



LEV DAVIDOVICH LANDAU  
(From a portrait)

E. M. LIFSHITZ  
Lev Davidovich Landau  
(1908–68)\*

Very little time has passed since the death of Lev Davidovich Landau on 1 April 1968, but fate wills that even now we view him at a distance, as it were. From that distance we perceive more clearly not only his greatness as a scientist, the significance of whose work becomes increasingly obvious with time, but also that he was a great-hearted human being. He was uncommonly just and benevolent. There is no doubt that therein lie the roots of his popularity as a scientist and teacher, the roots of that genuine love and esteem which his direct and indirect pupils felt for him and which were manifested with such exceptional strength during the days of the struggle to save his life following the terrible accident.

To him fell the tragic fate of dying twice. The first time it happened was six years earlier on 7 January 1962 when on the icy road, en route from Moscow to Dubna, his car skidded and collided with a lorry coming from the opposite direction. The epic story of the subsequent struggle to save his life is primarily a story of the selfless labour and skill of numerous physicians and nurses. But it is also a story of a remarkable feat of solidarity. The calamitous accident agitated the entire community of physicists, arousing a spontaneous and instant response. The hospital in which Landau lay unconscious became a centre to all those—his students and colleagues—who strove to make whatever contributions they could to help the physicians in their desperate struggle to save Landau's life.

‘Their feat of comradeship commenced on the very first day. Illustrious scientists who, however, had no idea of medicine, academicians, corresponding members of the scientific academies, doctors, candidates, men of the same generation as the 54-year-old Landau as well as his pupils and *their* still more youthful pupils—all volunteered to act as messengers, chauffeurs, intermediaries, suppliers, secretaries, members of the watch and, lastly, porters and labourers. Their spontaneously established headquarters was located in the office of the Physician-in-Chief of Hospital No. 50 and it became a round-the-clock organizational centre

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for an unconditional and immediate implementation of any instruction of the attending physicians.

'Eighty-seven theoreticians and experimenters took part in this voluntary rescue team. An alphabetical list of the telephone numbers and addresses of any one and any institution with which contact might be needed at any instant was compiled, and it contained 223 telephone numbers! It included other hospitals, motor transport bases, airports, customs offices, pharmacies, ministries, and the places at which consulting physicians could most likely be reached.

'During the most tragic days when it seemed that "Dau is dying"—and there were at least four such days—8–10 cars could be found waiting at any time in front of the seven-storey hospital building. . . .

'When everything depended on the artificial respiration machine, on 12 January, a theoretician suggested that it should be immediately constructed in the workshops of the Institute of Physical Problems. This was unnecessary and naive, but how amazingly spontaneous! The physicists obtained the machine from the Institute for the Study of Poliomyelitis and carried it in their own hands to the ward where Landau was gasping for breath. They saved their colleague, teacher, and friend.

'The story could be continued without limit. This was a real fraternity of physicists. . . . '\*

And so, Landau's life was saved. But when after three months he regained consciousness, it was no longer the same man whom we had known. He was not able to recover from all the consequences of his accident and never again completely regained his abilities. The story of the six years that followed is only a story of prolonged suffering and pain.

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Lev Davidovich Landau was born on 22 January 1908 in Baku, in the family of a petroleum engineer who worked on the Baku oil-fields. His mother was a physician and at one time had engaged in scientific work on physiology.

He completed his school course at the age of 13. Even then he already was attracted by the exact sciences, and his mathematical ability manifested itself very early. He studied mathematical analysis on his own and later he used to say that he hardly remembered a time when he did not know differentiation and integration.

His parents considered him too young to enter a university and for a year he attended the Baku Economic Technicum. In 1922 he enrolled at Baku University where he studied simultaneously in two departments: Physico-mathematical and Chemical. Subsequently he did not continue his chemical education but he remained interested in chemistry throughout his life.

In 1924 Landau transferred to the Physics Department of Leningrad University. In Leningrad, the main centre of Soviet physics at that time, he first made the acquaintance of genuine theoretical physics, which was then going through a turbulent period. He devoted himself to its study with all his youthful zeal and enthusiasm and worked so strenuously that often he became so exhausted that at night he could not sleep, still turning over formulae in his mind.

\* From D. Danin, *Literaturnaya Gazeta*, 21 July 1962, reprinted in this volume (p.84).

Later he used to describe how at that time he was amazed by the incredible beauty of the general theory of relativity (sometimes he even would declare that such a rapture on first making one's acquaintance with this theory should be a characteristic of any born theoretical physicist). He also described the state of ecstasy to which he was brought on reading the articles by Heisenberg and Schrödinger signalling the birth of the new quantum mechanics. He said that he derived from them not only delight in the true glamour of science but also an acute realization of the power of the human genius, whose greatest triumph is that man is capable of apprehending things beyond the pale of his imagination. And of course, the curvature of space-time and the uncertainty principle are precisely of this kind.

In 1927 Landau graduated from the university and enrolled for post-graduate study at the Leningrad Physicotechnical Institute where even earlier, in 1926, he had been a part-time research student. These years brought his first scientific publications. In 1926 he published a theory of intensities in the spectra of diatomic molecules,\* and as early as 1927, a study of the problem of damping in quantum mechanics, which first introduced a description of the state of a system with the aid of the density matrix.

His fascination with physics and his first achievements as a scientist were, however, at the time beclouded by a painful diffidence in his relations with others. This trait caused him a great deal of suffering and at times—as he himself confessed in later years—led him to despair. The changes which occurred in him with the years and transformed him into a buoyant and gregarious individual were largely a result of his characteristic self-discipline and feeling of duty toward himself. These qualities, together with his sober and self-critical mind, enabled him to train himself and to evolve into a person with a rare ability—the ability to be happy. The same sobriety of mind enabled him always to distinguish between what is of real value in life and what is unimportant triviality, and thus also to retain his mental equilibrium during the difficult moments which occurred in his life too.

In 1929, on an assignment from the People's Commissariat of Education, Landau travelled abroad and for one and a half years worked in Denmark, Great Britain and Switzerland. To him the most important part of his trip was his stay in Copenhagen where, at the Institute of Theoretical Physics, theoretical physicists from all Europe gathered round the great Niels Bohr and, during the famous seminars headed by Bohr, discussed all the basic problems of the theoretical physics of the time. This scientific atmosphere, enhanced by the charm of the personality of Bohr himself, decisively influenced Landau in forming his own outlook on physics and

\* He did not know, however, at the time that these results had been already published a year earlier by Hönl and London.

subsequently he always considered himself a disciple of Niels Bohr. He visited Copenhagen two more times, in 1933 and 1934. Landau's sojourn abroad was the occasion, in particular, of his work on the theory of the diamagnetism of an electron gas and the study of the limitations imposed on the measurability of physical quantities in the relativistic quantum region (in collaboration with Peierls).

On his return to Leningrad in 1931 Landau worked in the Leningrad Physicotechnical Institute and in 1932 he moved to Kharkov, where he became head of the Theoretical Division of the newly organized Ukrainian Physicotechnical Institute, an offshoot of the Leningrad Institute. At the same time he headed the Department of Theoretical Physics at the Physics and Mechanics Faculty of the Kharkov Mechanics and Machine Building Institute and in 1935 he became Professor of General Physics at Kharkov University.

The Kharkov period was for Landau a time of intense and varied research activity.\* It was there that he began his teaching career and established his own school of theoretical physics.

Twentieth-century theoretical physics is rich in illustrious names of trail-blazing creators, and Landau was one of these creators. But his influence on scientific progress was far from exhausted by his personal contribution to it. He was not only an outstanding physicist but also a genuinely outstanding educator, a born educator. In this respect one may take the liberty of comparing Landau only to his own teacher—Niels Bohr.

The problems of the teaching of theoretical physics as well as of physics as a whole had first attracted his interest while still quite a young man. It was there, in Kharkov, that he first began to work out programmes for the 'theoretical minimum'—programmes of the basic knowledge in theoretical physics needed by experimental physicists and by those who wish to devote themselves to professional research work in theoretical physics. In addition to drafting these programmes, he gave lectures on theoretical physics to the scientific staff at the Ukrainian Physicotechnical Institute as well as to students of the Physics and Mechanics Faculty. Attracted by the ideas of reorganizing instruction in physics as a whole, he accepted the Chair of General Physics at Kharkov State University (and subsequently, after the war, he continued to give lectures on general physics at the Physicotechnical Faculty of Moscow State University).

It was there also, in Kharkov, that Landau had conceived the idea and began to implement the programme for compiling a complete Course of Theoretical Physics and Course of General Physics. All his life long, Lan-

\* The extent of Landau's scientific activities at the time can be grasped from the list of studies he completed during the year 1936 alone: theory of second-order phase transitions [28, 29], theory of the intermediate state of superconductors [30], the transport equation in the case of Coulomb interaction [23], the theory of unimolecular reactions [22], properties of metals at very low temperatures [24], theory of the dispersion and absorption of sound [21, 27], theory of photoelectric effects in semiconductors [20].

dau dreamed of writing books on physics at every level—from school textbooks to a course of theoretical physics for specialists. In fact, by the time of his fateful accident, nearly all the volumes of the *Course of Theoretical Physics* and the first volumes of the *Course of General Physics* and *Physics for Everyone* had been completed. He also had drafted plans for the compilation of textbooks on mathematics for physicists, which should be ‘a guide to action’, should instruct in the practical applications of mathematics to physics, and should be free of the rigours and complexities unnecessary to this course. He did not have time to begin to translate this programme into reality.

Landau always attached great importance to the mastering of mathematical techniques by the theoretical physicist. The degree of this mastery should be such that, insofar as possible, mathematical complications would not distract attention from the physical difficulties of the problem—at least whenever standard mathematical techniques are concerned. This can be achieved only by sufficient training. Yet experience shows that the current style and programmes for university instruction in mathematics for physicists often do not ensure such training. Experience also shows that after a physicist commences his independent research activity he finds the study of mathematics too ‘boring’.

Therefore, the first test which Landau gave to anyone who desired to become one of his students was a quiz in mathematics in its ‘practical’ calculational aspects.\* The successful applicant could then pass on to the study of the seven successive sections of the programme for the ‘theoretical minimum’, which includes basic knowledge of all the domains of theoretical physics, and subsequently take an appropriate examination. In Landau’s opinion, this basic knowledge should be mastered by any theoretician regardless of his future specialization. Of course, he did not expect anyone to be as universally well-versed in science as he himself. But he thus manifested his belief in the integrity of theoretical physics as a single science with unified methods.

At first Landau himself gave the examination for the ‘theoretical minimum’. Subsequently, after the number of applicants became too large, this duty was shared with his closest associates. But Landau always reserved for himself the first test, the first meeting with each new young applicant. Anyone could meet him—it was sufficient to ring him up and ask him for an interview.

Of course, not every one who began to study the ‘theoretical minimum’ had sufficient ability and persistence to complete it. Altogether, between

\* The requirements were: ability to evaluate any indefinite integral that can be expressed in terms of elementary functions and to solve any ordinary differential equation of the standard type, knowledge of vector analysis and tensor algebra as well as of the principles of the theory of functions of a complex variable (theory of residues, Laplace method). It was assumed that such fields as tensor analysis and group theory would be studied together with the fields of theoretical physics to which they apply.

1934 and 1961, 43 persons passed this test. The effectiveness of this selection can be perceived from the following indicative facts alone: of these persons 7 already have become members of the Academy of Sciences and an additional 16, doctors of sciences.

In the spring of 1937 Landau moved to Moscow where he became head of the Theoretical Division of the Institute of Physical Problems which had not long before been established under the direction of P. L. Kapitza. There he remained to the end of his life; in this Institute, which became a home to him, his varied activity reached its full flowering. It was there, in a remarkable interaction with experimental research, that Landau created what may be the outstanding accomplishment of his scientific life—the theory of quantum fluids.

It was there also that he received the numerous outward manifestations of the recognition of his contributions. In 1946 he was elected a full Member of the USSR Academy of Sciences. He was awarded a number of orders (including two Orders of Lenin) and the honorific title of Hero of Socialist Labour—a reward for both his scientific accomplishments and his contribution to the implementation of important practical State tasks. He was awarded the State Prize three times and in 1962, the Lenin Prize. There also was no lack of honorific awards from other countries. As far back as 1951 he was elected member of the Danish Royal Academy of Sciences and in 1956, member of the Netherlands Royal Academy of Sciences. In 1959 he became honorary fellow of the British Institute of Physics and Physical Society and in 1960, Foreign Member of the Royal Society of Great Britain. In the same year he was elected to membership in the National Academy of Sciences of the United States and the American Academy of Arts and Sciences. In 1960 he became recipient of the F. London Prize (United States) and of the Max Planck Medal (West Germany). Lastly, in 1962 he was awarded the Nobel Prize in Physics ‘for his pioneering theories for condensed matter, especially liquid helium’.

Landau’s scientific influence was, of course, far from confined to his own disciples. He was deeply democratic in his life as a scientist (and in his life as a human being, for that matter; pomposity and deference to titles always remained foreign to him). Anyone, regardless of his scientific merits and title, could ask Landau for counsel and criticism (which were invariably precise and clear), on one condition only: the question must be businesslike instead of pertaining to what he detested most in science: empty philosophizing or vapidity and futility cloaked in pseudo-scientific sophistries. He had an acutely critical mind; this quality, along with his approach from the standpoint of profound physics, made discussion with him extremely attractive and useful.

In discussion he used to be ardent and incisive but not rude; witty and ironic but not caustic. The nameplate which he hung on the door of his office at the Ukrainian Physicotechnical Institute bore the inscription:

L. LANDAU  
BEWARE, HE BITES!

With years his character and manner mellowed somewhat, but his enthusiasm for science and his uncompromising attitude toward science remained unchanged. And certainly his sharp exterior concealed a scientifically impartial attitude, a great heart and great kindness. However harsh and unsparing he may have been in his critical comments, he was just as intense in his desire to contribute with his advice to another man's success, and his approval, when he gave it, was just as ardent.

These traits of Landau's personality as a scientist and of his talent actually elevated him to the position of a supreme scientific judge, as it were, over his students and colleagues.\* There is no doubt that this side of Landau's activities, his scientific and moral authority which exerted a restraining influence on frivolity in research, has also markedly contributed to the lofty level of our theoretical physics.

His constant scientific contact with a large number of students and colleagues also represented to Landau a source of knowledge. A unique aspect of his style of work was that, ever since long ago, since the Kharkov years, he himself almost never read any scientific article or book but nevertheless he was always completely au courant with the latest news in physics. He derived this knowledge from numerous discussions and from the papers presented at the seminar held under his direction.

This seminar was held regularly once a week for nearly 30 years, and in the last years its sessions became gatherings of theoretical physicists from all Moscow. The presentation of papers at this seminar became a sacred duty for all students and co-workers, and Landau himself was extremely serious and thorough in selecting the material to be presented. He was interested and equally competent in every aspect of physics and the participants in the seminar did not find it easy to follow his train of thought in instantaneously switching from the discussion of, say, the properties of 'strange' particles to the discussion of the energy spectrum of electrons in silicon. To Landau himself listening to the papers was never an empty formality: he did not rest until the essence of a study was completely elucidated and all traces of 'philology'—unproved statements or propositions made on the principle of 'why might it not'—therein were eliminated. As a result of such discussion and criticism many studies were condemned as 'pathology' and Landau completely lost interest in them. On the other hand, articles that really contained new ideas or findings were included in the so-called 'gold fund' and remained in Landau's memory for ever.

In fact, usually it was sufficient for him to know just the guiding idea

\* This position is symbolized in A. A. Yuzefovich's well-known friendly cartoon, 'Dau said', reproduced elsewhere in the present volume.



of a study in order to reproduce all of its findings. As a rule, he found it easier to obtain them on his own than to follow in detail the author's reasoning. In this way he reproduced for himself and profoundly thought out most of the basic results obtained in all the domains of theoretical physics.\* This probably also was the reason for his phenomenal ability to answer practically any question concerning physics that might be asked of him.

Landau's scientific style was free of the—unfortunately fairly widespread—tendency to complicate simple things (often on the grounds of generality and rigour which, however, usually turn out to be illusory). He himself always strove towards the opposite—to simplify complex things, to uncover in the most lucid manner the genuine simplicity of the laws underlying the natural phenomena. This ability of his, this skill at 'trivializing' things as he himself used to say, was to him a matter of special pride.

The striving for simplicity and order was an inherent part of the structure of Landau's mind. It manifested itself not only in serious matters but also in semi-serious things as well as in his characteristic personal sense of humour.† Thus, he liked to classify everyone, from women according to the degree of their beauty, to theoretical physicists according to the significance of their contribution to science. This last classification was based on a logarithmic scale of five: thus, a second-class physicist supposedly accomplished 10 times as much as a third-class physicist ('pathological types' were ranked in the fifth class). On this scale Einstein occupied the position  $\frac{1}{2}$ , while Bohr, Heisenberg, Schrödinger, Dirac and certain others were ranked in the first class. Landau modestly ranked himself for a long time in class  $2\frac{1}{2}$  and it was only comparatively late in his life that he promoted himself to the second class.

He always worked hard (never at a desk, usually reclining on a divan at home). The recognition of the results of one's work is to a greater or lesser extent important to any scientist; it was, of course, also essential to Landau. But it can still be said that he attached much less importance to questions of priority than is ordinarily the case. And at any rate there is no doubt that his drive for work was inherently motivated not by desire for fame but by an inexhaustible curiosity and passion for exploring the laws of nature in their large and small manifestations. He never omitted a chance to repeat the elementary truth that one should never work for extraneous purposes, work merely for the sake of making a great discovery, for then nothing would be accomplished anyway.

The range of Landau's interests outside physics also was extremely

\* Incidentally, this explains the absence of certain needed references in Landau's papers, which usually was not intentional. However, in some cases he could leave out a reference on purpose, if he considered the question too trivial; and he did have his own rather high standards on that matter.

† It is characteristic, however, that this trait was not a habit of Landau in his, so to speak, everyday outside life, in which he was not at all pedantically accurate and a 'zone of disorder' would quite rapidly arise around him.

wide. In addition to the exact sciences he loved history and was well-versed in it. He was also passionately interested in and deeply impressed by every genre of fine arts, though with the exception of music (and ballet).

Those who had the good fortune to be his students and friends for many years knew that our Dau, as his friends and comrades nicknamed him,\* did not grow old. In his company boredom vanished. The brightness of his personality never grew dull and his scientific power remained strong. All the more senseless and frightful was the accident which put an end to his brilliant activity at its zenith.

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Landau's articles, as a rule, display all the features of his characteristic scientific style: clarity and lucidity of physical statement of problems, the shortest and most elegant path towards their solution, no superfluities. Even now, after many years, the greater part of his articles does not require any revisions.

The brief review below is intended to provide only a tentative idea of the abundance and diversity of Landau's work and to clarify to some extent the place occupied by it in the history of physics, a place which may not always be obvious to the contemporary reader.

A characteristic feature of Landau's scientific creativity is its almost unprecedented breadth, which encompasses the whole of theoretical physics, from hydrodynamics to the quantum field theory. In our century, which is a century of increasingly narrow specialization, the scientific paths of his students also have been gradually diverging, but Landau himself unified them all, always retaining a truly astounding interest in everything. It may be that in him physics has lost one of the last great universalists.

Even a cursory examination of the bibliography of Landau's works shows that his life cannot be divided into any lengthy periods during which he worked only in some one domain of physics. Hence also the survey of his works is given not in chronological order but, insofar as possible, in thematic order. We shall begin with the works devoted to the general problems of quantum mechanics.

These include, in the first place, several of his early works. In the course of his studies of the radiation-damping problem he was the first to introduce the concept of incomplete quantum-mechanical description accomplished with the aid of quantities which were subsequently termed the density matrix [2]. In this article the density matrix was introduced in its energy representation.

Two articles [6, 7] are devoted to the calculation of the probabilities of quasi-classical processes. The difficulty of this problem stems from the fact that, by virtue of the exponential nature (with a large imaginary exponent) of the quasi-classical wave functions, the integrand in the matrix elements is a rapidly fluctuating quantity; this greatly complicates even an estimate of

\* Landau himself liked to say that this name originated from the French spelling of his name: Landau = *L'âne Dau* (the ass Dau).

the integral; in fact, until Landau's work all studies of problems of this kind were erroneous. Landau was the first to provide a general method for the calculation of quasi-classical matrix elements and he also applied it to a number of specific processes.

In 1930 Landau (in collaboration with R. Peierls) published a detailed study of the limitations imposed by relativistic requirements on the quantum-mechanical description [5]; this article caused lively discussions at the time. Its basic result lies in determining the limits of the possibility of measuring the particle momentum within a finite time (and in clarifying the topic of individual indeterminacy of the coordinate). This implied that in the relativistic quantum region it is not feasible to measure any dynamical variables characterizing the particles in their interaction, and that the only measurable quantities are the momenta (and polarizations) of free particles. Therein also lies the physical root of the difficulties that arise when methods of conventional quantum mechanics, employing concepts which become meaningless in the relativistic domain, are applied there. Landau returned to this problem in his last published article [98], in which he expressed his conviction that the  $\psi$ -operators, as carriers of unobservable information, and along with them the entire Hamiltonian method, should disappear from a future theory.

One of the reasons for this conviction was the results of the research into the foundations of quantum electrodynamics which Landau carried out during 1954-5 (in collaboration with A. A. Abrikosov, I. M. Khalatnikov and I. Ya. Pomeranchuk) [76-79, 83]. These studies were based on the concept of the point interaction as the limit of 'smeared' interaction when the smearing radius tends to zero. This made it possible to deal directly with finite expressions. Further, it proved possible to carry out the summation of the principal terms of the entire series of perturbation theory and this led to the derivation of asymptotic expressions (for the case of large momenta) for the fundamental quantities of quantum electrodynamics—the Green functions and the vertex part. These relations, in their own turn, were used to derive the relationship between the true charge and mass of the electron, on the one hand, and their 'bare' values, on the other. Although these calculations proceeded on the premise of smallness of the 'bare' charge, it was argued that the formula for the relation between true and bare charges retains its validity regardless of the magnitude of the bare charge. Then analysis of this formula shows that at the limit of point interaction the true charge becomes zero—the theory is 'nullified'.\* (A review of the pertinent questions is provided in the articles [82, 86].)

Only the future will show the extent of the validity of the programme planned by Landau for constructing a relativistic quantum field theory. He

\* In connection with the search for a more rigorous proof of this statement, the article [98] contains the assertion, characteristic of Landau, that 'the brevity of life does not allow us the luxury of spending time on problems which will lead to no new results'.

himself was energetically working in this direction during the last few years prior to his accident. As part of this programme, in particular, he had worked out a general method for determining the singularities of the quantities that occur in the diagram technique of quantum field theory [96].

In response to the discovery in 1956 of parity non-conservation in weak interactions, Landau immediately proposed the theory of a neutrino with fixed helicity ('two-component neutrino') [90]\*, and also suggested the principle of the conservation of 'combined parity', as he termed the combined application of spatial inversion and charge conjugation. According to Landau, the symmetry of space would in this way be 'saved'—the asymmetry is transferred to the particles themselves. This principle indeed proved to be more widely applicable than the law of parity conservation. As is known, however, in recent years processes not conserving combined parity have also been discovered; the meaning of this violation is at present still unclear.

A 1937 study [31] by Landau pertains to nuclear physics. This study represents a quantitative embodiment of the ideas proposed not long before by Bohr: the nucleus is examined by methods of statistical physics as a drop of 'quantum fluid'. It is noteworthy that this study did not make use of any far-reaching model conceptions, contrary to the previous practice of other investigators. In particular, the relationship between the mean distance between the levels of the compound nucleus and the width of the levels was established for the first time.

The absence of model conceptions is characteristic also of the theory of proton-proton scattering developed by Landau (in collaboration with Ya. A. Smorodinskiĭ). The scattering cross-section in their study was expressed in terms of parameters whose meaning is not restricted by any specific assumptions concerning the particle interaction potential.

The study (in collaboration with Yu. B. Rumer) [36] of the cascade theory of electron showers in cosmic rays is an example of technical virtuosity; the physical foundations of this theory had been earlier formulated by a number of investigators, but a quantitative theory was essentially lacking. That study provided the mathematical apparatus which became the basis for all subsequent work in this domain. Landau himself took part in the further refinement of the shower theory by contributing two more articles, one on the particle angular distribution [41] and the other on secondary showers [42].

Of no smaller virtuosity was Landau's work dealing with the elaboration of Fermi's idea of the statistical nature of multiple particle production in collisions [72]. † This study also represents a brilliant example of the methodological unity of theoretical physics in which the solution of a problem is accomplished by using the methods from a seemingly completely different

\* Simultaneously and independently, this theory was proposed by Salam and by Lee and Yang.

† This has been more fully described in a review article [85] written in collaboration with S. Z. Belen'kii.

domain. Landau showed that the process of multiple production includes the stage of the expansion of a 'cloud' whose dimensions are large compared with the mean free path of particles in it; correspondingly, this stage should be described by equations of relativistic hydrodynamics. The solution of these equations required a number of ingenious techniques as well as a thorough analysis. Landau used to say that this study cost him more effort than any other problem that he had ever solved.

Landau always willingly responded to the requests and needs of the experimenters, e.g. by publishing the article [54] which established the energy distribution of the ionization losses of fast particles during passage through matter (previously only the theory of mean energy loss had existed).

Turning now to Landau's work on macroscopic physics, we begin with several articles, representing his contribution to the physics of magnetism.

According to classical mechanics and statistics, a change in the pattern of movement of free electrons in a magnetic field cannot result in the appearance of new magnetic properties of the system. Landau was the first to elucidate the character of this motion in a magnetic field for the quantum case, and to show that quantization completely changes the situation, resulting in the appearance of diamagnetism of the free electron gas ('Landau diamagnetism' as this effect is now termed)[4]. The same study qualitatively predicted the periodic dependence of the magnetic susceptibility on the intensity of the magnetic field when this intensity is high. At the time (1930) this phenomenon had not yet been observed by anyone, and it was experimentally discovered only later (the De Haas–Van Alphen effect); a quantitative theory of this effect was presented by Landau in a later paper [37]

A short article published in 1933 [11] is of a significance greatly transcending the problem stated in its title—a possible explanation of the field dependence of the magnetic susceptibility of a particular class of substances at low temperatures. This article was the first to introduce the concept of antiferromagnetism (although it did not use this term) as a special phase of magnetic bodies differing in symmetry from the paramagnetic phase; accordingly, the transition from one state to the other must occur at a rigorously definite point.\* This article examined the particular model of a layered antiferromagnet with a strong ferromagnetic coupling in each layer and a weak antiferromagnetic coupling between the layers; a quantitative investigation of this case was carried out and the characteristic features of magnetic properties in the neighbourhood of the transition point were established. The method employed here by Landau was based on ideas which he subsequently elaborated in the general theory of second-order phase transitions.

\* Roughly a year earlier Néel (whose work was unknown to Landau) had predicted the possibility of existence of substances which, from the magnetic standpoint, consist of two sublattices with opposite moments. Néel, however, did not assume that a special state of matter is involved here, and instead he simply thought that a paramagnet with a positive exchange integral at low temperatures gradually turns into a structure consisting of several magnetic sublattices.

Another paper concerns the theory of ferromagnetism. The idea of the structure of ferromagnetic bodies as consisting of elementary regions spontaneously magnetized in various directions ('magnetic domains', as the modern term goes) was expressed by P. Weiss as early as in 1907. However, there was no suitable approach to the question of the quantitative theory of this structure until Landau (in collaboration with E. M. Lifshitz) [17] showed in 1935 that this theory should be constructed on the basis of thermodynamic considerations and determined the form and dimensions of the domains for a typical case. The same study derived the macroscopic equation of the motion of the domain magnetization vector and, with its aid, developed the principles of the theory of the dispersion of the magnetic permeability of ferromagnets in an alternating magnetic field; in particular, it predicted the effect now known as ferromagnetic resonance.

A short communication published in 1933 [9] expressed the idea of the possibility of the 'autolocalization' of an electron in a crystal lattice within the potential well produced by virtue of the polarization effect of the electron itself. This idea subsequently provided the basis for the so-called polaron theory of the conductivity of ionic crystals. Landau himself returned once more to these problems in a later study (in collaboration with S. I. Pekar) [65] dealing with the derivation of the equations of motion of the polaron in the external field.

Another short communication [13] reported on the results obtained by Landau (in collaboration with G. Placzek) concerning the structure of the Rayleigh scattering line in liquids or gases. As far back as the early 1920s Brillouin and Mandel'shtam showed that, owing to scattering by sound vibrations, this line must split into a doublet. Landau and Placzek drew attention to the attendant necessity of the existence of scattering by entropy fluctuations, not accompanied by any change in frequency; as a result, a triplet should be observed instead of a doublet.\*

Two of Landau's works pertain to plasma physics. One of these two [23] was the first to derive the transport equation with allowance for Coulomb interaction between particles; the slowness of decrease of these forces rendered inapplicable in this case the conventional methods for constructing transport equations. The other work [59], dealing with plasma oscillations, showed that, even under conditions when collisions between particles in the plasma can be disregarded, high-frequency oscillations will still attenuate ('Landau damping').†

His work to compile one of the successive volumes of the *Course of Theoret-*

\* No detailed exposition of the conclusions and results of this study was ever published in article form. It is partly presented in the book by Landau and Lifshitz, *Electrodynamics of Continuous Media*, Pergamon, Oxford 1960, §96; 2nd ed., 1984, §120.

† It is interesting that this work was carried out by Landau as his response to the 'philology' present, in his opinion, in previous studies dealing with this subject (e.g., the unjustified replacement of divergent integrals by their principal values). It was to prove his rightness that he occupied himself with this question.

*ical Physics* was to Landau a stimulus for a thorough study of hydrodynamics. Characteristically, he independently pondered and derived all the basic notions and results of this branch of science. His fresh and original perception led, in particular, to a new approach to the problem of the onset of turbulence and he elucidated the basic aspects of the process of the gradual development of unsteady flow with increase in the Reynolds number following the loss of stability by laminar motion and predicted qualitatively various alternatives possible in this case [50]. On investigating the qualitative properties of supersonic flow around bodies, he arrived at the unexpected discovery that in supersonic flow there must exist far from the body not one—as had been the conventional assumption—but two shock waves, one following the other [58]. Even in such a ‘classical’ field as the jet theory he succeeded in finding a new and previously unnoticed exact solution for an axially symmetric ‘inundated’ jet of a viscous incompressible fluid [49].

In Landau’s scientific creative accomplishments an eminent position is occupied—both from the standpoint of direct significance and in terms of the consequent physical applications—by the theory of second-order phase transitions [28, 29]; a first outline of the ideas underlying this theory is already contained in an earlier communication [16].\* The concept of phase transitions of various orders had first been introduced by Ehrenfest in a purely formal manner, with respect to the order of the thermodynamic derivatives which could undergo a discontinuity at the transition point. The question of exactly which of these transitions can exist in reality, and what is their physical nature, had remained open, and previous interpretations had been fairly vague and unsubstantiated. Landau was the first to point to the profound connection between the possibility of existence of a continuous (in the sense of variation in the body’s state) phase transition and the jump-like (discontinuous) change in some symmetry property of the body at the transition point. He also showed that far from just any change in symmetry is possible at that transition point and provided a method which makes it possible to determine the permissible types of change in symmetry. The quantitative theory developed by Landau was based on the assumption of the regularity of the expansion of thermodynamic quantities in the neighbourhood of the transition point. It is now clear that such a theory, which fails to allow for possible singularities of these quantities at the transition point, does not reflect all the properties of phase transitions. The question of the nature of these singularities was of great interest to Landau and during the last years of his activity he worked a great deal on this difficult problem without, however, succeeding in arriving at any definite conclusions.

The phenomenological theory of superconductivity developed in 1950 by Landau (in collaboration with V. L. Ginzburg) [71] also was constructed

\* Landau himself applied this theory to the scattering of X-rays by crystals [32] and—in collaboration with I. M. Khalatnikov—to the absorption of sound in the neighbourhood of the transition point [80].

in the spirit of the theory of phase transitions; subsequently it became, in particular, the basis for the theory of superconducting alloys. This theory involves a number of variables and parameters whose meaning was not completely clear at the time it was originally developed and became understandable only after the appearance in 1957 of the microscopic theory of superconductivity, which made possible a rigorous substantiation of the Ginzburg–Landau equations and a determination of the region of their applicability. In this connection, the story (recounted by V. L. Ginzburg\*) of an erroneous statement contained in the original article by Ginzburg and Landau is instructive. The basic equation of the theory, defining the effective wave function  $\Psi$  of superconducting electrons, contains the field vector potential  $\mathbf{A}$  in the term

$$\frac{1}{2m} - \nabla - \frac{e^*\mathbf{A}}{c} \Psi,$$

which is completely analogous to the corresponding term in the Schrödinger equation. It might be thought that in the phenomenological theory the parameter  $e^*$  should represent some effective charge which does not have to be directly related to the charge of the free electron  $e$ . Landau, however, refuted this hypothesis by pointing out that the effective charge is not universal and would depend on various factors (pressure, composition of specimen, etc.); then in an inhomogeneous specimen the charge  $e^*$  would be a function of coordinates and this would disturb the gauge invariance of the theory. Hence the article stated that ‘. . . there is no reason to consider the charge  $e^*$  as different from the electronic charge’. We now know that in reality  $e^*$  coincides with the charge of the Cooper electron pair, i.e.  $e^* = 2e$  and not  $e$ . This value of  $e^*$  could, of course, have been predicted only on the basis of the idea of electron pairing which underlies the microscopic theory of superconductivity. But the value  $2e$  is as universal as  $e$  and hence Landau’s argument in itself was valid.

Another of Landau’s contributions to the physics of superconductivity was to elucidate the nature of the so-called intermediate state. The concept of this state was first introduced by Peierls and F. London (1936) to account for the observed fact that the transition to the superconducting state in a magnetic field is gradual. Their theory was purely phenomenological, however, and the question of the nature of the intermediate state had remained open. Landau showed that this state is not a new state and that in reality a superconductor in that state consists of successive thin layers of normal and superconducting phases. In 1937 Landau [30] considered a model in which these layers emerge to the surface of the specimen; using an elegant and ingenious method he succeeded in completely determining the shape and

\* See *Soviet Physics Uspekhi* 11, 135, 1969.



dimensions of the layers in such a model. \* In 1938 he proposed a new variant of the theory, according to which the layers repeatedly branch out on emerging to the surface; such a structure should be thermodynamically more favourable, given sufficiently large dimensions of the specimen. †

But the most significant contribution that physics owes to Landau is his theory of quantum liquids. The significance of this new discipline at present is steadily growing; there is no doubt that its development in recent decades has produced a revolutionary effect on other domains of physics as well—on solid-state physics and even on nuclear physics.

The superfluidity theory was created by Landau during 1940–1 soon after Kapitza's discovery towards the end of 1937 of this fundamental property of helium II. Prior to it, the premises for understanding the physical nature of the phase transition observed in liquid helium had been essentially lacking and it is not surprising that the previous interpretations of this phenomenon now seem even naive. ‡ The completeness with which the theory of helium II had been constructed by Landau from the very beginning is remarkable: already his first classic paper [44] on this subject contained practically all the principal ideas of both the microscopic theory of helium II and the macroscopic theory constructed on its basis—the thermodynamics and hydrodynamics of this fluid; see also [51].

Underlying Landau's theory is the concept of quasi-particles (elementary excitations) constituting the energy spectrum of helium II. Landau was in fact the first to pose the question of the energy spectrum of a macroscopic body in such a very general form, and it was he, too, who discovered the nature of the spectrum for a quantum fluid of the type to which liquid helium ( $\text{He}^4$  isotope) belongs—or, as it is now termed, of the Bose type. In his 1941 work Landau assumed that the spectrum of elementary excitations consists of two branches: phonons, with a linear dependence of energy  $\varepsilon$  on momentum  $\mathbf{p}$ , and 'rotons', with a quadratic dependence, separated from the ground state by an energy gap. Subsequently he found that such a form of spectrum is not satisfactory from the theoretical standpoint (as it would be unstable) and careful analysis of the more complete and exact experimental data that had by then become available led him in 1946 to establish the now famous spectrum containing only one branch in which the 'rotons' correspond to a minimum on the curve of  $\varepsilon(\mathbf{p})$ . The macroscopic concepts of the theory of superfluidity are widely known. Basically they reduce to the idea of two motions simultaneously occurring in the fluid—'normal' motion and 'superfluid' motion, which may be visualized as motions of two 'fluid com-

\* Landau himself wrote concerning this matter that 'amazingly enough an exact determination of the shape of the layers proves to be possible' [30].

† A detailed description of this work was published in 1943 [47].

‡ Thus, Landau himself in his work on the theory of phase transitions [29] considered whether helium II is a liquid crystal, even though he emphasized the dubiousness of this assumption.

ponents'. \* Normal motion is accompanied by internal friction, as in conventional fluids. The determination of the viscosity coefficient represents a kinetic problem which requires an analysis of the processes of the onset of an equilibrium in the 'gas of quasi-particles'; the principles of the theory of the viscosity of helium II were developed by Landau (in collaboration with I. M. Khalatnikov) in 1949 [67, 68]. Lastly, yet another investigation (carried out in collaboration with I. Ya. Pomeranchuk [62] dealt with the problem of the behaviour of extraneous atoms in helium; it was shown, in particular, that any atom of this kind will become part of the 'normal component' of the fluid regardless of whether the impurity substance itself does or does not display the property of superfluidity—contrary to the incorrect view previously held in the literature.

The liquid isotope  $\text{He}^3$  is a quantum liquid of another type—the Fermi type as it is now termed. Although its properties are not as striking as the properties of liquid  $\text{He}^4$ , they are no less interesting from the standpoint of basic theory. A theory of liquids of this kind was developed by Landau and presented by him in three papers published during 1956–8. The first two of these [87, 88] established the nature of the energy spectrum of Fermi liquids, considered their thermodynamic properties and established the kinetic equation for the relaxation processes occurring in these liquids. His study of the kinetic equation led Landau to predict a special type of vibrational process in liquid  $\text{He}^3$  in the neighbourhood of absolute zero, which he termed zeroth sound. The third paper [93] presented a rigorous microscopic substantiation of the transport equation, whose earlier derivation had contained a number of intuitive assumptions.

Concluding this brief and far from complete survey, it only remains to be repeated that to physicists there is no need to emphasize the significance of Landau's contribution to theoretical physics. His accomplishments are of lasting significance and will for ever remain part of science.

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#### Papers by L. D. Landau

1. On the theory of the spectra of diatomic molecules (*Z. Phys.*, **40**, 621, 1926)
2. The damping problem in wave mechanics (*Z. Phys.*, **45**, 430, 1927)
3. Quantum electrodynamics in configuration space (*Z. Phys.*, **62**, 188, 1930; with R. Peierls)
4. Diamagnetism of metals (*Z. Phys.*, **64**, 629, 1930)

\* Some of the ideas of the 'two-component' macroscopic description of liquid helium were introduced independently of Landau by L. Tisza (although without providing a clear physical interpretation of them). His detailed article published in France in 1940 was, owing to wartime conditions, not received in the USSR until 1943 and the brief note of 1938 in the *Comptes rendus* of the Paris Académie des Sciences had unfortunately remained unnoticed. A criticism of the quantitative aspects of Tisza's theory was provided by Landau in the article [64].

5. Extension of the uncertainty principle to relativistic quantum theory (*Z. Phys.*, **69**, 56, 1931; with R. Peierls)
6. A theory of energy transfer on collisions (*Phys. Z. Sowjet.*, **1**, 88, 1932)
7. A theory of energy transfer II (*Phys. Z. Sowjet.*, **2**, 46, 1932)
8. On the theory of stars (*Phys. Z. Sowjet.*, **1**, 285, 1932)
9. Electron motion in crystal lattices (*Phys. Z. Sowjet.*, **3**, 664, 1933)
10. On the second law of thermodynamics and the universe (*Phys. Z. Sowjet.*, **4**, 114, 1933; with M. Bronstein)
11. A possible explanation of the field dependence of the susceptibility at low temperatures (*Phys. Z. Sowjet.*, **4**, 675, 1933)
12. Internal temperature of stars (*Nature*, **132**, 567, 1933; with G. Gamow)
13. Structure of the undisplaced scattering line (*Phys. Z. Sowjet.*, **5**, 172, 1934; with G. Placzek)
14. On the theory of the slowing down of fast electrons by radiation (*Phys. Z. Sowjet.*, **5**, 761, 1934)
15. On the production of electrons and positrons by a collision of two particles (*Phys. Z. Sowjet.*, **6**, 244, 1934; with E. Lifshitz)
16. On the theory of specific heat anomalies (*Phys. Z. Sowjet.*, **8**, 113, 1935)
17. On the theory of the dispersion of magnetic permeability in ferromagnetic bodies (*Phys. Z. Sowjet.*, **8**, 153, 1935; with E. Lifshitz)
18. On the relativistic correction of the Schrödinger equation for the many-body problem (*Phys. Z. Sowjet.*, **8**, 487, 1935)
19. On the theory of the accommodation coefficient (*Phys. Z. Sowjet.*, **8**, 489, 1935)
20. On the theory of the photoelectromotive force in semiconductors (*Phys. Z. Sowjet.*, **9**, 477, 1936; with E. Lifshitz)
21. On the theory of sound dispersion (*Phys. Z. Sowjet.*, **10**, 34, 1936; with E. Teller)
22. On the theory of uni-molecular reactions (*Phys. Z. Sowjet.*, **10**, 67, 1936)
23. The transport equation in the case of Coulomb interactions (*Phys. Z. Sowjet.*, **10**, 154, 1936)
24. On the properties of metals at very low temperatures (*Phys. Z. Sowjet.*, **10**, 649, 1936; with I. Pomeranchuk)
25. Scattering of light by light (*Nature*, **138**, 206, 1936; with A. Akhiezer and I. Pomeranchuk)
26. On the origin of stellar energy (*C. R. Acad. Sci. URSS*, **17**, 305, 1937; *Nature*, **141**, 333, 1938)
27. On the absorption of sound in solids (*Phys. Z. Sowjet.*, **11**, 18, 1937; with G. Rumer)
28. On the theory of phase transitions I (*Phys. Z. Sowjet.*, **11**, 26, 1937)
29. On the theory of phase transitions II (*Phys. Z. Sowjet.*, **11**, 545, 1937)
30. On the theory of superconductivity (*Phys. Z. Sowjet.*, **11**, 129, 1937)
31. On the statistical theory of nuclei (*Phys. Z. Sowjet.*, **11**, 556, 1937)
32. X-ray scattering by crystals in the neighbourhood of the Curie point (*Phys. Z. Sowjet.*, **12**, 123, 1937)
33. The scattering of X-rays by crystals with variable lamellar structure (*Phys. Z. Sowjet.*, **12**, 579, 1937)
34. Production of showers by heavy particles (*Nature*, **140**, 682, 1937; with G. Rumer)

35. Stability of neon and carbon with respect to  $\alpha$ -particle disintegration (*Phys. Rev.*, **52**, 1251, 1937)
36. The cascade theory of electronic showers (*Proc. Roy. Soc.*, **A166**, 213, 1938; with G. Rumer)
37. On the de Haas–van Alphen effect (*Proc. Roy. Soc.*, **A170**, 363, 1939)
38. On the polarisation of electrons by scattering (*Phys. Rev.*, **57**, 548, 1940)
39. On the ‘radius’ of the elementary particles (*J. Phys. USSR*, **2**, 485, 1940; *Phys. Rev.*, **58**, 1006, 1940)
40. On the scattering of mesotrons by ‘nuclear forces’ (*J. Phys. USSR*, **2**, 483, 1940)
41. The angular distribution of the shower particles (*J. Phys. USSR*, **3**, 237, 1940)
42. On the theory of secondary showers (*J. Phys. USSR*, **4**, 375, 1941)
43. On the scattering of light by mesotrons (*J. Phys. USSR*, **4**, 455, 1941; with J. Smorodinski)
44. The theory of superfluidity of helium II (*J. Phys. USSR*, **5**, 71, 1941)
45. A theory of the stability of strongly charged lyophobic sols and the coalescence of strongly charged particles in electrolytic solutions (*JETP*, **15**, 663, 1945; *Acta Phys.-chim. URSS*, **14**, 633, 1941; with B. Deryagin)
46. Dragging of a liquid by a moving plate (*Acta Phys.-chim. URSS*, **17**, 42, 1942; with B. Levich)
47. On the theory of the intermediate state of superconductors (*J. Phys. USSR*, **7**, 99, 1943)
48. On the relation between the liquid and the gaseous states of metals (*Acta Phys.-chim. URSS*, **18**, 194, 1943; with J. Zeldovich)
49. A new exact solution of the Navier–Stokes equations (*C. R. Acad. Sci. URSS*, **43**, 286, 1944)
50. On the problem of turbulence (*C. R. Acad. Sci. URSS*, **44**, 311, 1944)
51. On the hydrodynamics of helium II (*J. Phys. USSR*, **8**, 1, 1944)
52. On the theory of slow combustion (*Acta Phys.-chim. URSS*, **19**, 77, 1944)
53. On the theory of scattering of protons by protons (*J. Phys. USSR*, **8**, 154, 1944; with J. Smorodinsky)
54. On the energy loss of fast particles by ionisation (*J. Phys. USSR*, **8**, 201, 1944)
55. On a study of the detonation of condensed explosives (*C. R. Acad. Sci. URSS*, **46**, 362, 1945; with K. P. Staniukovich)
56. The determination of the flow velocity of the detonation products of some gaseous mixtures (*C. R. Acad. Sci. URSS*, **47**, 199, 1945; with K. P. Staniukovich)
57. Determination of the flow velocity of the detonation products of condensed explosives (*C. R. Acad. Sci. URSS*, **47**, 271, 1945; with K. P. Staniukovich)
58. On shock waves at large distances from the place of their origin (*J. Phys. USSR*, **9**, 496, 1945)
59. On the vibrations of the electronic plasma (*J. Phys. USSR*, **10**, 25, 1946)
60. On the thermodynamics of photoluminescence (*J. Phys. USSR*, **10**, 503, 1946)
61. On the theory of superfluidity of helium II (*J. Phys. USSR*, **11**, 91, 1947)
62. On the motion of foreign particles in helium II (*Dokl. Akad. Nauk SSSR*, **59**, 669, 1948; with I. Pomeranchuk)
63. On the angular momentum of a system of two photons (*Dokl. Akad. Nauk SSSR*, **60**, 207, 1948)
64. On the theory of superfluidity (*Phys. Rev.*, **75**, 884, 1949)

65. The effective mass of the polaron (*JETP\**, **18**, 419, 1948; with S. I. Pekar)
66. On the theory of energy transfer during collisions III (*JETP*, **18**, 750, 1948; with E. Lifshitz)
67. The theory of the viscosity of helium II: I. Collisions of elementary excitations in helium II (*JETP*, **19**, 637, 1949; with I. M. Khalatnikov)
68. The theory of the viscosity of helium II. II. Calculation of the viscosity coefficient (*JETP*, **19**, 709, 1949; with I. M. Khalatnikov)
69. On the electron-positron interaction (*JETP*, **19**, 673, 1949; with V. B. Beretskii)
70. The equilibrium form of crystals (A. F. Ioffe Festschrift, Moscow 1950, p. 44)
71. On the theory of superconductivity (*JETP*, **20**, 1064, 1950; with V. L. Ginzburg)
72. On multiple production of particles during collisions of fast particles (*Izv. Akad. Nauk SSSR, Ser. fiz.*, **17**, 51, 1953)
73. The limits of applicability of the theory of Bremsstrahlung by electrons and of the creation of pairs at large energies (*Dokl. Akad. Nauk SSSR*, **92**, 535, 1953; with I. Pomeranchuk)
74. Electron-cascade processes at ultra-high energies (*Dokl. Akad. Nauk SSSR*, **92**, 735, 1953; with I. Pomeranchuk)
75. Emission of  $\gamma$ -quanta during the collision of fast  $\pi$ -mesons with nucleons (*JETP*, **24**, 505, 1953; with I. Pomeranchuk)
76. The removal of infinities in quantum electrodynamics (*Dokl. Akad. Nauk SSSR*, **95**, 497, 1954; with A. A. Abrikosov and I. M. Khalatnikov)
77. An asymptotic expression for the electron Green function in quantum electrodynamics (*Dokl. Akad. Nauk SSSR*, **95**, 773, 1954; with A. A. Abrikosov and I. M. Khalatnikov)
78. An asymptotic expression for the photon Green function in quantum electrodynamics (*Dokl. Akad. Nauk. SSSR*, **95**, 1177, 1954; with A. A. Abrikosov and I. M. Khalatnikov)
79. The electron mass in quantum electrodynamics (*Dokl. Akad. Nauk SSSR*, **96**, 261, 1954; with A. A. Abrikosov and I. M. Khalatnikov)
80. On the anomalous absorption of sound near a second-order phase transition point (*Dokl. Akad. Nauk SSSR*, **96**, 469, 1954; with I. M. Khalatnikov)
81. A study of flow singularities using the Euler-Tricomi equation (*Dokl. Akad. Nauk SSSR*, **96**, 725, 1954; with E. M. Lifshitz)
82. On the quantum theory of fields (*Niels Bohr and the Development of Physics*, Pergamon Press, Oxford 1955, p. 52)
83. On point interactions in quantum electrodynamics (*Dokl. Akad. Nauk SSSR*, **102**, 489, 1955; with I. Pomeranchuk)
84. The gauge transformation of the Green function for charged particles (*Soviet Phys.-JETP*, **2**, 69, 1956; with I. M. Khalatnikov)
85. A hydrodynamic theory of multiple formation of particles (*Nuovo Cim. Suppl.*, **3**, 15, 1956; with S. Z. Belen'kii)
86. On the quantum theory of fields (*Nuovo Cim. Suppl.*, **3**, 80, 1956; with A. A. Abrikosov and I. Khalatnikov [= Khalatnikov])
87. The theory of a Fermi liquid (*Soviet Phys.-JETP*, **3**, 920, 1957)
88. Oscillations in a Fermi liquid (*Soviet Phys.-JETP*, **5**, 101, 1957)

\* Zhurnal éksperimental'noi i teoreticheskoi fiziki.

89. Conservation laws in weak interactions (*Soviet Phys.-JETP* **5**, 336, 1957)
90. Possible properties of the neutrino spin (*Soviet Phys.-JETP*, **5**, 337, 1957)
91. Hydrodynamic fluctuations (*Soviet Phys.-JETP*, **5**, 512, 1957; with E. M. Lifshitz)
92. The properties of the Green function for particles in statistics (*Soviet Phys.-JETP*, **7**, 182, 1958)
93. On the theory of the Fermi liquid (*Soviet Phys.-JETP*, **8**, 70, 1959)
94. Possibility of formulation of a theory of strongly interacting fermions (*Phys. Rev.*, **111**, 321, 1958; with A. A. Abrikosov, A. D. Galanin, L. P. Gorkov, I. Ya. Pomeranchuk, and K. A. Ter-Martirosyan)
95. Numerical methods of integrating differential equations by the mesh method (*Proc. All Soviet Math. Conf. (Moscow 1956)* Moscow 1958, p. 92; with N. N. Meiman and I. M. Khalatnikov)
96. On analytical properties of vertex parts in quantum field theory (*Nucl. Phys.*, **13**, 181, 1959; *Soviet Phys.-JETP*, **10**, 45, 1960)
97. Small binding energies in quantum field theory (*Soviet Phys.-JETP*, **12**, 1294, 1961)
98. Fundamental problems (*Theoretical Physics in the Twentieth Century, a Memorial Volume to Wolfgang Pauli*, Interscience, New York 1960, p. 245).