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On the Concept of Spontaneously Broken Gauge Symmetry in Condensed Matter Physics

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We discuss the concept of spontaneous breaking of gauge symmetry in superconductors and superfluids and, in particular, the circumstances under which the absolute phase of a superfluid can be physically meaningful and experimentally relevant. We argue that the study of this question pushes us toward the frontiers of what we understand about the quantum measurement process, and underline the need for a new theoretical framework that keeps pace with modern technological capabilities.

1. INTRODUCTION

The concept of spontaneously broken gauge symmetry (hereafter SBGS) is nowadays generally believed to be the key to understanding the phenomena of superconductivity and superfluidity. In this note we will raise and discuss some of the conceptual problems associated with this idea. For reasons of space we will consider explicitly only the case of a Bose superfluid and restrict ourselves to questions connected with the breaking of the *global* $U(1)$ symmetry (i.e., we will neglect the local gauge-invariance aspects which give the phenomenon its name). We do not attempt to comment on analogous questions which might arise in a particle-physics context.

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2. THE IDEA OF SBGS

We review very briefly the basic concept of SBGS, assuming that the reader already has some familiarity with this subject.[1] We consider a neutral Bose system described by field operators $\psi(\mathbf{r})$, $\psi^\dagger(\mathbf{r})$ which satisfy the standard Bose commutation relations

$$[\psi(\mathbf{r}), \psi^\dagger(\mathbf{r}')] = \delta(\mathbf{r}-\mathbf{r}'), \qquad [\psi(\mathbf{r}), \psi(\mathbf{r}')] = [\psi^\dagger(\mathbf{r}), \psi^\dagger(\mathbf{r}')] = 0 \tag{1}$$

The Hamiltonian is assumed to contain only (a) one- and two-particle potential energy terms which are functions only of the local particle density $N(\mathbf{r}) \equiv \psi^\dagger(\mathbf{r})\,\psi(\mathbf{r})$ and (b) kinetic-energy terms containing the product $\nabla\psi^\dagger(\mathbf{r})\cdot\nabla\psi(\mathbf{r})$. Clearly, any such Hamiltonian will be invariant under the *global* $U(1)$ transformation

$$\psi(\mathbf{r}) \to \psi(\mathbf{r}) \exp i\chi \tag{2}$$

where χ is a fixed real number independent of $\mathbf{r}$ and t. As a result, there exists a corresponding "Noether charge" which is conserved, namely the total particle number

$$\hat{N} \equiv \int \psi^\dagger(\mathbf{r})\,\psi(\mathbf{r})\,d\mathbf{r} \equiv \int N(\mathbf{r})\,d\mathbf{r} \tag{3}$$

We note for future reference that if we define an overall "phase" operator by the formal prescription

$$\int \psi(\mathbf{r})\,d\mathbf{r} = \hat{A} \exp i\hat{\phi} \qquad (\hat{A}, \hat{\phi} \text{ Hermitian operators}) \tag{4}$$

then we have the "number-phase commutation relation"

$$[\hat{N}, \hat{\phi}] = -i \tag{5}$$

(A similar *local* commutation relation can be derived,[2] but is of no importance for our present discussion.)

The idea of SBGS may be understood by analogy with the behavior of an isotropic Heisenberg ferromagnet. In this latter case, the $O(3)$ symmetry of the Hamiltonian is "spontaneously broken" below the Curie temperature, in that the magnetization picks out a given direction and the thermodynamic equilibrium state is no longer invariant under $O(3)$. Similarly, in the equilibrium Bose superfluid below the λ-transition the $U(1)$ symmetry (2) is spontaneously broken: the system chooses a state in which the expectation value of $\psi(\mathbf{r})$ is no longer zero (as it is in the normal state) but rather tends in the thermodynamic limit to a finite value:

$$\langle \psi(\mathbf{r}) \rangle = \text{const} \neq 0 \tag{6}$$

More generally, one assumes that in the physically relevant (not necessarily equilibrium) states of the liquid which correspond to superfluid behavior one can define an "order parameter" $\Psi(\mathbf{r}, t)$ by the prescription

$$\Psi(\mathbf{r}, t) \equiv \langle \psi(\mathbf{r} : t) \rangle \neq 0 \tag{7}$$

where $\langle \psi(\mathbf{r} : t) \rangle$ means the expectation value of the operator $\psi(\mathbf{r})$ at time t. We note that the order parameter is a *complex* object and therefore has (prima facie, at least—but see below) a definite *absolute* phase as well as a definite phase variation in space and time [just as the direction of magnetization at a given point in a ferromagnet has (prima facie!) an absolute significance]; it is this feature on which we will particularly focus in what follows. Once given the concept of an order parameter, one can proceed to associate appropriate energies with its space and time variations and to derive the phenomena of superfluidity or superconductivity in the standard way: see, e.g., Ref. 3. In the following we shall, however, usually not be concerned with continuous space variations of $\Psi(\mathbf{r}, t)$ and will therefore take it to be constant within any given "bulk" region.

3. HOW SERIOUSLY SHOULD WE TAKE IT?

First, a rather trivial but apparently not universally appreciated point: it is perfectly possible to define the concept of an order parameter $\Psi(\mathbf{r}, t)$ without invoking the idea of SBGS. To do this, we *assume* that the one-particle density matrix has a single eigenvalue $N_0 \sim N$ while all other eigenvalues are $O(1)$. We then diagonalize this density matrix for the given time t and call the one-particle state corresponding to the large eigenvalue [which for a general (nonequilibrium) situation is not necessarily the zero-momentum state or even an eigenstate of any particular Hamiltonian] $\phi_0(\mathbf{r} : t)$. Then we simply define

$$\Psi(\mathbf{r}, t) \equiv \sqrt{N_0}\, \phi_0(\mathbf{r}, t) \tag{8}$$

For all *practical* purposes the quantity defined by (8) has the same properties as that defined by (7). However, the definition (8) has, arguably, two advantages: First, while $\Psi(\mathbf{r}, t)$ still has an "absolute" phase, it is immediately and explicitly clear that this phase has no physical significance, since it simply corresponds to a particular convention for the choice of the hases of the single-particle wave functions, which is completely arbitrary. Second, the definition (8) focuses attention on the assumption that one *and only one* single-particle state is macroscopically occupied.

To see the significance of this last point, let us consider an intriguing problem which was raised and discussed a few years ago by Siggia and Ruckenstein (SR).[4] In the last decade or so it has become possible to prepare atomic hydrogen in a strongly spin-polarized state, so that the electronic spin degree of freedom is effectively frozen out and the system effectively behaves like a collection of bosons of spin 1/2 (corresponding to the *nuclear* spin degree of freedom, which is not frozen out at the relevant temperatures) (see, e.g., Ref. 5). Suppose now we could prepare the system at some temperature T such that $T_\lambda < T$, $\mu_n B \ll T$, (where T_λ = Bose condensation temperature, μ_n = nuclear moment, B = magnetic field), so that both $\uparrow$ and $\downarrow$ nuclear spin states have an appreciable (and nearly equal) population. We then imagine "quenching" the system through T_λ, the cooling power being provided by collisions (e.g., with the cell walls) *which conserve nuclear spin.* Presumably, Bose condensation then occurs in both the nuclear spin populations. What is the correct description of the resulting state of the system?

SR's approach to this problem is based on the concept of SBGS and runs crudely as follows: If Bose condensation occurs in both the nuclear spin populations, there must be an order parameter $\Psi_\uparrow \equiv \langle \psi_\uparrow \rangle$ for the $\uparrow$ spins and another one $\Psi_\downarrow \equiv \langle \psi_\downarrow \rangle$ for the $\downarrow$ spins (as usual we neglect for present purposes any spatial variation). But since $\Psi_\uparrow$ and $\Psi_\downarrow$ are complex numbers, their relative phase must be well defined, that is

$$\langle \psi_\uparrow \rangle = \exp(i\,\Delta\phi)\langle \psi_\downarrow \rangle \tag{9}$$

But this means that it is possible to find a direction in the xy-plane such that when $\uparrow$ and $\downarrow$ are defined with respect to this direction we have $\langle \psi_\downarrow \rangle = 0$, $\langle \psi_\uparrow \rangle \neq 0$: i.e., the magnetization of the sample spontaneously develops a nonzero component in the xy-plane!

We want to stress that this apparently bizarre conclusion may well be physically correct, once various small symmetry-breaking terms in the Hamiltonian are taken into account (cf. Ref. 4). However, the point we want to emphasize in the present context is that this conclusion does *not* follow simply from the observation that Bose condensation occurs in both the $\downarrow$ and $\uparrow$ bands. Essentially, SR's ansatz for the final state Φ of the many-body system (taken for simplicity to be at $T = 0$) is schematically

$$\Phi = \text{const}(a\phi_\uparrow + b\phi_\downarrow)^N \tag{10}$$

whereas the mere fact of Bose condensation in both bands would be equally compatible with (for example) the state

$$\Phi = \text{const}\ \phi_\uparrow^{N/2} \phi_\downarrow^{N/2} \tag{11}$$

which clearly corresponds to zero transverse magnetization. Whether the true ground state is actually more like (10) or (11) (or something different from either) would seem to depend on fairly delicate considerations which are qualitatively similar to those we shall encounter in the next section and Sec. 4 in the context of Josephson junctions, and which may be specific to the particular system considered.

We now turn to the principal difficulty in taking the idea of SBGS literally for an isolated system: If we do so, then the absolute phase ϕ defined by Eq. (4) has a well-defined expectation value, and it therefore immediately follows from (5) that *the total particle number cannot be well defined.* In fact, it is easy to show from (5) that the existence of a well-defined absolute phase for an isolated superfluid system requires the wave function in the particle-number representation to be of the form

$$\Phi = \sum_N a_N \psi_N, \qquad a_N = |a_N| e^{iN\phi} \tag{12}$$

i.e., a *coherent superposition* (not a mixture!) of states corresponding to different total particle number (or baryon number). The rest of this paper will be devoted to the question of whether, and how, we can make sense of this apparently exotic idea. Of course, as is well known,[6] it is always *possible* to form a state of definite particle number by the prescription

$$|N\rangle = \int e^{iN\phi} |\phi\rangle \, d\phi \tag{13}$$

where $|\phi\rangle$ is an eigenstate of the phase. The question is (a) whether it is *necessary* to do this, and (b) whether it may sometimes be misleading to do it.

It should be emphasized that the answer to the question we have just raised—in effect, "does an isolated superfluid possess a phase?"—can have, in principle, experimental consequences. This may be illustrated by an intriguing thought experiment proposed by P. W. Anderson[7]: Imagine that we collect and condense two sets of helium atoms at opposite ends of the Earth. We then bring the two buckets of helium so formed close together and at some moment connect them ("instantaneously": this is a gedanken experiment, not a practical one!) through a Josephson superleak. We know from the general theory of Josephson junctions[8] (see next section) that *if* there is a definite relative phase $\Delta\phi$ between the condensate in the two buckets, a Josephson current proportional to $\sin\Delta\phi$ will flow between them. Now, which of the following two statements, if either, about the behavior of the system immediately following the connection is correct?

(a) No Josephson current flows on any trial.

(b) In general, on each particular trial, a Josephson current flows, but the corresponding relative phase is random and unpredictable.

Anderson favors conclusion (b), and certainly if one believed the hypothesis of SBSG interpreted as above [i.e., as implying Eq. (12)] one would be inexorably forced to this conclusion.[4]

4. JOSEPHSON COUPLING AND DECOUPLING

To examine the question raised at the end of the last section, and related questions, it is convenient to start by considering the *equilibrium* state of two similar superfluid systems coupled by a weak link (Josephson junction). If the systems are sufficiently large, we can neglect to a first approximation any "capacitative" energy between them (i.e., any energy which depends on the relative number of particles on the two sides of the link, cf. below) and write the ground-state many-body wave function schematically in the form

$$\Phi \sim (a\psi_L + b\psi_R)^N \tag{14}$$

where $\psi_L(\psi_R)$ is the Schrödinger amplitude for a particle to be on the left (right) of the junction. Here we will not worry about the absolute phase (for which there is no possible reference standard, cf. above), but rather concentrate on the relative phase of two superfluids that are eventually decoupled; this *relative* phase $[\arg(a/b)]$ will be denoted ϕ. Thus (14) can be conveniently rewritten

$$\Phi \sim (|a|\, e^{i\phi/2}\psi_L + |b|\, e^{-i\phi/2}\psi_R)^N \tag{15}$$

If, now, we define an operator $\Delta\hat{N} \equiv \sum_i \Delta N_i$ such that ψ_R, ψ_L are, respectively, eigenstates of ΔN_i with eigenvalues $+1$, -1, then it is easy to show (cf. Ref. 9) by explicit calculation that the matrix elements of the operator $\Delta\hat{N}$ between states of the form (15) are identical to those of the

[4] Anderson's own reasons for favoring (b) are somewhat more subtle and seem to involve, implicitly, the considerations mentioned in Sec. 5 below.

operator $-i\partial/\partial\phi$, i.e., within the relevant manifold ΔN and ϕ are conjugate variables:

$$[\Delta N, \phi] = -i \tag{16}$$

In general, the Josephson coupling energy depends on ϕ, in the simplest cases as $-\cos\phi$. Thus, if there are no competing energies, the ground state is of the form (15) with $\phi = 0$. However, in general there will be a term in the Hamiltonian which depends on the conjugate variable ΔN; in the case of a realistic (charged) superconductor the main contributor to this term is the capacitance energy which is associated with any finite volume, while for the truly neutral superfluid the compressibility or gravitational energy gives rise to a term of similar form (but much smaller). Thus the general form of the relevant terms is

$$H_{\text{eff}}(\phi, \Delta N) = -E_J \cos\phi + \tfrac{1}{2}\kappa^{-1}(\Delta N)^2 \tag{17}$$

where κ is some coefficient related to the capacitance, compressibility, or gravitational effects (e.g., for a superconductor it is $C/4e^2$, the factor of 4 coming from the fact that the "bosons" are Cooper *pairs*). While in recent years ultrasmall-capacitance junctions have been fabricated which may have $\kappa^{-1} \gtrsim E_J$, the usual situation is that $\kappa E_J \gg 1$; under this condition the ground state is essentially that of a simple harmonic oscillator, with large fluctuations in N and ϕ very well defined:

$$\overline{\phi^2} \sim \overline{((\Delta N)^2)}^{-1} \sim (\kappa E_J)^{-1} \ll 1 \tag{18}$$

Suppose now that equilibrium has been established and we "suddenly" decouple the two bulk superfluids, e.g., by cutting the Josephson weak link with a high-power laser. The coefficient of $\cos\phi$ is now effectively zero (or exponentially small compared to its previous value). Does the quantity ϕ stay well defined? There are two rather different physical effects which we need to consider. First, just as in a harmonic oscillator in which the spring constant is suddenly set to zero, the "kinetic-energy" term (here the term in ΔN^2) will tend to broaden the wave packet in coordinate space.[7] It is straightforward to estimate this effect[10]; we find that in the limit $\kappa E_J \gg 1$ the time taken for the spread in ϕ to become of order 2π (indicating total lack of definition of the phase) is given by the estimate

$$\tau \sim 2\pi\sqrt{2}\,\hbar(\kappa^3/E_J)^{1/4} \tag{19}$$

For a pair of superconductors with $E_J \sim 5$ eV, $C \sim 1$ nF (a fairly large value in practice) this gives $\tau \sim 25$ ns. For a pair of buckets of superfluid helium with free surfaces of area 1 cm^2 and a Josephson coupling energy of the order of 1 K (cf. Ref. 11), we find that dephasing due to gravitational forces takes a much larger time, ~ 100 years, while for the corresponding closed geometry compressibility forces would give a time about three orders of magnitude smaller[5] (about one month).

The second effect which we have to consider is the dephasing effect of a random environment. While a known, c-number force which couples linearly to ΔN would give only a fixed and calculable precession of ϕ, even linear coupling to a quantum (and hence intrinsically random) environment will lead to some degree of dephasing. Of course, this effect is also present in the coupled situation, but this may be effectively opposed by the Josephson coupling energy. [It should be noted, incidentally, that in practice the very act of severing contact (e.g., by physically cutting a Josephson junction) may itself cut off the most effective dissipation mechanismus such as the tunneling of normal quasiparticles.[(12)]]

At the time of writing we have not carried out a concrete calculation of this effect for the case of physical interest, where the isolated system is described by Eq. (17) (although it should not be difficult to do so, and indeed the results may be implicit in some of the existing literature on quantum dissipation). However, we have carried out a detailed calculation[(10)] for a closely related problem, namely that of a "spin–boson" system for which the tunneling matrix element Δ is suddenly switched off at $t=0$, and the result is instructive. We recall that with the "standard" notation (see, e.g., Ref. 13) the effects of the environment are entirely described by a spectral function $J(\omega)$, which for low ω can be assumed to go like ω^s; the *coupled* equilibrium system shows a finite degree of coherence at $T=0$ (that is, a finite expectation value of the operator σ_x in the notation of Ref. 13) if and only if $J(\omega)$ either tends to zero with ω faster than ω itself ($s>1$, "superohmic" case) or is equal to $\alpha\omega$ ($s=1$, "ohmic" case) with the dimensionless parameter α less than 1 (see, e.g., Ref. 14). Our calculation assumes that for $t<0$ the equilibrium state is described approximately by the variational wave function of Ref. 15 (which conforms to the above statements), and studies the evolution following decoupling ($\Delta \to 0$) at $t=0$. At zero temperature, we find that for the *ohmic* case with $\alpha<1$ the quanity $\langle \sigma_x \rangle$ decays to zero algebraically with time [as $(\Delta_r t)^{-2\alpha}$, where Δ_r is the renormalized tunneling matrix element[(13)]], whereas in the "superohmic" case $\langle \sigma_x \rangle$ tends to a finite value for $t \to \infty$, i.e., the coherence is never totally destroyed by the environment. These asymptotic laws are modified at nonzero temperatures if $t \gg \hbar/kT$ (the former laws are still valid if $\omega_c^{-1} \ll t \ll \hbar/kT$, where ω_c is an upper cutoff in the spectrum

[5] Anderson (Ref. 7) quotes a dephasing time, for the superfluid helium case, of order 10^8 years. It is not clear to us on what assumptions this estimate is based.

function). In the ohmic case with $\alpha < 1$, $\langle \sigma_x \rangle$ decays exponentially with time [as $\langle \sigma_x \rangle \sim \exp(-\pi\alpha kTt/\hbar)$, if $kT \ll \Delta_r$]. As to the superohmic cases, $\langle \sigma_x \rangle$ decays algebraically with time, if $s = 2$, and tends to a constant value, if $s = 3$.

The asymptotic laws that we have obtained for the case of a two-level system yield qualitative information on the long-time behavior of a decoupled Josephson function if one exploits the analogy, $\hat{\sigma}_z \cong \Delta\hat{N}$ and $\hat{\sigma}_x \cong \cos\hat{\phi}$. The degree of definition of the relative phase $\hat{\phi}$ is closely related to the coherence between the states with different relative number of particles (which is given by the value of the off-diagonal terms of the reduced density matrix in the representation of $\Delta\hat{N}$ eigenstates), in the same way as, in the spin–boson problem, a good degree of coherence between eigenstates of σ_z must exist for σ_x to be well defined.

The qualitative upshot of these considerations is that for any given physical system the relative phase ϕ becomes ill-defined in the limit $t \to \infty$, but that the time (call it τ_ϕ) for this to occur may well be long enough to allow, in principle at least, interesting experiments. This raises at least two different questions:

(1) If we confine ourselves to times much shorter than τ_ϕ, is it possible to set up a "standard of phase"?

(2) At times $t \gg \tau_\phi$, what exactly does "ill-defined" mean?

We will discuss the second question in the next section: the first is left for a future occasion.

5. CLASSICAL AND QUANTUM IGNORANCE: DOES "MEASUREMENT" CREATE A RELATIVE PHASE?

In Sec. 3 we saw that, whatever is the case at short times, at sufficiently long times the relative phase of two superfluids which were once in contact (and hence, presumably, *a fortiori* that of two which have never been in contact) is ill defined. This apparently innocuous statement raises at least two very intriguing questions, to illustrate which we focus on the thought experiment described in Sec. 2, in which the two superfluids are suddenly brought into contact and one asks whether or not a Josephson current flows between them.

The first question concerns the relation between "classical ignorance" and "quantum ignorance." If we had neglected the dephasing effects explored in Sec. 3 and assumed, instead, that the two systems had been subjected to some classical but unknown potential difference during the period of their separation, then we could hardly avoid the conclusion that the answer (b) of Sec. 2 is correct. Is answer (b) still correct if the dephasing is due rather to the mechanisms of Sec. 3? We believe it is not: i.e., that in this case no Josephson current will flow on any trial [answer (a)] and the system, starting in effect in an incoherent mixture of relative number eigenstates, will simply radiate photons, etc., until it attains the ground state of its new Hamiltonian including the Josephson coupling (such a ground state corresponding, of course, to $\phi = 0$ and hence no current). An intriguing aspect of this conjecture is that it prima facie suggests that the reduced density matrix $\rho_{NN'}$ (which would be identical in the two cases considered) does *not* give all the information needed to predict the result of all possible experiments on the (physically isolated) system, contrary to what a naive reading of many textbooks of quantum theory might lead one to assume. This illustrates the ambiguities of applying standard quantum measurement theory to this kind of case (cf. below).

However, before drawing conclusions about the results of actual experiments, one should face up to the following question: Even if we allow that prior to the reconnection the state of the two systems was correctly described by an incoherent mixture of number eigenstates, does the act of "looking to see" whether a Josephson current flows itself force the system into an eigenstate of current and hence of relative phase? If we take at face value the statements so often made in quantum measurement theory, that "measurement projects the system into an eigenstate of the measured quantity," then it would seem that the answer is yes; and hence (since the concept of the "occurrence" or not of an event in the absence of a specification of how it is to be observed is alien to quantum mechanics) that answer (b) of Sec. 2 is indeed after all correct. Yet if one thinks about it seriously, this answer is bizarre in the extreme. In principle there is nothing to stop us from considering a case where the magnitude of the Josephson current is of order of, say, kiloamps. If we agree that in the absence of a "measurement" of the current the system is (initially at least) in a mixture of eigenstates of relative number and hence not in an eigenstate of current (relative phase), can it really be that by placing, let us say, a miniscule compass needle next to the system, with a weak light beam to read off its position, we can force the system to "realize" a definite macroscopic value of the current? Common sense certainly rebels against this conclusion, and we believe that in this case common sense is right. The problem is that we are implicitly trying to apply the quantum measurement nostrums developed in the 20's and 30's in the context of experiments of the Stern–Gerlach type, where only microscopic objects have to be described by quantum mechanics, to experiments of a type only feasible in the 90's (if then!) where we seriously wonder about the effects of quantum mechanics on *macroscopic* bodies. What is needed is a quantum measurement theory for the 90's—one in which all the assumptions about relative energy and time scales, etc., which are implicit in Stern–Gerlach type analysis, are made explicit and if necessary revised.

In this paper we have raised many questions and given few answers. To summarize, the absolute phase of a superfluid is not a necessary, or indeed a meaningful, concept. However, under certain conditions, the relative phase of two superfluids can be meaningful, even when they are physically separated; but these conditions are extremely stringent. The question whether a measurement can "create" a relative phase when none previously existed remains unresolved and would seem to require a more realistic approach than currently exists to the concept of measurement in the context of macroscopic quantum phenomena. We hope nevertheless that we have reinforced the case (cf. Ref. 7) that the study of phase coherence in superfluids leads to many intriguing questions about the meaning of the quantum formalism. It is a pleasure to dedicate this paper to John Bell, who has probably done more than anyone else to make us all think about these issues, on the occasion of his 60th birthday.

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