective reality independent of the act of observation. As Pascual Jordan put it[3]:

“Observations not only disturb what has to be measured, they produce it….We compel [the electron] to assume a definite position…. We ourselves produce the results of measurements.”

Jordan’s statement is something of a truism for contemporary physicists. Underlying it, we have all been taught, is the disruption of what is being measured by the act of measurement, made unavoidable by the existence of the quantum of action, which generally makes it impossible even in principle to construct probes that can yield the information classical intuition expects to be there.

Einstein didn’t like this. He wanted things out there to have properties, whether or not they were measured[4]:

“We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it.”

The EPR paper describes a situation ingeniously contrived to force the quantum theory into asserting that properties in a space-time region **B** are the result of an act of measurement in another space-time region **A**, so far from **B** that there is no possibility of the measurement in **A** exerting an influence on region **B** by any known dynamical mechanism. Under these conditions, Einstein maintained that the properties in **A** must have existed all along.

Spooky actions at a distance

Many of his simplest and most explicit statements of this position can be found in Einstein's correspondence with Max Born.[5] Throughout the book (which sometimes reads like a Nabokov novel), Born, pained by Einstein's distaste for the statistical character of the quantum theory, repeatedly fails, both in his letters and in his later commentary on the correspondence, to understand what is really bothering Einstein. Einstein tries over and over again, without success, to make himself clear. In March 1948, for example, he writes:

"That which really exists in B should …not depend on what kind of measurement is carried out in part of space A; it should also be independent of whether or not any measurement at all is carried out in space A. If one adheres to this program, one can hardly consider the quantum-theoretical description as a complete representation of the physically real. If one tries to do so in spite of this, one has to assume that the physically real in B suffers a sudden change as a result of a measurement in A. My instinct for physics bristles at this."

Or, in March 1947:

"I cannot seriously believe in [the quantum theory] because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance."

The "spooky actions at a distance" (spukhafte Fernwirkungen) are the acquisition of a definite value of a property by the system in region **B** by virtue of the measurement carried out in region **A**. The EPR paper presents a wavefunction that describes two correlated particles, localized in regions **A** and **B**, far apart.

In this particular two-particle state one can learn (in the sense of being able to predict with certainty the result of a subsequent measurement) either the position or the momentum of the particle in region **B** as a result of measuring the corresponding property of the particle in region **A**. If "that which really exists" in region **B** does not depend on what kind of measurement is carried out in region **A**, then the particle in region **B** must have had both a definite position and a definite momentum all along.

Because the quantum theory is intrinsically incapable of assigning values to both quantities at once, it must provide an incomplete description of the physically real. Unless, of couse, one asserts that it is only by virtue of the position (or momentum) measurement in **A** that the particle in **B** acquires its position (or momentum): spooky actions at a distance.

At a dramatic moment Pauli appears in the *Born-Einstein Letters*, writing Born from Princeton in 1954 with his famous tact on display:

"Einstein gave me your manuscript to read; he was *not at all* annoyed with you, but only said you were a person who will not listen. This agrees with the impression I have formed myself insofar as I was unable to recognize Einstein whenever you talked about him in either your letter or your manuscript. It seemed to me as if you had erected some dummy Einstein for yourself, which you then knocked down with great pomp. In particular, Einstein does not consider the concept of 'determinism' to be as fundamental as it is frequently held to be (as he told me emphatically many times)… In the same way, he *disputes* that he uses as criterion for the admissibility of a theory the question: *Is it rigorously deterministic?* "

Pauli goes on to state the real nature of Einstein's "philosophical prejudice" to Born, emphasizing that "Einstein's point of departure is 'realistic' rather than 'deterministic'." According to Pauli the proper grounds for challenging Einstein's view are simply that:

"One should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle. But it seems to me that Einstein's questions are ultimately always of this kind."

Faced with spooky actions at a distance, Einstein preferred to believe that things one cannot know anything about (such as the momentum of a particle with a definite position) do exist all the same.

In April 1948 he wrote to Born:

"Those physicists who regard the descriptive methods of quantum mechanics as definitive in principle would…drop the requirement for the independent existence of the physical reality present in different parts of space; they would be justified in pointing out that the quantum theory nowhere makes explicit use of this requirement. I admit this, but would point out: when I consider the the physical phenomena known to me, and especially those which are being so successfully encompassed by quantum mechanics, I still cannot find any fact anywhere which would make it appear likey that [the] requirement will have to be abandoned. I am therefore inclined to believe that the description of quantum mechanics…has to be regarded as an incomplete and indirect description of reality…"

A fact is found

The theoretical answer to this challenge to provide "any fact anywhere" was given in 1964 by John S.Bell, in a famous paper[6] in the short-lived journal *Physics.* Using a gedanken experiment invented[7] by David Bohm, in which "properties one cannot know anything about" (the simultaneous values of the spin of a particle along several distinct directions) are required to exist by EPR line of reasoning, Bell showed ("Bell's theorem") that the nonexistence of these properties is a direct consequence of the quantitative numerical predictions of the quantum theory. The conclusion is quite independent of whether or not one believes that the quantum theory offers a complete description of physical reality.

If the data in such an experiment are in agreement with the numerical predictions of the quantum theory, then Einstein's philosophical position has to be wrong.

In the last few years, in a beautiful series of experiments, Alain Aspect and his collaborators at the University of Paris's Institute of Theoretical and Applied Optics in Orsay provided[8] the experimental answer to Einstein's challenge by performing a version of the EPR experiment under conditions in which Bell's type of analysis applied.

They showed that the quantum-theoretic predictions were indeed obeyed. Thirty years after Einstein's challenge, a fact -not a metaphysical doctrine- was provided to refute him.

Attitudes toward this particular 50-year sequence of intellectual history and scientific discovery vary widely.[9] From the very start Bohr certainly took it seriously. Leon Rosenfeld describes[10] the impact of the EPR argument:

"This onslaught came down upon us as a bolt from the blue. Its effect on Bohr was remarkable….A new worry could not have come at a less propitious time. Yet, as soon as Bohr had heard my report of Einstein's argument, everything else was abandoned."

Bell's contribution has become celebrated in what might be called semi-popular culture. We read, for example, in *The Dancing Wu Li Masters* that[11]:

"Some physicists are convinced that [Bell's theorem] is the most important single work, perhaps, in the history of physics."

And indeed, Henry Stapp, a particle theorist at Berkeley, writes that[12]:

"Bell's theorem is the most profound discovery of science."

At the other end of the spectrum, Abraham Pais, in his recent biography of Einstein, writes[13] of the EPR article that "bolt from the blue" the basis for "the most profound discovery of science" :

"The only part of this article which will ultimately survive, I believe, is…a phrase ['No reasonable definition of reality could be expected to permit this'] which so poignantly summarizes Einstein's views on quantum mechanics in his later years."

I think it is fair to say that more physicists would side with Pais than with Stapp, but between the majority position of near indifference and the minority position of wild extravagance is an attitude I would characterize as balanced. This was expressed to me most succintly by a distinguished Princeton physicist on the occasion of my asking how he thought Einstein would have reacted to Bell's theorem.

He said that Einstein would have gone home and thought about it hard for several weeks that he couldn't guess what he would then have said, except that it would have been extremely interesting. He was sure that Einstein would have been very bothered by Bell's theorem.

Then he added:

"Anybody who's not bothered by Bell's theorem has to have rocks in his head."

To this moderate point of view I would only add the observation that contemporary physicists come in two varieties.

Type 1 physicists are bothered by EPR and Bell's theorem.

Type 2 (the majority) are not, but one has to distinguish two subvarieties.

Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein) or to contain physical assertions that can be shown to be false.

Type 2b are not bothered and refuse to explain why. Their position is unassailable. (There is a variant of type 2b who say that Bohr straightened out[14] the whole business, but refuse to explain how.)

A gedanken demonstration

To enable you to test which category you belong to, I shall describe, in black-box terms, a very simple version of Bell's gedanken experiment, deferring to the very end any reference whatever either to the underlying mechanism that makes the gadget work or to the quantum-theoretic analysis that accounts for the data. Perhaps this backwards way of proceeding will make it easier for you to lay aside your quantum theoretic prejudices and decide afresh whether what I describe is or is not strange.[15]

What I have in mind is a simple gedanken demonstration. The apparatus comes in three pieces. Two of them (**A** and **B**) function as detectors.

They are far apart from each other (in the analogous Aspect experiments over 10 meters apart). Each detector has a switch that can be set to one of three positions; each detector responds to an event by flashing either a red light or a green one. The third piece (**C**), midway between **A** and **B**, functions as a source. (See **figure 1**.)

There are no connections between the pieces, no mechanical connections, no electromagnetic connections, nor any other known kinds of relevant connections. (I promise that when you learn what is inside the black boxes you will agree that there are no connections.)

The detectors are thus incapable of signaling to each other or to the source via any known mechanism, and with the exception of the "particles" described below, the source has no way of signaling to the detectors. The demonstration proceeds as follows:

The switch of each detector is independently and randomly set to one of its three positions, and a button is pushed on the source; a little after that, each detector flashes either red or green. The settings of the switches and the colors that flash are recorded, and then the whole thing is repeated over and over again.

The data consist of a pair of numbers and a pair of colors for each run. **A** run, for example, in which **A** was set to 3, **B** was set to 2, **A** flashed red, and **B** flashed green, would be recorded as "**32RG**", as shown in **figure 2**.

Because there are no built-in connections between the source **C** and the detectors **A** and **B**, the link between the pressing of the button and the flashing of the light on a detector can only be provided by the passage of something (which we shall call a "particle", though you can call it anything you like) between the source and that detector. This can easily be tested; for example, by putting a brick between the source and a detector. In subsequent runs, that detector will not flash. When the brick is removed, everything works as before.

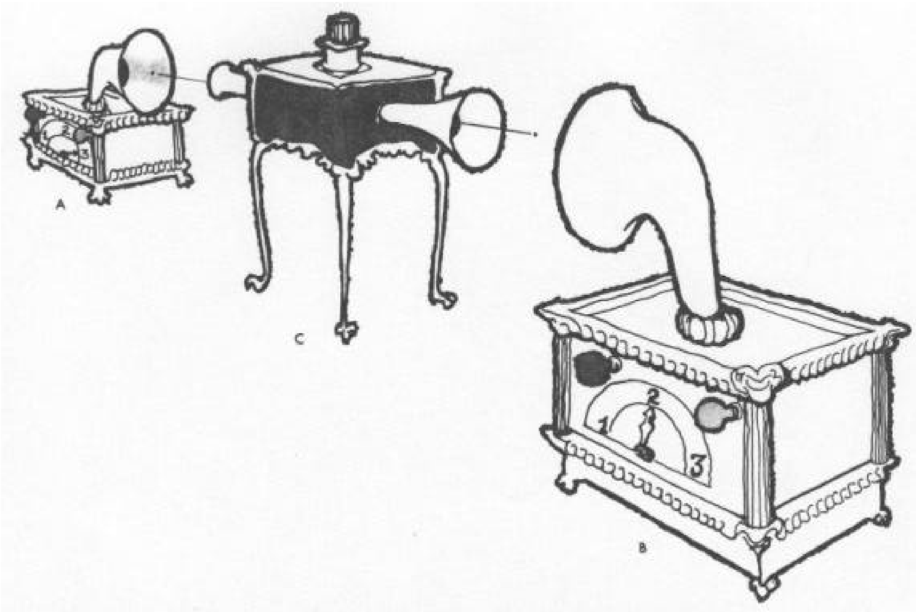
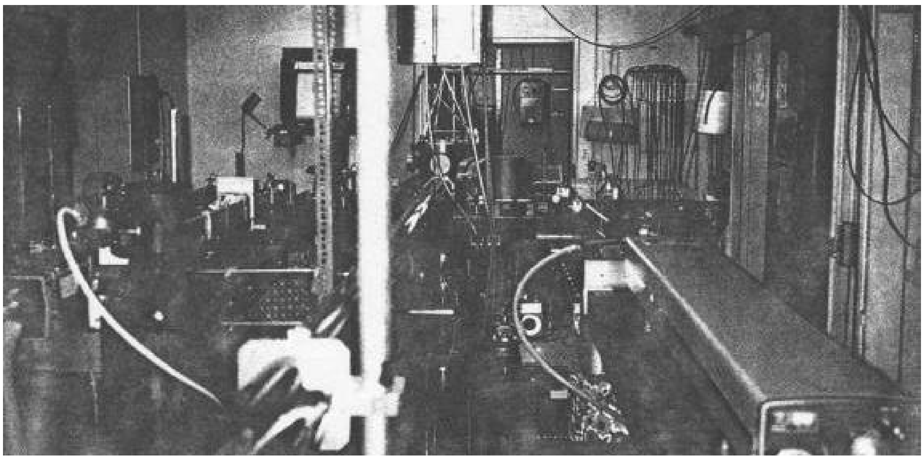


Figure 1 - An EPR apparatus.

The experimental setup consist of two detector, **A** and **B**, and a source of something ("particles" or whatever) **C**. To start a run, the experimenter pushes the button on **C**; something passes from **C** to both detectors. Shortly after the button is pushed each detector flashes one of its lights. Putting a brick between the source and one of the detectors prevents that detectors from flashing, and moving the detectors farther away from the source increases the delay between when the button is pushed and when the lights flash. The switch settings on the detectors vary randomly from one run to another. Note that there are no connections between the three parts of the apparatus, other than via whatever it is that passes from **C** to **A** and **B**.

The photo below shows a realization of such an experiment in the laboratory of Alain Aspect in Orsay, France. In the center of the lab is a vacuum chamber where individual calcium atoms are excited by the two lasers visible in the picture. The re-emitted photons travel 6 meters through the pipes to be detected by a two-channel polarizer.



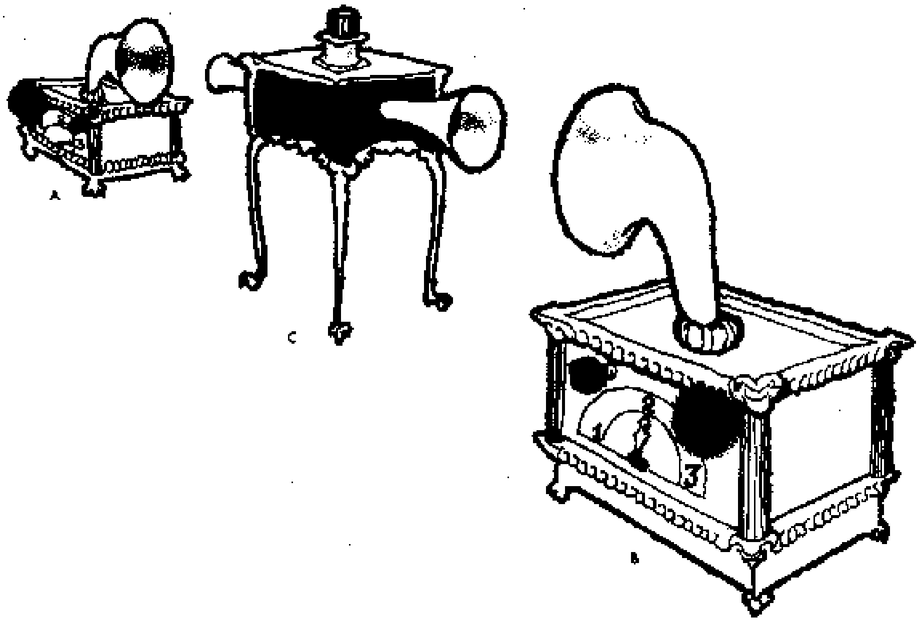


Figure 2 - The result of a run.
Shortly after the experimenter pushed the button on the source in figure 1, the detectors flash one lamp each. The experimenter records the switch settings and the colors of the lamps and then repeats the experiment. Here, for example, the record reads **32RG** –the switches are in positions 3 and 2 and the lamps flashed R and G, respectively.

31RR	12GR	23GR	13RR	33RR	12RR	22RR	32RG	13GG
22GG	23GR	33RR	13GG	31RG	31RR	33RR	32RG	32RR
31RG	33GG	11RR	12GR	33GG	21GR	21RR	22RR	31RG
33GG	11GG	23RR	32GR	12GR	12RG	11GG	31RG	21GR
12RG	13GR	22GG	12RG	33RR	31GR	21RR	13GR	23GR

Figure 3 – Data produced by the apparatus.
This is a fragment of an enormous set of data generated by many, many runs: each entry shows the switch settings and the colors of the lights that flashed for a run. The switch settings are changed randomly from run to run.

31RR	12GR	23GR	13RR	**33RR**	12RR	**22RR**	32RG	13GG
22GG	23GR	**33RR**	13GG	31RG	31RR	**33RR**	32RG	32RR
31RG	**33GG**	**11RR**	12GR	**33GG**	21GR	21RR	**22RR**	31RG
33GG	11GG	23RR	32GR	12GR	12RG	**11GG**	31RG	21GR
12RG	13GR	**22GG**	12RG	**33RR**	31GR	21RR	13GR	23GR

Figure 4 – Switches set the same.
The data of figure 3, but highlighted to pick out those runs in which both detectors had the same switch settings as they flashed. Note that in such runs the lights always flash the same colors.

31**RR**	12**GR**	23**GR**	13**RR**	33**RR**	12**RR**	22**RR**	32**RG**	13**GG**
22**GG**	23**GR**	33**RR**	13**GG**	31**RG**	31**RR**	33**RR**	32**RG**	32**RR**
31**RG**	33**GG**	11**RR**	12**GR**	33**GG**	21**GR**	21**RR**	22**RR**	31**RG**
33**GG**	11**GG**	23**RR**	32**GR**	12**GR**	12**RG**	11**GG**	31**RG**	21**GR**
12**RG**	13**GR**	22**GG**	12**RG**	33**RR**	31**GR**	21**RR**	13**GR**	23**GR**

Figure 5 – Switches set any way.
The data of figure 3, but highlighted to emphasize only the colors of the lights that flashed in each run, no matter how the switches were set when the lights flashed. Note that the pattern of colors is completely random.

Typical data from a large number of runs are shown in **figure 3**. There are just two relevant features:

I) **If one examines only those runs in which the switches have the same setting (figure 4), then one finds that the lights always flash the same colors.**

II) **If one examines all runs , without any regard to how the switches are set (figure 5), then one finds that the pattern of flashing is completely random. In particular, half the time the lights flash the same colors, and half the time different colors.**

That is all there is to the gedanken demonstration.
Should you be bothered by these data unless you have rocks in your head ?

How could it work ?

Consider only those runs in which the switches had the same setting when the particles went through the detectors. In all such runs the detectors flash the same colors. If they could communicate, it would be child's play to make the detectors flash the same colors when their switches had the same setting, but they are completely unconnected. Nor can they have been preprogrammed always to flash the same colors, regardless of what is going on, because the detectors are observed to flash different colors in at least some of those runs in which their switches are differently set, and the switch settings are independent random events.

How, then, are we to account for the first feature of the data? No problem at all. Born, in fact, in a letter of May 1948, offers[5] such an explanation to Einstein:

"It seems to me that your axiom of the 'independence of spatially separated objects A and B' is not as convincing as you make out. It does not take into account the fact of coherence; objects far apart in space which have a common origin need not be independent. I believe that this cannot be denied and simply has to be accepted. Dirac has based his whole book on this."

In our case the detectors are triggered by particles that have a common origin at the source **C**. It is then easy to dream up any number of explanations for the first feature of the data.

Suppose, for example, that what each particle encounters as it enters its detector is a target (**figure 6**) divided into eight regions, labeled RRR, RRG, RGR, RGG, GRR, GRG, GGR, and GGG. Suppose each detector is wired so that if a particle lands in the GRG bin, the detector flips into a mode in which the light flashes G if the switch is set to 1, R if it is set to 2, and G if it is set to 3; RGG leads to a mode with R for 1 and G for 2 and 3, and so on. We can then easily account for the fact that the lights always flash the same colors when the switches have the same settings by assuming that in each run the source always fires its particles into bins with the same labels.

Evidently this is not the only way. One could imagine that particles come in eight varieties: cubes, spheres, tetrahedra,… All settings produce R when a cube is detected, a sphere results in R for settings 1 and 2, G for setting 3, and so forth. The first feature of the data is then accounted for if the two particles produced by the source in each run are always both of the same variety.

Common to all such explanations is the requirement that each particle should, in one way or another, carry to its detector a set of instructions for how it is to flash for *each* of the three possible switch settings, and that in *any* run of the experiment both particles should carry the same instruction sets:

I) A set of instructions that covers *each* of the three possible settings is required because there is no communication between the source and the detectors other than the particles themselves. In runs in which the switches have the same setting, the particles cannot know whether that setting will be 11, 22, or 33. For the detectors always to flash the same colors when the switches have the same setting, the particles must carry instructions that specify colors for each of the three possibilities.

II) The absence of communication between source and detectors also requires that the particles carry such instruction sets in *every* run of the experiment –even those in which the switches end up with different settings- because the particles always have to be prepared: any run may turn out to be one in which the switches end up with the same settings.

This generic explanation is pictured schematically in **figure 7**.

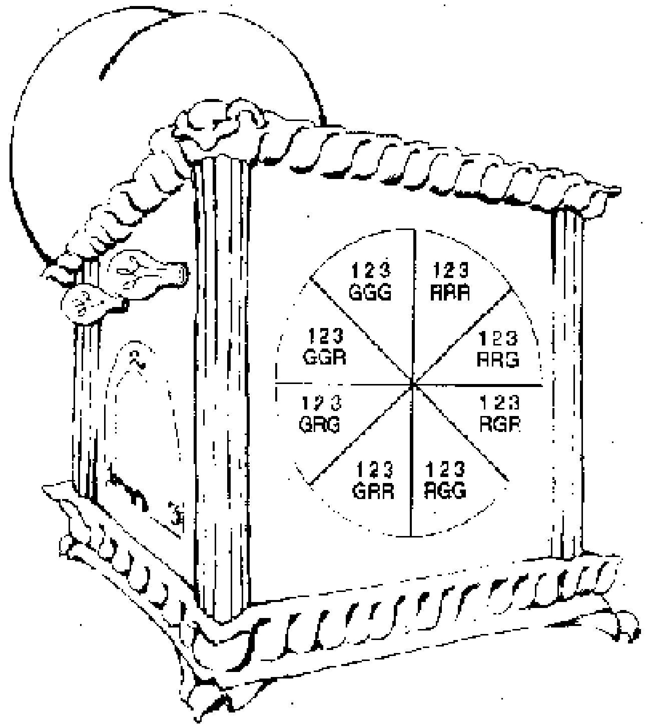


Figure 6 – Model of a detector to produce data like those in figure 4.
Particles from the source fall with equal probability into any of the eight bins; for each bin the color flashed depends on the switch as indicated on the back of the box.

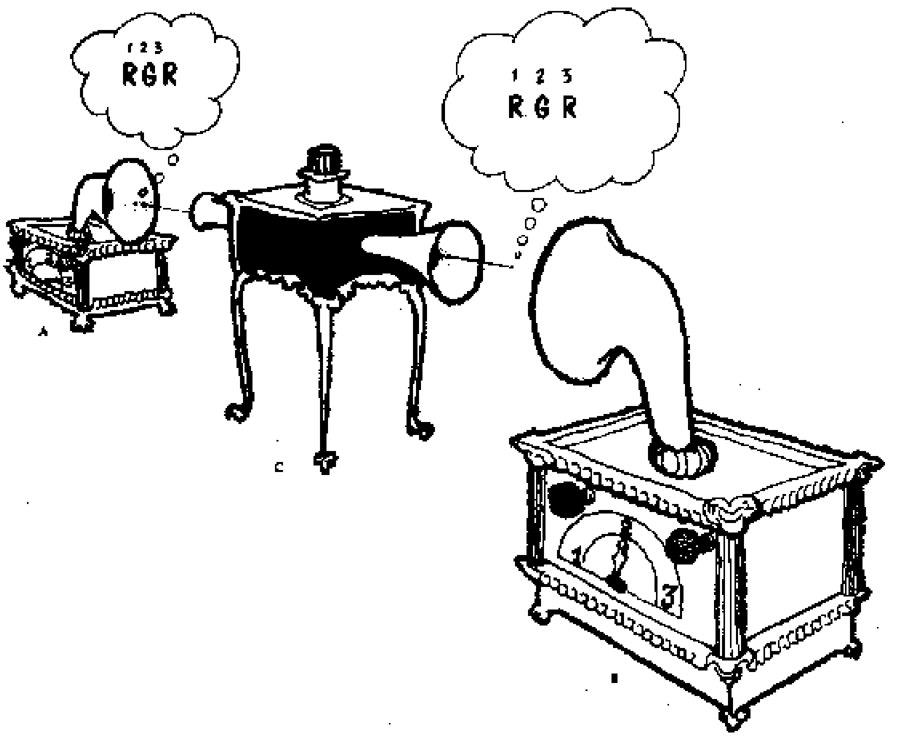


Figure 7 – Instruction sets.
To guarantee that the detectors of figure 6 flash the same color when the switches are set the same, the two particles must in one way or another carry instruction sets specifyng how their detectors are to flash for each possible switch setting. The results of any one run reveal nothing about the instructions beyond the actual data; so in this case, for example, the first instruction (1R) is “something one cannot know anything about”, and I’ve only guessed at it, assuming that “it exists all the same”.

Alas, this explanation –the only one, I maintain, that someone not steeped in quantum mechanics will ever be able to come up with (though it is an entertaining game to challenge people to try)- is untenable.

It is inconsistent with the second feature of the data: **There is no conceivable way to assign such instruction sets to the particles from one run to the next that can account for the fact that in all runs taken together, without regard to how the switches are set, the same colors flash half the time.**

Pause to note that we are about to show that "something one cannot know anything about" –the third entry in an instruction set- cannot exist. For even if instruction sets did exist, one could never learn more than two of the three entries (revealed in those runs where the switches ended up with two different settings). Here is the argument.

Consider a particular instruction set, for example, RRG. Should both particles be issued the instruction set RRG, then the detectors will flash the same colors when the switches are set to 11, 22, 33, 12, or 21; they will flash different colors for 13, 31, 23, or 32.

Because the switches at each detector are set randomly and independently, each of these nine cases is equally likely, so the instruction set RRG will result in the same colors flashing ***5/9*** of the time.

Evidently the same conclusion holds for the sets RGR, GRR, GGR, GRG and RGG, because the argument uses only the fact that one color appears twice and the other once. All six such instructions sets also result in the same colors flashing ***5/9*** of the time.

But the only instruction sets left are RRR and GGG, and these each result in the same colors flashing *all* of the time.

Therefore if instructions sets exist, the same colors will flash in at least ***5/9*** of all the runs, regardless of how the instruction sets are distributed from one run of the demonstration to the next.
This is Bell's theorem (also known as Bell's inequality) for the gedanken demonstration.
But in the actual gedanken demonstration the same colors flash only ½ the time.
The data described above violate this Bell's inequality, and therefore there can be no instruction sets.

If you don't already know how the trick is done, may I urge you, before reading how the gedanken demonstration works, to try to invent some other explanation for the first feature of the data that does not introduce connections between the three parts of the apparatus or prove to be incompatible with the second feature.

One way to do it

Here is one way to make such a device:
Let the source produce two particles of spin ½ in the singlet state, flying apart toward the two detectors. (Granted, this is not all that easy to do, but in the Orsay experiments described below, the same effect is achieved with correlated photons).

Each detector contains a Stern-Gerlach magnet, oriented along one of three directions ($a^{(1)}$, $a^{(2)}$, or $a^{(3)}$), perpendicular to the line of flight of the particles, and separated by 120°, as indicated in **figure 8**.

The three settings of the switch determine which orientation is used. The light on one detector flashes red or green, depending on whether the particle is deflected toward the north (spin up) or south (spin down) pole of the magnet as it passes between them; the other detector uses the opposite color convention.

That's it. Clearly there are no connections between the source and the detectors or between the two detectors. We can nevertheless account for the data as follows:

When the switches have the same setting, the spins of both particles are measured along the same direction, so the lights will always flash the same colors if the measurements along the same direction always yield opposite values. But this is an immediate consequence of the structure of the spin singlet state, which has the form:

$$|\psi\rangle = (1/\sqrt{2})\ [\ |+ -\rangle - |- +\rangle\ ] \qquad \textbf{(1)}$$

independent of the direction of the spin quantization axis, and therefore yields +– or –+ with equal probability, but never ++ or –– ,whenever the two spins are measured along any common direction.

To establish the second feature of the data, note that the product $m_1 m_2$ of the results of the two spin measurements (each of which can have the values +½ or –½) will have the value –¼ when the lights flash the same colors and +¼ when they flash different colors. We must therefore show that the product vanishes when averaged over all the nine distinct pairs of orientations the two Stern-Gerlach magnets can have.

For a given pair of orientation, $a^{(i)}$ and $a^{(j)}$, the mean value of this product is just the expectation value in the state ψ of the corresponding product of (commuting) hermitian observables $a^{(i)} \cdot S^{(1)}$ and $a^{(j)} \cdot S^{(2)}$.

Thus the second feature of the data requires:

$$0 = \sum_{ij} \langle\psi| [a^{(i)} \cdot S^{(1)}][a^{(j)} \cdot S^{(2)}] |\psi\rangle \qquad \textbf{(2)}$$

But equation 2 is an immediate consequence of the linearity of quantum mechanics, which lets one take the sums inside the matrix element, and the fact that the three unit vectors around an equilateral triangle sum to zero:

$$\sum_i a^{(i)} = \sum_j a^{(j)} = 0 \qquad \textbf{(3)}$$

This completely accounts for the data. It also unmasks the gedanken demonstration as a simple embellishment of Bohm's version of the EPR experiment. If we kept only runs in which the switches had the same setting, we would have precisely the Bohm-EPR experiment. The assertion that instruction sets exist is then blatant quantum-theoretic nonsense, for it amounts to the insistence that each particle has stamped on it in advance the outcome of the measurements of three different spin components corresponding to noncommuting observables $S \cdot a^{(i)}$, i=1,2,3. According to EPR, this is merely a limitation of the quantum-theoretic formalism, because instruction sets are the only way to account for the first feature of the data.

Bell's analysis adds to the discussion those runs in which the switches have different settings, extracts the second feature of the data as a further elementary prediction of quantum mechanics, and demonstrates that any set of data exhibiting this feature is incompatible with the existence of the instruction sets apparently required by the first feature, quite independently of the formalism used to explain the data, and quite independently of any doctrines of quantum theology.

The experiments

The experiments of Aspect and his colleagues at Orsay confirm that the quantum-theoretic predictions for this experiment are in fact realized, and that the conditions for observing the results of the experiment can in fact be achieved. (A distinguished colleague once told me that the answer to the EPR paradox was that correlations in the singlet state could never be maintained over macroscopic distances –that anything, even the passage of a cosmic ray in the next room, would disrupt the correlations enough to destroy the effect).

In these experiments the two spin ½ particles are replaced by a pair of photons and the spin measurements become polarization measurements.

The photon pairs are emitted by calcium atoms in a radiative cascade after suitable pumping by lasers. Because the initial and final atomic states have J=0, quantum theory predicts (and experiment confirms) that the photons will be found to have the same polarizations (lights flashing the same colors in the analogous gedanken experiment) if they are measured along the same direction –feature number 1.

But if the polarizations are measured at 120° angles, then theory predicts (and experiment confirms) that they will be the same only a quarter of the time [$\frac{1}{4} = \cos^2(120°)$].

This is precisely what is needed to produce the statistics of feature number 2 of the gedanken demonstration: the randomly set switches end up with the same setting (same polarizations measured) 1/3 of the time, so in all runs the same colors will flash 1/3 x 1 + 2/3 x (1/4) = ½ the time.

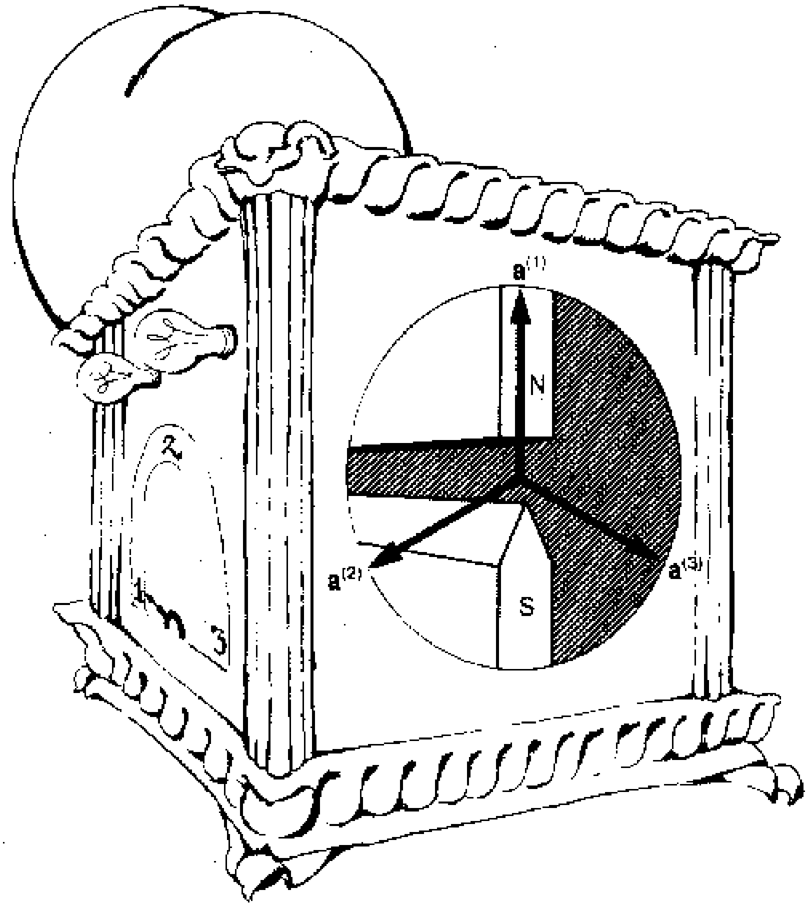


Figure 8 – A realization of the detector to produce the data of figure 3.
The particles have a magnetic moment and can be separated into "spin up" and "spin down" particles by the Stern-Gerlach magnet inside the detector. Setting the switch to positions 1,2, or 3 rotates the north pole of the magnet along the coplanar unit vectors $a^{(1)}$, $a^{(2)}$, or $a^{(3)}$, separated by 120°. The vector sum of the three unit vectors is, of course, zero. The switch positions on the two detectors correspond to the same orientations of the magnetic field. One detector flashes red for spin up, green for spin down; the other uses the opposite color convention.

The people in Orsay were interested in a somewhat modified version of Bell's argument in which the angles of greatest interest were multiples of 22.5°, but they collected data for many different angles, and, except for EPR specialists, the conceptual differences between the two cases are minor.[16]

There are some remarkable features to these experiments. The two polarization analyzers were placed as far as 13 meters apart without producing any noticeable change in the results, thereby closing the loophole that the strange quantum correlations might somehow diminish as the distance between regions **A** and **B** grew to macroscopic proportions. At such separations it is hard to imagine that a polarization measurement of photon #1 could, in any ordinary sense of the term, "disturb" photon #2.

Indeed, at these large separations, a hypothetical disturbance originating when one photon passed through its analyzer could only reach the other analyzer in time to affect the outcome of the second polarization measurement if it traveled at a superluminal velocity.

In the third paper of the Orsay group's series, bizarre conspiracy theories are dealt a blow by an ingenious mechanism for rapidly switching the directions along which the polarizations of each photon are measured.

Each photon passes to its detector through a volume of water that supports an ultrasonic standing wave. Depending on the instantaneous amplitude of the wave, the photon either passes directly into a polarizer with one orientation or is Bragg reflected into another with a different orientation.

The standing waves that determine the choice of orientation at each detector are independently driven and have frequencies so high that several cycles take place during the light travel time from one detector to the other. (This corresponds to a refinement of the gedanken demonstration in which, to be absolutely safe, the switches are not given their random settings until *after* the particles have departed from their common source).

What does it mean ?

What is one to make of all this? Are there "spooky actions at a distance" ?
A few years ago I received the text of a letter from the executive director of a California think-tank to the Under-Secretary of Defense for Research and Engineering, alerting him to the EPR correlations:

"If in fact we can control the faster-than-light nonlocal effect, it would be possible…to make an untappable and unjammable command-control-communication system at very high bit rates for use in the submarine fleet. The important point is that since there is no ordinary electromagnetic signal linking the encoder with the decoder in such a hypothetical system, there is nothing for the enemy to tap or jam. The enemy would have to have actual possession of the "black box" decoder to intercept the message, whose reliability would not depend on separation from the encoder nor on ocean or weather conditions…."

Heady stuff indeed! But just what is this nonlocal effect? Using the language of the gedanken demonstration, let us talk about the "N-color" of a particle (N can be 1, 2, or 3) as the color (red or green) of the light that flashes when the particle passes through a detector with its switch set to N.

Because instruction set cannot exist, we know that a particle cannot at the same time carry a definite 1-color, 2-color and 3-color to its detector. On the other hand, for any particular N (say 3), we can determine the 3-color of the particle heading for detector **A** before it gets there by arranging things so that the other particle first reaches detector **B**, where its 3-color is measured.

If the particle at **B** was 3-colored red, the particle at **A** will turn out to be 3-colored red, and green at **B** means green at **A**.

Three questions now arise:

I) Did the particle at **A** have its 3-color prior to the measurement of the 3-color of the particle at **B**? The answer cannot be yes, because, *prior* to the measurement of the 3-color at **B**, it is altogether possible that the roll of the dice at **B** or the whim of the **B**-operator will result in the 2-color or the 1-color being measured at **B** instead. Barring the most paranoid of conspiracy theories, "prior to the measurement of the 3-color at **B**" is indistinguishable from "prior to the measurement of the 2- (or 1-) color at **B**". If the 3-color already existed, so also must the 2- and 1-colors have existed. But instruction sets (which consist of a specification of the 1-, 2-, and 3-colors) do not exist.

II) Is the particle at **A** 3-colored red *after* the measurement at **B** shows the color red? The answer is surely yes, because under these circumstances it is invariably a particle that will cause the detector at **A** to flash red.

III) Was something (the value of its 3-color) transmitted to the particle at **A** as a result of the measurement at **B**?

Orthodox quantum metaphysicians would, I believe, say no, nothing has changed at **A** as the result of the measurement at **B**; what has changed is our knowledge of the particle at **A**. (Somewhat more spookily, they might object to the naive classical assumption of localizability or separability implicit in the phrases "at **A**" and "at **B**").

This seems very sensible and very reassuring: N-color does not characterize the particle at all, but only what we know about the particle. But does that last sentence sound as good when "particle" is changed to "photon" and "N-color" to "polarization"? And does it really help you to stop wondering why the lights always flash the same colors when the switches have the same settings?

What is clear is that if there is spooky action at a distance, then, like other spooks, it is absolutely useless except for its effect, begnin or otherwise, on our state of mind.

For the statistical pattern of red and green flashes at detector **A** is entirely random, however the switch is set at detector **B**. Whether the particles arriving at **A** all come with definite 3-colors (because the switch at B was stuck at 3) or definite 2-colors (because the switch was stuck at 2) or no colors at all (because there was a brick in front of the detector at **B**) –all this has absolutely no effect on the statistical distribution of colors observed at **A**. The manifestation of this "action at a distance" is revealed only through a comparison of the data independently gathered at **A** and at **B**.

This is a most curious state of affairs, and while it is wrong to suggest that EPR correlations will replace sonar, it seems to me something is lost by ignoring them or shrugging them off.

The EPR experiment is as close to magic as any physical phenomenon I know of, and magic should be enjoyed. Whether there is physics to be learned by pondering it is less clear. The most elegant answer I have found[17] to this last question comes from one of the great philosophers of our time, whose view of the matter I have taken the liberty of quoting in the form of the poetry it surely is:

We have always had a great deal of difficulty
understanding the world view
that quantum mechanics represents.

At least I do,
because I'm an old enough man
that I havent' got to the point
that this stuff is obvious to me.

Okay, I still get nervous with it….

You know how it always is,
every new idea,
it takes a generation or two
until it becomes obvious
that there's no real problem.

I cannot define the real problem,

therefore I suspect there's no real problem,
but I'm not sure
there's no real problem.

Nobody in the 50 years since Einstein, Podolsky and Rosen has ever put it better than that.

[Some of the views expressed above were developed in the course of occasional technical studies of EPR correlations supported by the National Science Foundation under grant No. DMR 83-14625.]

References

1. Daniel Greenberger, discussion remarks at the Symposium on Fundamental Questions in Quantum Mechanics, SUNY, Albany, April 1984.
2. A.Einstein, B.Podolsky, N.Rosen, Phys. Rev. **47**, 777 (1935).
3. Quoted by M. Jammer, *The Philosophy of Quantum Mechanics*, Wiley, New York (1974) p.151.
4. A.Pais, Rev. Mod. Phys. **51**, 863 (1979).
5. *The Born-Einstein Letters*, with comments by M.Born, Walker, New York (1971).
6. J.S.Bell, Physics **1**, 195 (1964).
7. D.Bohm, *Quantum Theory*, Prentice-Hall, Englewood Cliffs, N.J. (1951) pp. 614-619.
8. A.Aspect, P.Grangier, G.Roger, Phys. Rev. Lett. **47**, 460 (1981). A.Aspect, P.Grangier, G.Roger, Phys. Rev. Lett. **49**, 91 (1982). A.Aspect, J.Dalibard, G.Roger, Phys. Rev. Lett. **49**, 1804 (1982).
9. For a discussion of the views of today's physicists toward the meaning of the quantum theory, see the interesting and provocative essay "Cognitive Repression in Contemporary Physics" by E.F.Keller, Am. J. Phiys. **47**, 718 (1977).
10. L.Rosenfeld in Niels Bohr, *His Life and Work as Seen by His Friends and Colleagues*, S.Rozental ed., North Holland, Amsterdam (1967) pp. 114-36.

11. G.Zukav, *The Dancing Wu-Li Masters –An Overview of the New Physics*, Morrow, New York (1979) p.282. On the same page it is also said that "Bell's theorem is a mathematical construct which as such is indecipherable to the non-mathematician", a view that I hope the rest of this article will dispel.
12. H.Stapp, Nuovo Cimento **40B**, 191 (1977).
13. A.Pais, *"Subtle is the Lord…" The Science and the Life of Albert Einstein,* Oxford U.P., New York (1982) p.456.
14. N.Bohr, Phys. Rev. **48**, 696 (1935).
15. What follows is a somewhat refined version of an argument I published a few years ago in Am. J. Phys. **49**, 940 (1981), incorporating some improvements suggested by Richard Friedberg. For other elementary treatments see J.S.Bell's beautiful essay, *"Bertlemann's Socks and the Nature of Reality"*, J. Phys. (Paris) **42**, C2-41 (1981), B.d'Espagnat's article in the November 1979 Scientific American, or d'Espagnat's recent book, *In Search of Reality*, Springer-Verlag, New York (1983).
16. For a survey of other attempts to realize the EPR experiment, and the variants of Bell's original argument used to interpret experimental tests, see J.F.Clauser, A.Shimony, Repts. Prog. Phys. **41**, 1881 (1978).
17. R.P.Feynman, Int. J. Theor. Phys. **21**, 471 (1982).