themselves. The fact that pili often are virulence and colonization factors in Gram-negative organisms supports our conclusion that many DNA transfer events mediated by filamentous phages may occur on host mucosal surfaces.

Our results also emphasize the co-evolution of genetic elements mediating the transfer of virulence genes with the pathogenic bacterial species they infect. Thus, a virulence factor (TCP) is the receptor for a bacteriophage encoding another virulence factor (CT), both of which are coordinately regulated by the same virulence regulatory gene (*toxR*). In this case, the natural habitat of both phage and pathogen is the gastrointestinal tract. It is apparent that this host compartment provides the necessary environmental signals required for the expression of essential gene products mediating interactions between all three participants, namely, bacterium, phage, and mammalian host.

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Minimal Energy Requirements in Communication

Rolf Landauer

The literature describing the energy needs for a communications channel has been dominated by analyses of linear electromagnetic transmission, often without awareness that this is a special case. This case leads to the conclusion that an amount of energy equal to $kT\ln 2$, where kT is the thermal noise per unit bandwidth, is needed to transmit a bit, and more if quantized channels are used with photon energies $h\nu > kT$. Alternative communication methods are proposed to show that there is no unavoidable minimal energy requirement per transmitted bit. These methods are invoked as part of an analysis of ultimate limits and not as practical procedures.

Information is inevitably tied to a physical representation, such as a mark on a paper, a hole in a punched card, an electron spin pointing up or down, or a charge present or absent on a capacitor. This representation leads us to ask whether the laws of physics restrict the handling of information and in particular whether there are minimal energy dissipation requirements associated with information handling. The subject has three distinct but interrelated branches dealing, respectively, with the measurement process, the communications channel, and computation. Concern with the measurement process can be dated back to Maxwell's demon (*1*). In the development of that subject, the notion that information is physical was introduced by Szilard (*2*), although it was not widely accepted for many decades. Concern with the communications channel became a subject of intense concern after Shannon's work (*3*). It is the newest of the three branches, computation, that has caused us to reexamine the perceived wisdom in the two earlier areas (*1*, *4–8*). It was pointed out long ago (*9*) that the steps in the computational process that inevitably demand an energy consumption with a known and specifiable lower bound are those that discard information. It was also understood long ago (*9*) that operations that do throw away information, such as the logical AND and the logical OR, can be imbedded in larger operations that perform a logical 1:1 mapping and do not discard information. Nevertheless, a real understanding of what is now called reversible computation came from the work of Bennett (*10*, *11*), who showed that computation can always be conducted through a series of logical 1:1 mappings. Bennett furthermore showed that physical implementations exist that allow this mapping to be utilized to perform computation with arbitrarily little dissipation per step, if done sufficiently slowly. Bennett's discussion envisioned classical machinery with viscous frictional forces proportional to the velocity of motion. It is these forces that can be made as small as desired, through slow computation.

The notion of logically reversible operations, which do not discard information, provides the unifying thread between the three fields of measurement, communications, and computation. In the measurement process, transfer of information from the system to be measured to the meter does not require any minimal and unavoidable dissi-

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pation (9). The dissipation comes later, if we reset the meter by itself for subsequent reuse and discard the information in it (1, 11).

Communication can also be done with logically and physically reversible operations. As a result, there is no unavoidable lower limit on the energy dissipation per transmitted bit. The term reversible is used in the physical chemist's sense in the discussion of thermodynamic cycles, assuming frictional forces that can be made as small as we wish. I do not use it in the physicist's sense, requiring a total absence of frictional forces.

Our concern is with theoretical limits, not necessarily related to serious technological promise. Our proposed methods for communicating with very little energy expenditure are not put forth as practical candidates. In that connection, however, it is appropriate to recall that reversible computers were originally conceptual proposals with an unrealistic flavor. In recent years, however, closely related schemes have been suggested to save power in real CMOS (complementary metal-oxide semiconductor) logic (12).

The prevailing view about the minimal energy required for bit transmission is based on Shannon's work (3). Shannon provided general expressions that relate channel capacity to the relative probabilities for various received messages, as a function of the transmitted message. These probabilities define the likelihood of erroneous transmission. Shannon's general expressions are not in question, but problems arise when specific models are used for the error probabilities and their generality is overinterpreted. Shannon understood these limitations; later workers have been less cautious. Quoting from Shannon, "An important special case occurs when the noise is added to the signal and is independent of it (in the probability sense)."

In that case, Shannon finds the well-known result

$$C = W \log_2 \frac{P+N}{N} \quad (1)$$

where C is the channel capacity, W is the bandwidth, P is the average received power, and N is the average noise power. Thermal noise, for a classical transmission line with additive equilibrium noise, is given by $N = kTW$, where k is Boltzmann's constant and T is temperature. Equation 1 yields a maximum for C/P, at small P, given by

$$C = P/kT \ln 2 \quad (2)$$

This equation suggests that at least $kT\ln 2$ energy per transmitted bit is required, although it is not clear that this energy has to be dissipated. This result views noise as an error that maps the intended signal into other nearby signals. However, the linearity of the system, reflected in the addition of P and N in Eq. 1, does not lead to the best way to handle digital signals. These signals are best handled in bistable system in which 0 and 1 are states of local stability. Small noise pulses will cause a temporary deviation from the ideal desired 0 or 1 state, followed by a restoration back to the state of local stability.

If the frequencies ω in the signal are such that the photon energies $\hbar\omega$ ($\hbar$ is Planck's constant h divided by 2π) are comparable to or larger than kT, then the quantization of the signal becomes important. The literature on quantum channel capacity [summarized in (13)], just like that dealing with the classical case, is focused on the linear electromagnetic transmission channel. The work of Caves and Drummond (13) represents a remarkable exception in this long history. They emphasize the limitations that lead to their results.

The rest of the literature is variable in the clarity with which it describes the limits of its results. Brillouin (14), for example, believes he is giving a general law of nature when he states ". . . one bit of information can never be obtained for less than $k\ln 2$ in negentropy costs." Levitin (15) reaffirmed Brillouin's conclusion in recent years but has somewhat more careful language in describing the range of applicability. Bremermann and Marko (16) are two further examples, of many, who give the $kT\ln 2$ result (and its quantum extensions) very broad interpretation.

Classical channel. The fact that there is no minimal energy requirement of $kT\ln 2$ per transmitted bit should be apparent from several different perspectives. First of all, computation can be carried out with arbitrarily little dissipation (4, 10, 11). Within a computer, bits are transmitted; therefore, there can be no minimal unavoidable energy penalty for the motion of bits within the computer. A second argument (17): The difference between memory and communication is only a matter of perspective. What is viewed as the rest frame? If there is no unavoidable lower bound to the energy cost of storing a bit, then there should not be any for communication. This statement is closely allied to my own version: we can communicate by shipping the memory (18). These general arguments may not satisfy the reader who asks about connecting into and out of the memory, so I describe a more specific apparatus. First, a classical link discussed in earlier work (5, 19) demonstrated that communication with low energy dissipation does not require physical long-range transport of matter. Therefore, a physical memory structure does not actually have to be shipped. The basic element is a time-modulated potential containing a particle (Fig. 1). This scheme is an adaptation of a proposal for the use of subharmonic parametric excitation to carry out logic (20), which was subsequently elaborated by many investigators (21). The potential V in Fig. 1 is assumed to be heavily damped as a result of viscous frictional forces proportional to particle velocity. The potential will change slowly with time, so as to minimize frictional forces on the moving particle. The mere time dependence of the potential, in the absence of motion of the particle, is not a source of energy dissipation. The time-dependent forces can be generated, for example, by moving controlled charges toward and away from a charged information-bearing particle in the well. Coupling particles in different wells with springs and choosing different phases for the time modulation of different wells can be used to accomplish all the logic functions of a computer (5, 22), but that will not be needed here. The basic unit of interest to us couples two wells (Fig. 2). The unit on the left, with the particle locked securely in its left or right pocket, exerts a biasing force on the particle on the right, in a well about to become bistable. Thus, the particle on the right will likely end up in the new, right-hand pocket.

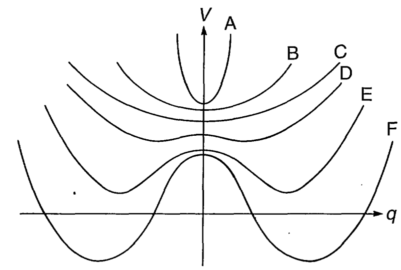

Fig. 1. Potential V as a function of particle position q changing with time. The potential starts with a single minimum (curve A), ends up in a deeply bistable state (curve F), and then returns to the single minimum. The relative vertical displacement of curves is selected for clarity and has no significance.

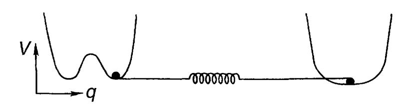

Fig. 2. Particle in deeply bistable potential well on the left, coupled to a particle on the right. The particle on the right is in a well about to undergo a transition to a bistable state. The spring is symbolic; it is the relative displacements from the center of the respective potentials that are coupled.

Consider in more detail the particle in Fig. 1, subject to a biasing force pushing it to the right. Curve C in Fig. 1, at the onset of bifurcation, is relatively flat. The biasing force displaces the minimum to the right, and as the bifurcation proceeds, this minimum evolves into a deeper pocket, favored over the left pocket, which would start to form at a later stage of the bifurcation. When this metastable left pocket starts to form, the Boltzmann factor, $\exp(-V/kT)$, gives a nonvanishing probability for trapping the particle in this undesired left pocket. Without following the exact kinetics, it is still clear that the particle has some chance of eventually ending up in the undesired left pocket, after we have reached its maximum depth (curve F, Fig. 1). This error probability, however, can be made as small as desired by suitable design choices (*5*, *22*). Such minimization requires a large enough biasing force and a sufficiently slow change of V with time to allow the particle trapped in the incorrect metastable state to escape during the initial stages of the bifurcation.

After the transfer of information from the left well of Fig. 2 to the right one has been accomplished, the left well can then be restored to its monostable state to receive a new bit. In that connection, another precaution must be described. Consider Fig. 3, which shows the transferred old bit in the right well. The potential in the middle well, where the bit originated, has started to return to the monostable state, with a lowered barrier between its two pockets. Assume that immediately to its left there is a new bit to be transferred into the central well and that this new bit is of opposite polarity to the old one. The biasing forces exerted on the central well from its two neighbors cancel (Fig. 3). As the barrier in the central well is lowered, the particle has some chance of thermally activated escape into the other pocket of the well. The ability to control the escape velocity by use of a sufficiently slow potential modulation rate is lost; the escape velocity is controlled by the barrier height. More detailed considerations show that the energy dissipation is that of a particle released into a volume twice its original size, $kT\ln 2$. To eliminate such undesired losses, the far left well in Fig. 3 must be in its monostable state until the central well has been restored to its monostable state. Only after that can the left well receive a new bit. To prevent this error, as the left well (Fig. 2) is restored to its monostable state, the well on its left is in turn also brought to a monostable well, acting as a buffer. The bias force on the well being restored therefore comes primarily from its right neighbor and favors the occupied pocket. Only after the central well (Fig. 3) is restored to the monostable state can its left neighbor become bistable, allowing it to receive a new bit. Passage of a bit along such a chain clearly constitutes a communication link. It is not passive like a transmission line or optical fiber: Active machinery is required for the propagation. In order to minimize friction, the wells have to be modulated slowly; therefore, the information velocity is low. This restriction does not imply a low information rate; the linear density of particles can be as high as desired. It can be arbitrarily high, as long as we do not inquire as to how such time-dependent wells are actually constructed.

The time-dependent potentials and the springs do not have to be perfect; for example, the potential can be slightly asymmetric. The parameters involved can be chosen to let the machinery function as intended, as long as the deviations fall within designated small error bounds (*5*, *22*).

Quantum mechanical channel. We now turn to a description of a quantum mechanical procedure, enlarging on an earlier suggestion (*23*). The procedure is quantum mechanical but assumes that the bits are classical and each is in a clearly defined 0 or 1 state. This formulation ignores the recently fashionable possibility of utilizing quantum mechanically coherent superpositions of 0 and 1 (*24*, *25*). Consideration of this extra freedom could possibly lead to more optimistic results and certainly will not give more restrictive results. Even without that additional benefit, the conventional limits for quantized linear electromagnetic channels do not apply when we consider alternative communication possibilities. Caves and Drummond (*13*) point out that

$$C = \frac{\pi}{\ln 2}\sqrt{2P/3\hbar} \qquad (3)$$

for the linear boson channel, where C is the bit transmission rate and P is the power. Thus, $P/C \sim C$. The minimal energy per bit is proportional to the bit rate, or roughly the quantum size. The following example will show, instead, that no matter how high C is, no unavoidable minimal energy expenditure is needed. It is optimistically assumed, as in other quantum mechanical analyses in this field, that Hamiltonian and frictionless systems are available.

Again using symmetrical bistable wells, take a particle in the left pocket as a 0 and a particle in the right pocket as a 1. If the barrier between the pockets is high enough, the information can be stable against both tunneling and thermal agitation for a long time. Basically, the proposal is to ship the wells with their bits, which means that the source of the well potential is in motion. For example, if the information-bearing particle is charged and externally controlled charges produce the well field, then these external charges are in motion.

A deeply bistable symmetrical well has an even wave function for its ground state and an odd wave function for its first excited state (Fig. 4). A particle in the left pocket results from a superposition of the two exhibited wave functions, giving almost complete cancellation in the right pocket. The particle is not in the ground state, but we can make the elevation in energy above the ground state, proportional to the energy splitting of the two states, as small as we wish by making the barrier sufficiently high. (The two states become degenerate if the barrier is impenetrable.) Thus, information can be stored with as little energy elevation above the ground state as desired, and even this elevation is not necessarily an energy expenditure. After all, we do not need or even want a relaxation to the ground state. As is known from theories of totally quantum mechanical computation, the computation proceeds in excited states of the system, but that does not in itself require dissipation. Furthermore, if we do not ask how we can actually construct these wells, but stick to elementary quantum mechan-

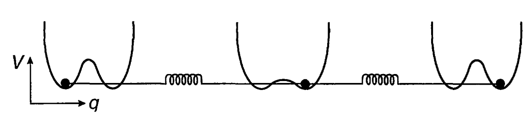

Fig. 3. Need to control influence of third well. Information in the central well has been transferred to right well, and the central well is returning to the monostable state. If the well on the far left is in a different information state, no bias will be exerted on the central well, allowing dissipative barrier crossing as the barrier disappears.

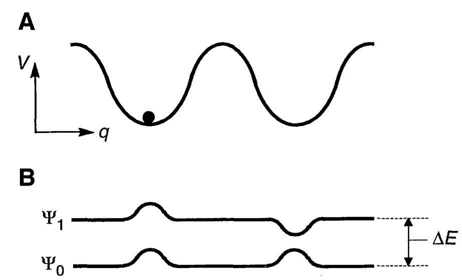

Fig. 4. (**A**) The system is in the left well, in a superposition of the symmetric ground state Ψ_0 and the antisymmetric first excited state Ψ_1. (**B**) The wave functions for the two lowest states, separated by energy ΔE.

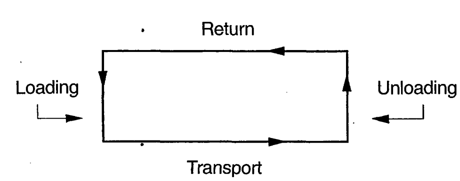

Fig. 5. Schematic characterization of communications link. Information is sent along bottom link. Standardized bits (say, all 0) are returned in the top link. On each side, the wells slow down (or even stop) for loading and unloading.

ics and statistical mechanics, then the wells can be as compact as desired. Thus, there is no restriction on the information density. Elementary quantum mechanics, per se, poses no restriction on bit density or on energy elevation in bit storage.

The communication link based on this theory resembles a ski lift (Fig. 5), but instead of chairs, it has moving bistable wells. There is a loading area where information is inserted into a well. As in a modern ski lift, the well is moved slowly (or even is stationary) during loading, and then is accelerated for transport. It slows down again, or halts, during unloading. Then it is left in a standardized state, say 0, and returned at higher speed to the loading area. To minimize the effects of acceleration and deceleration, narrow well pockets are used; thus, the states within a single pocket are separated by a large energy and are not very polarizable.

In the loading step (Fig. 6B), the incoming bit, which can be a 0 or 1, arrives in well α. The moving well on the communications link, well β, has just arrived (Fig. 5) in the 0 state. The state of α is copied into β , and later α is restored to the 0 state (Fig. 6C). It is the copying step (Fig. 6B) that requires the bit to be clearly in a 0 or 1 state rather than a quantum mechanical superposition; in the latter case, copying is impossible (*26*).

The copying is executed by a temporary modulation of the barrier in well β, which permits tunneling from the 0 to the 1 pocket if and only if α is in the 1 state. If the barrier in β is lowered so that tunneling can occur with appreciable probability, then the particle will oscillate back and forth between the two pockets with frequency $\Delta E/h$, where ΔE is the splitting between the two lowest states of the bistable well. If we allow the particle to tunnel for exactly half of a cycle, it will have moved to the other pocket; after that, the barrier is raised again. The potential during the copying step is a function of the position of both particles. The potential before any barrier lowering is symmetrical with respect to interchange of α and β (Fig. 7A). In that case, there is a barrier V for one particle tunneling and $2V$ for both tunneling simultaneously. These barriers are presumed high enough to prevent tunneling. A lowered barrier for β permits the desired copying if α is in the 1 state (Fig. 7B). How might the barrier lowering be accomplished? The particle in α can be taken to have an attractive potential for the particle in β. (A similar process can be used for a repulsive potential.) The two wells are not oriented as in Fig. 6, with the two directions for barrier crossing aligned with each other; instead, the direction of barrier crossing in α is perpendicular to that in β (Fig. 8). The 1 valley in α is placed closest to the β barrier, the 0 valley farther away. Thus, if α is in the 1 state, then the β barrier is lowered. Meanwhile, the barrier in α is kept high enough to prevent unintended tunneling; it may even have to be raised temporarily. As the α well approaches the β well, the force needed by the deterministic (that is, heavy and essentially classical) apparatus controlling the motion depends on the information state of α, but such a force results in energy demands that are compensated in later parts of the cycle: it is not energy dissipation. Note also that as the α particle approaches its target, tunneling will commence before α reaches its final position. This early rise in tunneling probability must be taken into account in the total time allowed for tunneling.

To reset the α bit to 0, we can use a similar procedure (Fig. 7C). Both the α and β wells are turned 90° from that shown in Fig. 8, so that the two controlled α pockets are equal distances from the controlling particle in β. Turning the wells is a process similar to the motion of the wells along the link; the sources of the well field are turned in a process that does not depend on the information content of the wells. The wells must be turned so that the controlling β particle is closer or farther from the α barrier, depending on the state of β. Then, the tunneling in the α well is allowed to proceed, if the barrier has been lowered. The end result, involving two sequentially controlled tunneling events, is an exchange between the states of α and β. This exchange is a unitary operation, and if we permit ourselves the generosity characteristic of much of the literature on quantum computation [for example, (*27*)], which requires the specification of only a unitary time-evolution operator and not its physical embodiment, then all of the details related to the time-dependent wells with controlled tunneling can be skipped: We avoid the intermediate stage in which the content of α is copied to β. Therefore, the information state can be a quantum mechanical superposition of 0 and 1.

So that only the desired particles interact, rather than those further away, short-range forces are required, and we do not try to explain how this can be achieved. Coulomb forces screened by metallic electrodes are a possibility, but a complete analysis and design for that does not exist. Another possibility replaces the bistable wells with bistable quadrupole moments in arrays of quantum dots (*28*). The internal barriers to electron transfer in such arrays must be controlled and changed with time.

At the receiving end, a similar procedure can be used to unload and restore the moving bit to the 0 state. The communications link cannot be expected to act perfectly. There are errors due to the fact that the machinery is likely to be imperfect. The tunneling barrier may not be exactly as intended, the time allowed for tunneling transfer may be too short or too long, or the distance between two interacting particles invoked for controlled tunneling (Fig. 7)

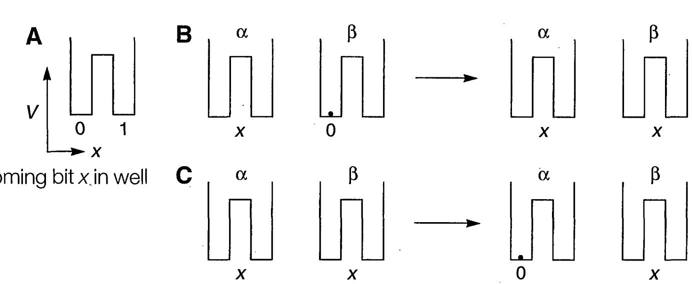

Fig. 6. Well states during information transfer cycle. (**A**) Bistable well used to hold information. (**B**) Incoming bit x (in state 0 or 1) in well α is copied into well β. (**C**) Incoming bit x in well α is reset to the 0 state.

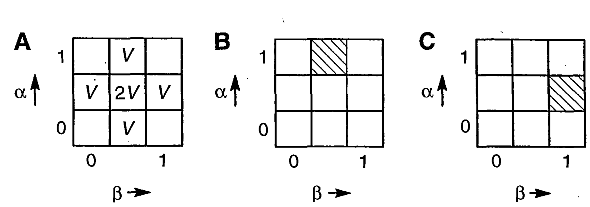

Fig. 7. (**A**) Barrier as a function of the positions of the two particles involved before time modulation begins. (**B**) Shaded region has reduced barrier during copying of α to β. The barrier in the receiving β well is lowered if and only if the particle in the controlling α well is in the 1 state. (**C**) Shaded region has reduced barrier during resetting for particle in well α.

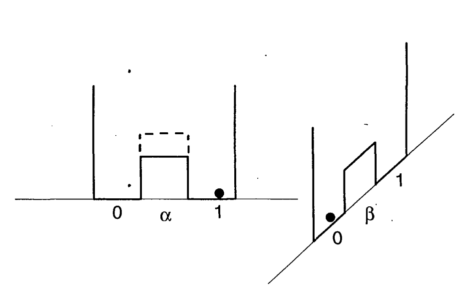

Fig. 8. Controlling α well is perpendicular to controlled β well. The position of α controls the barrier of β. The α barrier can be increased temporarily to lock the α particle firmly in its position.

may be incorrect. Even if the components act exactly as intended, there are still residual errors. For example, tunneling may occur unintentionally through the higher barrier. Another problem arises from the supposed cancellation of wave functions in realizing a 0 or 1 state (Fig. 4A). The two superposed eigenstates (Fig. 4B) have slightly different energies. Their exact wave functions within a pocket are slightly different; they cannot cancel exactly and cannot leave a totally unoccupied pocket. This difficulty can be minimized by using very narrow potential pockets, in which the next higher state within the pocket, having a node for the wave function within that pocket, is at a much higher energy. In the extreme limit, the behavior approaches that of a true two-level system, such as an electron spin, where no details beyond the spin-up and spin-down states exist.

Errors matter in two ways. First, an incorrect bit can be transmitted. This is not a serious problem, if the probability of error is small: a small error rate requires only a small amount of redundancy to regain error free transmission (*3*). The decoding that restores the original message is not dissipationless; after all, we are throwing away information about the error (*4*). However, the minimal required dissipation is proportional to the error rate. Additionally, if errors arise, then the slow accumulation of erroneous bits in the return link (Fig. 5) of the transport machinery must be prevented. The supposed 0-state bits returning to the loading stage (Fig. 5) may have become 1's. We can occasionally reset these bits on their return to the intended 0 state. One possible way to do this takes its clue from Lloyd's proposal (*25*) for error correction in quantum computation. The bit is biased so as to favor the 0 state, and then the barrier is slowly reduced. Eventually, a radiative decay from the metastable 1 state to the lower lying 0 state occurs. It can be assisted by coupling to oscillators (for example, a resonant cavity) at the emission frequency. The decay is a dissipative event, but once again, the dissipation is proportional to the error rate and can be far less than indicated in Eq. 3. Incidentally, the methods of Figs. 6 through 8 can be adapted to carry out computer logic in arrangements where tunneling is controlled by more than one input well.

Overview. The two proposals discussed above use active channels with machinery all along the length of the channel. Such a setup makes these schemes unappealing as serious technological candidates. The most obvious alternative to an active link, for the classical case, is the use of particles with sufficiently well-defined speed and direction, as in the well-known colliding billiard ball computer (*29*). This scheme assumes that friction and noise can be completely eliminated. Furthermore, the chaotic nature of billiard ball collisions makes it extremely sensitive to flaws in the machinery. Using billiard ball motion for a communications link is most easily done within a framework where the net rate of billiard ball transfer does not depend on the information content of the link. For example, 0 is denoted by a ball followed by an empty slot, and a 1 is represented by an empty slot followed by a ball. This scheme permits use of a return link (Fig. 5). Alternatively, if we are optimistic about the lifetime of a photon, then conceivably there are possible inventions that make use of the polarization or timing of single photons. Or perhaps there could be nonlinear optical links.

Even if the limits discussed here are practically unachievable, it is still important to understand such limits. For example, I do not believe that a genuinely useful form of quantum parallelism (*24*) will be achieved; nevertheless, computer scientists concerned with the minimal number of steps required in the execution of an algorithm must take its possibility into account.

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