Resource Letter MM-1: Magnetic monopoles

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Citation: American Journal of Physics **58**, 429 (1990); doi: 10.1119/1.16474
View online: https://doi.org/10.1119/1.16474
View Table of Contents: https://aapt.scitation.org/toc/ajp/58/5
Published by the American Association of Physics Teachers

RESOURCE LETTER

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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course content in specified fields. No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455.

Resource Letter MM-1: Magnetic monopoles

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(Received 20 June 1989; accepted for publication 30 December 1989)

This Resource Letter provides a guide to the literature on magnetic monopoles. The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of somewhat more specialized nature. The letter A indicates more specialized or advanced material. An asterisk (*) indicates those articles included in the accompanying Reprint Book.

I. INTRODUCTION

While the concept of elementary particles goes back to classical writings of Democritus and Lucretius, perhaps the first indication of an explicit property that might characterize the interactions of such particles was the notion of magnetic polarity. Early writings on magnetism did not refer to the possibility of microscopic poles, but from the modern point of view they give a basis for calling the monopole the oldest example of a hypothetical particle. Despite determined and ingenious experimental searches spanning many years, the existence of isolated magnetic poles has not been established. Instead a collection of ever more stringent limits on monopole flux and production rate have been produced.

The lack of observational confirmation notwithstanding, magnetic poles continue to challenge experimenters to find better ways to seek them, and theorists to test their consistency with increasingly elaborate pictures of the microscopic behavior of Nature. Such efforts have led to surprises and have illuminated some of the deepest and most puzzling aspects of these pictures.

Before proceeding, we review some basic facts and preview some results to be presented later on. Like an electric charge, a magnetic monopole may be described either by the way it responds to external electromagnetic fields or by the form of its own field. On the one hand it experiences a force,

$$\mathbf{F} = g(\mathbf{B} + \mathbf{E}\times\mathbf{v}/c),$$

where g is the pole strength and **B** and **E** are the magnetic and electric fields, respectively. On the other hand, a pole at rest is the source of a radial magnetic field,

$$\mathbf{B} = g\mathbf{r}/r^3.$$

A system of one charge and one pole is an intrinsic gyroscope, whose charge–pole axis responds to a torque by precessing about, rather than inclining itself toward, the axis of attraction. This property of monopole dynamics underlies the constraint that quantum theory imposes on magnetic pole strength.

The rarity of magnetic poles, if they exist at all, accounts for the difference between the electric and the magnetic properties of matter. Electric forces, which hold matter together, are much stronger than magnetic forces but tend to be screened. The superficially paradoxical consequence is that strong magnetic fields can be produced more easily than strong electric fields over macroscopic distances. For the same reason most matter is quite transparent to, and little disturbed by, magnetic fields. Thus a technique like nuclear magnetic resonance imaging may be used with negligible biological damage.

A pole responds to **H** rather than **B**, since otherwise it could extract an unlimited amount of energy by making repeated passes through a ferromagnet in its ground state. Therefore, poles will be attracted to the boundary nearest the opposite-sign pole of such a magnet.

Quantum mechanics constrains the minimum allowed pole strength to be comparable to that of the electric charge of a thulium nucleus, $Z \sim 69$, so a pole moving rapidly enough through matter would ionize strongly. This ionization would have a weak velocity dependence, in striking contrast to that of an equivalent electric charge with its Bragg peak just above the ionization threshold velocity.

A strong magnetic field exerts an enormous force on a pole's large magnetic charge, sufficient to rip it out of almost any material. A pole moving through a closed conducting circuit would create an electromotive force rising and falling without change of sign. For a superconducting circuit, the effect can be large enough to detect the passage of a solitary pole.

Since the minimum pole strength is so large, the pole must be geometrically much bigger than its Compton wavelength, and so in some approximation may be described as a classical object. However, for electrodynamics to be consistent with quantum theory the pole's magnetic field must be expressible as the curl of a vector potential, making a spread-out magnetic charge impossible. Thus the radius of a monopole sets a length scale below which, and by the uncertainty principle a mass scale above which, electrodynamics has to be modified.

The standard model of electroweak interactions also forbids spread-out monopoles, since the magnetic field in that theory is the sum of two pieces, one of which again is the curl of a vector potential. Verification of the electroweak model at energies above 100 GeV implies that the monopole mass must be at least 10 TeV, too large to be produced at any existing accelerator.

A number of classical field theories modify electrodynamics so as to produce monopoles. Perhaps the simplest is the combination of general relativity with electrodynamics, where the resulting monopole has a mass of a tenth of a milligram and is also a black hole. Such a massive, tiny object could easily be dislodged from any material by heating or jostling. Since more massive poles should decay to less massive ones, the mass of the lightest, hence stable, monopole could be anywhere in the range from 10^{17} down to 10 TeV.

Since an electrically charged Dirac particle in the lowest angular momentum state about a monopole has a wavefunction that diverges as $1/r$, new phenomena become possible, including deeply bound states, as well as catalysis of transitions in which nucleons decay.

To summarize, monopoles should exist, be far more massive than any previously observed elementary particle, have unique properties providing a host of distinctive measurements that can be used for their detection, and be exceedingly rare, perhaps produced in the very early expansion of our hot, young Universe.

Magnetic poles have four aspects whose study began at different times but all continue in the present. The first, the phenomenological or experimental stream, emerged in modern form with the work of Petrus Peregrinus more than 700 years ago. He observed that magnets have north and south poles which cutting fails to isolate, instead producing only smaller magnets, each with its own north and south pole. He also noted that magnets tend to line up so that poles conjugate, unlike attracting.

The second stream began with Oersted's discovery in 1820 that electric currents induce the deflection of magnets. This led Ampère to declare that all magnetism is due to circulating electricity, so that isolated poles need never occur. From that point on, thought experiments on the behavior of poles in various circumstances were used only to clarify concepts that could then be tested by experiment. Such studies are found already in Faraday's early work on electromagnetism.

In 1931, Dirac began the third stream with his discovery that the consistency of quantum mechanics requires the quantization, in integer multiples of a smallest unit, of the product of any electric charge with the strength of any isolated pole. This at once gave a rationale for the observed quantization of electric charge, and specified the minimum nonzero value of pole strength. Dirac's work led directly to experimental searches for poles in cosmic rays and at accelerators, as well as theoretical studies of old classical, as well as new quantum, issues connected with monopoles.

Already some time before Dirac's work it was found that a solution of the classical Einstein–Maxwell equations could be interpreted as a stable, electrically charged black hole. The duality-rotation invariance of the equations allows this object to bear magnetic instead of electric charge. This first model for internal monopole structure is still the one best grounded in established physics. It has since been realized that Dirac's quantization condition requires internal structure, models for which have been found in a variety of classical field theories.

In the following four sections we describe some of the important and instructive works in each of the four streams: phenomenological or experimental, quantum, structural, and didactic or pedagogical. There are now well over 3000 publications related to monopoles, so our selection is inclusive if not exhaustive only until the 1960s, when growth began in earnest. In the period since then we attempt to recognize works that have laid the foundations for major developments, adding recent citations so that the significant intermediate stages we have failed to include can be traced.

There are many advanced (A) and not enough elementary (E) and intermediate (I) articles among our citations, since especially the more recent developments have yet to receive the "honorable simplifications" that would make them accessible to the widest possible audience. We hope that our background remarks will help readers picture what lies behind the formalism, and that this collection will stimulate efforts to reveal that essence more fully.

II. EXPERIMENTS AND OBSERVATIONS

A. Phenomenological beginnings

*1. **"On the Magnet,"** Pierre de Maricourt, Letter to Siger de Foucaucourt (1269) in *The Letter of Petrus Peregrinus on the Magnet*, translated by Brother Arnold (McGraw, New York, 1904), Pt. I (E)

is an amazing work written while in the crusading King Charles of Anjou's siege party surrounding the Saracen town of Lucera, Italy. Peregrinus defines magnetic poles and observes that they are never seen in isolation. Somewhat spoiling the impression of modernity, he also includes in Pt. II recipes for perpetual motion machines!

2. **"Magnetism and Electricity,"** in J. Needham, *Science and Civilisation in China* (Cambridge U.P., Cambridge, 1962), Vol. 4, pp. 229–334 (E)

argues that much, if not all, of what Peregrinus says was known in China a couple of centuries earlier, as were details about compass needle deviation from geographical north–south lines. However, Peregrinus' approach puts things together in a systematic, "modern" way, and his description of a spherical lodestone that he names "terrella" is an eerie anticipation of Gilbert.

3. **"On the Loadstone and Magnetic Bodies,"** William Gilbert (1600), translated by P. F. Mottelay, in *Great Books of the Western World*, edited by R. M. Hutchins (Encyclopedia Britannica, Chicago, 1952), Vol. 28, pp. 3–12 (E)

is Gilbert's great work on magnetism and perhaps the first example of unification of scale in physics. He argues that the Earth itself is a magnet, differing only in size from a lodestone. This precedes by almost a century Newton's even more dramatic proposal for gravity, that the force between the Earth and bodies near its surface is the same one that guides planets in their orbits around the Sun.

*4. **"Law of Magnetic Force,"** C. A. Coulomb (1788), translated in *A Source Book in Physics*, edited by W. F. Magie (Harvard U.P., Cambridge, 1935), pp. 417–420 (E)

establishes for magnetic poles, using very long thin magnets to mimic the action of isolated poles, something already shown for electric charges: The force between them varies inversely as the square of the distance and is proportional to the product of pole strengths or charges as the case may be, repulsive for like and attractive for opposite polarities.

5. **"The Action of Currents on Magnets,"** H. C. Oersted (1820), translated by J. E. Kempe in Ref. 4, pp. 437–441 (E)

provides the first sign that electricity and magnetism are connected, leading directly to the work of Ampère.

B. Unified electrodynamics

*6. **"Electrodynamic Model of Magnetism,"** A. M. Ampère (1820), translated in Ref. 4, pp. 447–460 (E)

asserts that all magnetism is due to circuits of electricity, explaining at one stroke why magnets do not have isolated poles. After criticism by Faraday, Ampère comes to the position that an ordinary magnet contains only microscopic molecular currents, rather than a single band of current around the outside of the magnet. With this assertion he introduces the principle of magnetic ambiguity: From the outside it is impossible to tell whether a magnet is made of pole pairs or current loops. This principle has survived for a century and a half, during which time electrodynamics has evolved into the first gauge field theory.

7. **"Fictitious Magnetic Conduction-Current,"** in O. Heaviside, *Electromagnetic Theory* (Benn, London, 1893), Vol. 1, p. 25 (E)

strongly states the typical belief of the period, that magnetic currents do not exist.

8. **"On the Possible Existence of Magnetic Conductivity and Free Magnetism (in French),"** P. Curie, Sèances Soc. Phys. (Paris, 1894), pp. 76–77 (E)

suggests out of the blue that magnetic charge might exist, quoting a textbook by Vaschy as having already introduced magnetic currents. However, that text only used such currents didactically, explicitly rejecting their reality. Curie does not say how to look for these currents and charges, but merely suggests keeping an open mind on the subject. This seems to be the first post-Ampèrian proposal of isolated poles.

Experimental indications of monopoles were reported by Ehrenhaft for 20 years in some 60 papers containing increasingly specific claims. These claims were rebutted by a number of authors, who ascribed the indications of magnetic currents to the complexity of his experimental conditions, which included electric and magnetic fields as well as strong light beams all acting on multicomponent chemical solutions. Furthermore, it was troubling that Ehrenhaft's poles showed no sign of obeying the quantization condition.

9. **"Magnetophotophoresis and Electrophotophoresis (in German),"** F. Ehrenhaft, Phys. Z. **31**, 478–485 (1930) (I)

claims from the motion of aerosol-sized ferromagnetic particles the existence of magnetic monopoles.

10. **V. F. Mikhailov**, Phys. Lett. B **130**, 331–334 (1983) (I)

asserts confirmation of Ehrenhaft's result.

11. **"Why Does the Sun Sometimes Look Like a Magnetic Monopole?,"** J. M. Wilcox, Comments Astrophys. Space Phys. **4**, 141–147 (1972) (I)

reports that fitting magnetic field data from a solar probe requires a nonzero monopole contribution, as if the Sun were a repository of net north magnetic charge.

12. **"Search for Magnetic Monopoles in the Moon,"** K. H. Schatten, Phys. Rev. D **1**, 2245–2251 (1970) (I)

limits the net resident monopole content of the Moon using Explorer 35 magnetometer data.

C. Searches for monopoles with quantized charge

The first effort to look for the new kind of monopole came 20 years after Dirac introduced the quantization condition. The large ionization expected if the pole velocity were more than 1% of the speed of light was exploited in all the early searches, whether in cosmic rays or at accelerators.

*13. **"The Interaction of the Dirac Magnetic Monopole with Matter,"** W. V. R. Malkus, Phys. Rev. **83**, 899–905 (1951) (I)

makes the first theoretical analysis of monopole interactions with matter and describes the first experiment seeking to detect Dirac monopoles. A solenoid was used to concentrate the flux onto a photographic emulsion.

*14. **"Search for Dirac Monopoles,"** H. Bradner and W. M. Isbell, Phys. Rev. **114**, 603–604 (1959) (I)

present the first of the accelerator searches that now have become an obligatory commissioning exercise at each new machine.

The track etch technique has been widely used in monopole searches. Here, a dielectric develops a radiation-damaged region along a particle path, which is made visible by acid etching.

15. **"Search for Multiply Charged Dirac Magnetic Poles,"** R. L. Fleischer, I. S. Jacobs, W. M. Schwarz, and P. B. Price, Phys. Rev. **177**, 2029–2035 (1969) (I)

report a search in plastic detectors for tracks of monopoles extracted by a pulsed magnet from polar deep sea manganese nodules.

16. **"Search for Supermassive Magnetic Monopoles Using Mica Crystals,"** P. B. Price and M. H. Salamon, Phys. Rev. Lett. **56**, 1226–1229 (1986) (I)

describe a fourfold coincidence experiment on ancient mica that looked for monopoles bound to aluminum or manganese picked up in passing through the Earth's crust.

Abundant reactions in the upper atmosphere would swamp ionization signals from the rarer cosmic-ray monopoles and so have driven monopole detectors underground.

17. **"Search for Superheavy Magnetic Monopoles at the Baksan Underground Telescope,"** E. N. Alexeyev, M. M. Boliev, A. E. Chudakov, B. A. Makoev, S. P. Mikheyev, and Yu. V. Sten'kin, Lett. Nuovo Cimento **35**, 413–418 (1982) (I)

present results using this now favored venue.

One of the recent reports of a monopole observation was a single event based on a track in a balloon-borne package of photographic emulsion and Lexan plastic.

*18. **"Evidence for Detection of a Moving Magnetic Monopole,"** P. B.

Price, E. K. Shirk, W. Z. Osborne, and L. S. Pinsky, Phys. Rev. Lett. **35**, 487–490 (1975). (I)

Immediately a plausible prosaic alternative explanation of the event was offered, and soon serious errors were found in the data calibration.

*19. **"Analysis of a Reported Magnetic Monopole,"** L. W. Alvarez, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energy*, edited by W. T. Kirk (SLAC, Stanford, 1976), pp. 967–979. (I)

Magnetic monopoles in principle may be produced in pairs by collisions involving photons, leptons, or hadrons in accelerator beams or in cosmic rays.

20. **"Unexplained Multiphoton Phenomenon,"** G. B. Collins, J. R. Ficenec, D. M. Stevens, and W. P. Trower, Phys. Rev. D **8**, 982–983 (1973) (I)

review properties of five unexplained cosmic-ray-induced emulsion events found in the mid-1950s.

21. **"Magnetic Poles and Energetic Photon Showers in Cosmic Rays,"** M. A. Ruderman and D. Zwanziger, Phys. Rev. Lett. **22**, 146–148 (1969) (I)

explain these events as possible annihilation and bremsstrahlung photons from the recombination of magnetic pole–antipole pairs.

22. **"Search for Multiphoton Events from Proton–Nuclei Interactions at 300 GeV/c,"** D. M. Stevens, G. B. Collins, J. R. Ficenec, W. P. Trower, J. Fisher, and S. Iwata, Phys. Rev. D **14**, 2207–2218 (1976) (I)

fail to find such multiphoton events in an accelerator search.

23. **"Would Shower Cores or Relativistic Monopoles Produce Straight Lightning?"** D. R. Tompkins, Jr., Phys. Rev. D **4**, 1268–1274 (1971) (I)

points out that a heavily ionizing monopole is one of the few imaginable sources for a lightning stroke without the usual jaggedness, so that straight lightning could be used as a monopole indicator with quite high collecting power.

The orientation of the electric field vector in Cherenkov light from an electrically charged particle passing through matter is perpendicular to the Cherenkov cone, while for a magnetic pole it is tangent.

24. **"Search for the Dirac Monopole by Means of Vavilov–Cherenkov Radiation at the 70 GeV IHEP Proton Synchrotron,"** V. P. Zrelov, L. Kollarova, D. Kollar, V. P. Lupiltsev, P. Povlovic, J. Ruzicka, V. I. Sidorova, M. F. Shabashov, and R. Janik, Czech. J. Phys. B **26**, 1306–1318 (1976) (I)

made an experiment that exploits this unique monopole property.

25. **"Search for Ferromagnetically Trapped Magnetic Monopoles of Cosmic-Ray Origin,"** E. Goto, H. H. Kolm, and K. W. Ford, Phys. Rev. **132**, 387–396 (1963) (I,E)

describe an experiment to extract monopoles trapped in iron ore using a high-field pulsed magnet on samples from meteorites and Adirondack rock outcroppings.

26. **"New Limit on the Magnetic Monopole Density in Old Iron Ore,"** T. Ebisu and T. Watanabe, J. Phys. G **11**, 883–889 (1985) (I)

heated ore samples above the Curie point so that any monopoles would drop under gravity into their detector.

D. Searches depending on quantum effects

If a magnetic pole passes through a conducting loop it induces an electric current whose magnitude is proportional to the magnetic charge but depends weakly on the pole's position and the angle of penetration of the loop's plane. If the loop is made of a superconductor the induced current will persist.

*27. **Search for Magnetic Monopoles in the Lunar Sample,"** L. W. Alvarez, P. H. Eberhard, R. R. Ross, and R. D. Watt, Science **167**, 701–703 (1970) (I)

report a measurement on exotic material using a multipass superconducting induction detector.

*28. **"First Results from a Superconductive Device for Moving Magnetic Monopoles,"** B. Cabrera, Phys. Rev. Lett. **48**, 1378–1380 (1982) (I)

reports a signal in an induction detector, which in principle is unique to a monopole. Lack of confirmation from all but one of the many later equivalent detectors of equal or greater collecting power cause this event increasingly to be disregarded.

29. **"Observation of an Unexplained Event from a Magnetic Monopole Detector,"** A. D. Caplin, M. Hardiman, M. Koratzinos, and J. C. Schouten, Nature (London) **321**, 402–406 (1986). (I)

The invention of GUT magnetic poles leads directly to the idea that monopoles passing through matter would induce proton decay along their trajectories.

30. **"Experimental Limits on Magnetic Monopole Catalysis of Nucleon Decay,"** S. Errede, J. L. Stone, J. C. van der Velde, R. M. Bionta, G. Blewitt, C. B. Bratton, B. G. Cortez, G. W. Foster, W. Gajewski, M. Goldhaber, J. Greenberg, T. J. Haines, T. W. Jones, D. Kielczewska, W. R. Kropp, J. G. Learned, E. Lehmann, J. M. LoSecco, P. V. Ramana Murthy, H. S. Park, F. Reines, J. Schultz, E. Shumard, D. Sinclair, D. W. Smith, H. W. Sobel, L. R. Sulak, R. Svoboda, and C. Wuest, Phys. Rev. Lett. **51**, 245–258 (1983) (I)

use a large underground proton decay detector to search for catalyzed interactions.

Massive GUT monopoles traveling at less than a thousandth the speed of light could lose energy by Zeeman level splitting of gas molecules. These excited atoms could be detected after Penning collisional energy transfer to another gas species that subsequently ionizes.

31. **"Energy Loss of Slowly Moving Magnetic Monopoles in Matter,"** S. D. Drell, N. M. Kroll, M. T. Mueller, S. J. Parke, and M. A. Ruderman, Phys. Rev. Lett. **50**, 644–648 (1983) (I)

make these calculations for hydrogen and helium.

32. **"First Results from a Search for Magnetic Monopoles by a Detector Utilizing the Drell Mechanism and the Penning Effect,"** F. Kajino, S. Matsuno, Y. K. Yuan, and T. Kitamura, Phys. Rev. Lett. **52**, 1373–1376 (1984) (I)

use helium–methane-filled proportional counters in this measurement.

E. Indirect searches for magnetic monopoles

Observations of astrophysical phenomena can provide bounds on the size of aggregations of magnetic poles. First, the maximum monopole mass density can be estimated by equating it to the mass density of a critically closed Universe calculated from the measured Hubble expansion and gravitational constants. Second, the existence of galactic and intergalactic magnetic fields, estimated from observation of polarized starlight, places a limit on the monopole number since monopoles would be accelerated by the field and therefore drain its energy.

33. **"The Origin of Magnetic Fields,"** E. N. Parker, Astrophys. J. **160**, 383–404 (1970) (A)

presents a far-reaching discussion of the cosmic magnetic fields explicitly admitting the possible existence of magnetic poles and deriving the now famous "Parker limit" on their number.

34. **"Magnetic Monopoles and the Survival of Galactic Magnetic Fields,"** M. S. Turner, E. N. Parker, and T. Bogdan, Phys. Rev. D **26**, 1296–1305 (1982) (I)

reconsider the issue for very heavy monopoles, for which the limit is less stringent because of the competition between gravitational and magnetic forces on the poles. They also point out problems with an alternative point of view, that poles might comprise the principal source of the galactic field—a gigantic magnet, oscillating very slowly as the north and south poles build up alternately at opposite ends.

35. **"On the Stability of the Galactic Magnetic Field in the Presence of a Magnetic Monopole Halo,"** D. F. Chernoff, S. L. Shapiro, and I. Wasserman, Astrophys. J. **304**, 799–820 (1986) (I)

give a recent presentation of this minority view.

36. **"The Magnetic Monopole Flux and the Survival of Intracluster Magnetic Fields,"** Y. Raphaeli and M. S. Turner, Phys. Lett. B **121**, 115–119 (1983) (I)

use evidence on intracluster magnetic fields to obtain a more stringent but less reliable bound on monopole flux than that of Parker (Ref. 33).

Celestial bodies containing magnetic poles could exhibit their influence.

37. **"Monopole Catalysis of Nucleon Decay in Neutron Stars,"** E. W. Kolb, S. A. Colgate, and J. A. Harvey, Phys. Rev. Lett. **49**, 1373–1375 (1982) (I)

38. **"Catalyzed Nucleon Decay in Neutron Stars,"** S. Dimopoulos, J. P. Preskill, and F. Wilczek, Phys. Lett. B **119**, 320–322 (1982) (I)

imply a density limit on decay-catalyzing monopoles in the galaxy from the measured limit on x-ray flux from neutron stars.

39. **"Monopole Abundance in the Solar System and the Intrinsic Heat in the Jovian Planets,"** J. Arafune, M. Fukugita, and S. Yanagita, Phys. Rev. D **32**, 2586–2590 (1985) (I)

use the measured radiated heat of the large planets to limit the abundance of decay-catalyzing poles in the solar system.

III. THE DELICATE FIT OF MONOPOLES INTO QUANTUM MECHANICS

A. The quantization condition

*40. **"Quantized Singularities in the Electromagnetic Field,"** P. A. M. Dirac, Proc. R. Soc. London Ser. A **133**, 60–72 (1931) (A)

is the most important single paper on magnetic monopoles. Dirac begins with a remarkable review of his hole theory of the proton in the light of criticisms by Oppenheimer, Weyl, and Tamm. He abandons his previous position that a single Dirac equation describes negatively charged electrons and positively charged protons, and embraces the unavoidable consequence, that a positively charged antiparticle of the electron must exist. More than a year after publication, discovery of the positron was announced by Carl Anderson, who appears to have known nothing about Dirac's prediction despite being Oppenheimer's Caltech faculty colleague.

Dirac goes on to explore a formulation of quantum electrodynamics (QED) in which the change in phase of a charged-particle wavefunction between two points depends on the choice of the connecting path. By insisting that matrix elements of observable operators be unaffected by ambiguities in the choice of phase change, and imposing continuity conditions on the wavefunction, he concludes that the product of the magnitude of any isolated electric charge with that of any isolated magnetic pole must be an integral multiple N of a smallest unit, $qg = N\hbar c/2$. He also notes that if the monopole has a characteristic radius R, then its energy or mass must be large, $E \sim g^2/R$.

41. **"The Theory of Magnetic Monopoles,"** P. A. M. Dirac, Phys. Rev. **74**, 817–830 (1948) (A)

extends his original idea to include relativity.

42. **"Electric and Magnetic Charge Renormalization I and II,"** J. Schwinger, Phys. Rev. **151**, 1048–1054; 1055–1057 (1966) (A)

presents an argument that the "bare" electric and magnetic charges defined in perturbative QED, as well as the physically observable charges, must both obey the quantization condition on their product. This point, along with puzzles he raised earlier about ambiguities arising when charges and poles overlap, were at least hints of inconsistencies in the notion of point monopoles.

43. **"Sources and Magnetic Charge,"** J. Schwinger, Phys. Rev. **173**, 1536–1544 (1968) (A)

introduces the possibility of particles carrying both electric and magnetic charge, and shows that the quantization condition on pairs of such particles, $q_1g_2 - q_2g_1 = N\hbar c/2$, follows from the requirement of duality–rotation invariance. A difficulty associated with global rotation invariance might be overcome by restriction of magnetic charge to even multiples of the Dirac unit.

44. **"Exactly Soluble Nonrelativistic Model of Particles with Both Electric and Magnetic Charges,"** D. Zwanziger, Phys. Rev. **176**, 1480–1488 (1968) (A)

also generalizes the quantization condition, and adds a special scalar potential to enlarge the symmetry, like the "accidental degeneracy" of the hydrogen atom, permitting analytic solutions for the energy eigenfunctions. The kinetic momentum operators obey the Jacobi identity because all wavefunctions vanish for coincidence of two such particles, since the velocity-dependent magnetic interaction produces a centrifugal potential even in the lowest partial wave.

45. **"Possible Binding of a Magnetic Monopole to a Particle with Electric Charge and a Magnetic Dipole Moment,"** D. Sivers, Phys. Rev. D **2**, 2048–2054 (1970) (I)

builds on Malkus (Ref. 13), showing that a massive monopole and a nucleus with a large magnetic moment have a sufficient attraction to become attached.

46. **"A Magnetic Model of Matter,"** J. Schwinger, Science **165**, 757–761 (1969) (E,I)

explains many observed properties of strongly interacting particles by assuming they contain "dyons" (the now standard term for particles bearing both electric and magnetic charge) arranged so the total magnetic charge always vanishes.

*47. **"Connection of Spin and Statistics for Charge–Monopole Composites,"** A. S. Goldhaber, Phys. Rev. Lett. **36**, 1122–1125 (1976) (A)

resolves the apparent paradox of spinless electrically and magnetically charged bosons combining to form a dyon with half-integer spin by showing that such an object is actually a fermion. This analysis also provides another proof of Dirac's quantization condition, and excludes dyons with fractional electric charge in standard QED.

*48. **"Dyons of Charge $e\theta/2\pi$,"** E. Witten, Phys. Lett. B **86**, 283–287 (1979) (A)

observes that the presence of a vacuum angle θ, equivalent to inclusion of a term proportional to θ **E·B** in the Lagrangian density, implies that a monopole must carry a specific fractional electric charge. The conclusion is obtained in a particular context, quantization of the "charge rotor" collective degree of freedom of an 't Hooft–Polyakov monopole (Refs. 64–66), but was later seen to be independent of assumptions about internal structure of the pole. Fractional dyon charge presents a phase consistency problem as indicated by Schwinger (Ref. 43) and Goldhaber (Ref. 47).

49. **"Remarks on Dyons,"** F. Wilczek, Phys. Rev. Lett. **48**, 1146–1149 (1982) (A)

sketches the solution to the problem.

50. **"Field Corrections to Induced Statistics,"** A. S. Goldhaber, R. MacKenzie, and F. Wilczek, Mod. Phys. Lett. A **4**, 21–31 (1989) (A)

make the solution precise: Nonzero θ is not only a sufficient but also a necessary condition for fractional dyon charge.

B. Fermion interactions with monopoles

There are two related special features that arise when a charged Dirac particle interacts with a minimum-strength monopole. The lowest allowed angular momentum is zero, because of cancellation between the "electron" spin and the Poincaré–Thomson electromagnetic angular momentum. In the state with zero angular momentum there is a second cancellation, exact if the particle has Dirac's gyromagnetic ratio 2, between centrifugal repulsion and magnetic dipole attraction. Thus there is nothing to stop the electron from plunging directly into the monopole. Furthermore, in the vicinity of the pole, the electron's wavefunction diverges inversely with radius. This singularity leads to some degree of ambiguity in the definition of the interaction, hence to tantalizing possibilities for new and peculiar phenomena.

51. **"On the Theory of a Point Magnetic Pole (in German),"** P. P. Banderet, Helv. Phys. Acta **19**, 503–522 (1946) (A)

examines the Dirac equation for the motion of an electron in the field of a monopole and obtains the scattering wavefunctions, but glosses over the subtleties of the lowest partial wave (total angular momentum zero).

52. **"Motion of an Electron in the Field of a Magnetic Pole,"** Harish-Chandra, Phys. Rev. **74**, 883–887 (1948) (I,A)

argues for a boundary condition on the wavefunction at the coincidence point which implies that there is no bound state.

53. **"Scattering of a Dirac Particle with Charge Ze by a Fixed Magnetic Monopole,"** Y. Kazama, C. N. Yang, and A. S. Goldhaber, Phys. Rev. D **15**, 2287–2299 (1977) (A)
54. **"Dirac Particle in a Magnetic Field: Symmetries and Their Breaking by Monopole Singularities,"** A. S. Goldhaber, Phys. Rev. D **16**, 1815–1827 (1977) (I,A)

observe that the boundary condition is ambiguous, so that it amounts to an extra adjustable parameter needed to define fully the dynamics of an electron–monopole system.

55. **"Fermion-Monopole System Reexamined,"** H. Yamagishi, Phys. Rev. D **27**, 2383–2396 (1983) (A)
56. **"Does a Dyon Leak?,"** B. Grossman, Phys. Rev. Lett. **50**, 464–467 (1983) (A)

show how the choice of boundary condition determines the dyon charge produced by electron vacuum polarization.

IV. THE NECESSITY AND CONSEQUENCES OF MONOPOLE STRUCTURE

After 40 years it was realized that Dirac's condition, necessary to make monopoles consistent with quantum mechanics, is not sufficient. The large coupling required by the quantization condition implies that the pole must have internal dimensions large compared to its Compton wavelength. Inside the monopole, quantum electrodynamics cannot describe the structure.

If the unavoidable modification of QED extends standard Maxwell theory to include additional fields, then the large monopole size implies that it may be described by classical field equations. Thus a quantum consistency condition leads to stable classical objects!

A. Necessity

*57. **"The Spatial Extent of Magnetic Monopoles,"** C. J. Goebel in *Quanta*, edited by P. G. O. Freund, C. J. Goebel, and Y. Nambu (University of Chicago Press, Chicago, 1970), pp. 338–344 (I,A)

uses dispersion relations for monopole–photon scattering to deduce that a monopole field is inherently spread out because of the large magnetic charge.

58. **"Monopoles and Gauge Theories,"** A. S. Goldhaber, Ref. 131, pp. 1–15 (I,A)

arrives at the same conclusion from energy considerations.

B. Classical models

Although the first model (charged black hole) antedates Dirac's work, it was not until after 't Hooft and Polyakov discovered the existence of a monopole in a classical non-Abelian gauge theory with "spontaneously broken" or hidden gauge symmetry (hence nonzero masses for the electrically charged gauge bosons) that awareness of monopole structure as possible, if not inevitable, became widespread.

59. **"On the Gravitation of the Electric Field Given by Einstein's Theory (in German),"** H. Reissner, Ann. Phys. (Leipzig) **50**, 106–120 (1916) (I,A)
60. **"Energy of the Gravitational Field in Einstein's Theory,"** G. Nordström, K. Akad. Amsterdam Proc. **20**, 1238–1245 (1918) (A)

independently discovered a solution to the Einstein–Maxwell equations that has finite energy although formally singular: A point charge lies at zero radius, within the event horizon and therefore inaccessible to external observers. This solution is defined for a range of ratios of charge to mass. The maximum ratio, for which the object is completely stable, is such that Coulomb repulsion and gravitational attraction between two identical charges cancel exactly, leaving no net long-range force. The fact that the charge could just as well be magnetic as electric was noted by Rainich (Ref. 100). The name "black pole" seems appropriate for a stable, magnetically charged black hole.

61. **"A Class of Exact Solutions of Einstein's Field Equations,"** S. D. Majumdar, Phys. Rev. **72**, 390–398 (1947) (A)
62. **"A Static Solution of the Equations of the Gravitational Field for an Arbitrary Charge Distribution,"** A Papapetrou, Proc. R. Ir. Acad. A **51**, 191–204 (1947) (A)

each show that the total energy of a collection of same-sign black poles does not depend on pole location. Thus the force between poles vanishes for any separation, large or small. Such "floating monopoles" are the nearest thing known in three spatial dimensions to the solitons in one spatial dimension which can go through each other without changing speed or emitting radiation.

63. **"Some Solutions of the Classical Isotopic Gauge Field Equations,"** T. T. Wu and C. N. Yang, in *Properties of Matter Under Unusual Conditions*," edited by H. Mark and S. Fernbach (Interscience, New York, 1969), pp. 349–354 (A)

study a rotationally symmetric, static gauge field configuration that satisfies the source-free Yang–Mills equations everywhere except at the center, to produce a magnetic field that can be interpreted as a monopole. Here, the non-Abelian gauge field, but not the monopole character of the solution, is recognized; in Goldhaber (Ref. 115), vice versa. Neither shows how to get a realistic model for a stable monopole with finite mass.

*64. **"Magnetic Monopoles in Unified Gauge Theories,"** G. 't Hooft, Nucl. Phys. B **79**, 276–283 (1974) (I,A)
*65. **"Particle Spectrum in Quantum Field Theory,"** A. M. Polyakov, JETP Lett. **20**, 194–195 (1974) [Extended version in Sov. Phys. JETP **41**, 988–995 (1975)] (I,A)

is an uncanny repetition of the Reissner–Nordström and Majumdar–Papapetrou "black pole" discussions (Refs. 59–62), where again a model for monopole structure is independently discovered. Here, the Coulomb repulsion is overcome by an attractive scalar Higgs field that not only stabilizes the Wu–Yang object (Ref. 63), but defines its character, since at large distance and low-energy scales massive non-Abelian "charged photons" play no part, and the dynamics become indistinguishable from normal Maxwell theory. References 64 and 65 constitute the most important development in monopole theory after Dirac. Not only did they initiate awareness of monopole structure and its classical character, they also launched a wave of interest in "topological" configurations (including instantons, discovered a little later). This continues to flourish, and to nourish renewed links between physics and mathematics.

66. **"Poles with Both Magnetic and Electric Charges in Non-Abelian Gauge Theory,"** B. Julia and A. Zee, Phys. Rev. D **11**, 2227–2232 (1975) (A)

extend the 't Hooft–Polyakov monopole description to include an isovector electrostatic scalar potential, permitting time-independent solutions carrying arbitrary electric as well as quantized magnetic charge. The electric charge is arbitrary because its quantum unit becomes negligible in the classical limit.

67. **"Exact Classical Solution for the 't Hooft Monopole and the Julia–Zee Dyon,"** M. K. Prasad and C. M. Sommerfield, Phys. Rev. Lett. **35**, 760–762 (1975) (A)

68. **"The Stability of Classical Solutions,"** E. B. Bogomolny, Sov. J. Nucl. Phys. **24**, 449–454 (1976) (A)

69. **"Can One Dent a Dyon?"** S. Coleman, S. Parke, A. Neveu, and C. M. Sommerfield, Phys. Rev. D **15**, 544–545 (1977) (A)

show that for a system of 't Hooft–Polyakov monopoles there is a lower bound on the energy proportional to the total magnetic charge. Further, this Bogomolny–Prasad–Sommerfield (BPS) bound is saturated in the limit where the Higgs boson mass vanishes, provided a set of first-order equations has a solution. That solution is obtained for a dyon with unit monopole strength.

70. **"The Force Between 't Hooft–Polyakov Monopoles,"** N. S. Manton, Nucl. Phys. B **16**, 525–541 (1977) (I,A)

makes explicit the suggestions of these works that, like black poles, these monopoles have no static forces among them.

71. **"Magnetic Monopoles in SU(3) Gauge Theories,"** E. Corrigan, D. I. Olive, D. B. Fairlie, and J. Nuyts, Nucl. Phys. B **106**, 475–492 (1976) (A)

generalize to SU(3) the spherical symmetry of the 't Hooft–Polyakov monopole in SU(2), thus reducing the SU(3) static field equations from three dimensions to one radial dimension. This most symmetric form is a promising candidate for the lowest energy solution.

72. **"Spherically Symmetric Monopoles,"** D. Wilkinson and A. S. Goldhaber, Phys. Rev. D **16**, 1221–1231 (1977) (A)

give rules for constructing all spherically symmetric monopoles for any gauge group.

73. **"Topology of Cosmic Domains and Strings,"** T. W. B. Kibble, J. Phys. A **9**, 1387–1398 (1976) (A)

finds that, in any theory with hidden gauge symmetry, above a critical temperature where the symmetry is restored, the group space orientation of the Higgs field cannot be correlated in causally disconnected regions. Thus monopoles must form when the temperature falls below the critical value.

74. **"On the Concentration of Relic Magnetic Monopoles in the Universe,"** Ya. B. Zeldovitch and M. Yu. Khlopov, Phys. Lett. B **79**, 239–241 (1978) (A)

75. **"Cosmological Production of Superheavy Magnetic Monopoles,"** J. P. Preskill, Phys. Rev. Lett. **43**, 1365–1368 (1979) (A)

show that in traditional Big Bang cosmology with a grand unified theory such as SU(5) there would be too many monopoles to have escaped observation.

76. **"Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems,"** A. H. Guth, Phys. Rev. D **23**, 347–356 (1981) (A)

suggests that the entire observable Universe originated from a region of dimensions big enough to hold at most one monopole, so that instead of too many poles there are essentially none.

77. **"Thermal Production of Superheavy Magnetic Monopoles in the New Inflationary Universe Scenario,"** P. R. Lindblom and P. J. Steinhardt, Phys. Rev. D **31**, 2151–2154 (1985) (A)

concludes that there could be observable monopole production resulting from "conventional" heat generation as the newly inflated Universe begins to settle into equilibrium.

78. **"Solitons with Fermion Number 1/2,"** R. Jackiw and C. Rebbi, Phys. Rev. D **13**, 3398–3409 (1976) (A)

introduce the idea that a solition, a stable object with nontrivial twisting or topology for the field expectation values at different points in space, may polarize the vacuum to localize charges or quantum numbers not found for any configuration of elementary quanta. The 't Hooft–Polyakov monopole is a case in point.

79. **"Magnetic Monopoles as Gauge Particles,"** C. Montonen and D. I. Olive, Phys. Lett. B **72**, 117–120 (1977) (A)

propose a quantum version of the classical electric–magnetic duality where, in a spontaneously broken non-Abelian gauge theory just as a multiplet of vector bosons appears, so does a multiplet of monopoles representing the "dual" of the original gauge group. The 't Hooft–Polyakov monopole is proposed as an example, except no reason why it should have unit spin is provided.

80. **"Monopoles and Dyons in the SU(5) Model,"** C. P. Dokos and T. N. Tomaras, Phys. Rev. D **21**, 2940–2952 (1980) (A)

construct a monopole in the simplest grand unified theory, and observe that in the limit where the pole is treated as a classical static field configuration the scattering of light fermions does not conserve baryon number.

81. **"Anomalous Fermion Production by a Julia–Zee Dyon,"** A. S. Blaer, N. H. Christ, and J.-F. Tang, Phys. Rev. Lett. **47**, 1364–1367 (1981) (A)

use the simplicity of fermion dynamics in the lowest angular momentum state to give a transparent illustration of a subtle quantum effect, the anomalous violation of a naive conservation law resulting from consistency requirements for obedience to even more sacred conservation laws. In this case, energy and electric charge conservation enforce nonconservation of helicity when a highly charged dyon discharges by emission of fermion–antifermion pairs.

*82. **"Superheavy Magnetic Monopoles and the Decay of the Proton,"** V. A. Rubakov, Sov. Phys. JETP Lett. **33**, 644–646 (1981) (A)

83. **"Dyon–Fermion Dynamics,"** C. G. Callan, Jr., Phys. Rev. D **26**, 2058–2068 (1982) (A)

as well as Wilczek (Ref. 49), suggest that in grand unification theories magnetic monopoles should be catalysts for proton decay.

84. **"Monopoles, Gauge Fields, and Anomalies,"** A. S. Goldhaber, in *Fourth Workshop on Grand Unification*, edited by H. A. Weldon, P. Langacker, and P. J. Steinhardt (Birkhaüser, Boston, 1983), pp. 115–120 (A)

argues that the chiral anomaly of the standard model assures such catalysis, but not necessarily at a rate characteristic of strong interactions. As in Ref. 81, conservation of charges coupled to gauge fields (including that of the massive Z boson) forces nonconservation of an ungauged charge, in this case, baryon number.

*85. **"A Remark on the Scattering of BPS Monopoles,"** N. S. Manton, Phys. Lett. B **110**, 54–56 (1982) (I,A) builds on the verification of his own earlier conjecture (Ref. 70) that Bogomolny–Prasad–Sommerfeld monopoles "float," and finds that this slow scattering can be described in terms of geodesic, therefore radiationless, motion. These poles are the closest thing known to the original one-dimensional solitons that do not radiate when they scatter, since the expected quadrupole radiation is suppressed. The scattering results in exchange of electric charge between pairs of initially like, chargeless poles.

86. **"Low Energy Scattering of Non-Abelian Monopoles,"** M. F. Atiyah and N. J. Hitchin, Phys. Lett. A **107**, 21–25 (1985) (A)

apply recent beautiful developments in mathematics to obtain analytic results for the scattering.

87. **"Consequences of a Monopole with Dirac Magnetic Charge,"** G. Lazarides, Q. Shafi, and W. P. Trower, Phys. Rev. Lett. **49**, 1756–1758 (1982) (I,A)

observe that color magnetic flux, for example from an SU(5) monopole, if not screened might be channeled into a tube or tubes that terminate on other poles, thus tying them together in a kind of pole confinement.

88. **"Global Color is Not Always Defined,"** P. Nelson and A. Manohar, Phys. Rev. Lett. **50**, 943–945 (1983) (A)

89. **"Monopole Topology and the Problem of Color,"** A. P. Balachandran, G. Manno, N. Mukunda, J. S. Nilson, E. C. G. Sudarshan, and P. Zaccaria, Phys. Rev. Lett. **50**, 1553–1555 (1983) (A)

90. **"Chromodyons and Equivariant Gauge Transformations,"** A. Abouelsaood, Phys. Lett. B **125**, 467–469 (1983) (A)

91. **"What Becomes of Global Color?"** P. Nelson and S. Coleman, Nucl. Phys. B **237**, 1–31 (1984) (A)

explore what happens to the notion of dyon electric charge when the monopole has color magnetic as well as ordinary magnetic charge, as in Ref. 80. Once again, ordinary electric charge can be defined for the monopole, as can four of the eight color electric charges. However, the remaining four are peculiar: Undefinable for an isolated monopole, even for a widely separated pole–antipole pair they are not localized, instead being distributed along the lines of magnetic flux connecting pole and antipole.

92. **"Kaluza–Klein Monopole,"** R. D. Sorkin, Phys. Rev. Lett. **51**, 87–90 (1983) (A)

93. **"Magnetic Monopoles in Kaluza–Klein Theories,"** D. J. Gross and M. J. Perry, Nucl. Phys. B **226**, 29–48 (1983) (A)

provide once again a duo of works announcing a new classical field model of monopole structure. This object is bound by an extra scalar field as well as by gravitational attraction. Not a black hole, it has a smaller mass-to-charge ratio than the "black pole" but like it, and like the BPS monopole, exerts no long-range static force on another pole of the same type.

V. PEDAGOGY: LEARNING FROM AND ABOUT MONOPOLES

A. Classical electromagnetism

In unified electromagnetism it becomes possible to describe a magnet either in terms of pole pairs or current loops, as long as the magnet interior remains inaccessible. Often the pole description permits simpler calculation and analysis, but, in any case, the equivalence provides a powerful consistency check on understanding of electrodynamics.

94. **"Why Ampère Did Not Discover Magnetic Induction,"** L. P. Williams, Am. J. Phys. **54**, 306–311 (1986) (E,I)

describes the interaction between Ampère and Faraday. Not surprisingly, little was heard of possible isolated poles for a long time after Ampère's work.

95. **"On Some New Electro-Magnetical Motions, and on the Theory of Magnetism,"** M. Faraday (1821), reprinted in *Experimental Researches in Electricity* (Taylor, London, 1844), Vol. 2, pp. 127–147 (E)

is a study of magnetic motion, one of the first to use isolated magnetic poles heuristically or didactically. Faraday deduces from Ampère's ideas that a pole would be continually accelerated around a straight section of current-carrying wire, and confirms this by experiment, using for the pole one end of a long magnet.

96. **"Law of Magnetic Force,"** in *Treatise on Electricity and Magnetism*, J. C. Maxwell (Clarendon, Oxford, 1873), Vol. 2, pp. 3–7 (E)

makes clear that isolated poles do not occur, and uses Coulomb's law to define the unit of magnetic charge as length times the square root of force. Ten years later, this definition gave rise to a debate in the Philosophical Magazine involving such notable physicists as Clausius, Helmholtz, Larmor, Lodge, and J. J. Thomson, for all of whom an isolated magnetic pole was only a convenient mathematical construct not to be sought in Nature.

97. **"Electricity—Remarks on Birkeland's Experiment (in French),"** H. Poincaré, C. R. Acad. Sci. **123**, 530–533 (1896) (I)

computes the deflection of a cathode-ray beam, already thought to consist of electrically charged particles, passing near the end of a long thin magnet, whose force is equivalent to that from an isolated magnetic pole. The calculation exploits the conservation of the total angular momentum obtained by adding up the usual orbital part and an extra contribution, pointing from the charge q toward the pole g, of magnitude equal to the produce qg/c. A mechanical model for this is a rapidly spinning wheel whose axle is a straight light rod of variable length terminated by point masses representing the charge and the pole.

*98. **"On Momentum in the Electric Field,"** J. J. Thomson, Philos. Mag. **8**, 331–356 (1904) (I)

develops his 1893 discovery that the electromagnetic field carries momentum with a density proportional to the Poynting power density vector, or, in modern language, that the electromagnetic energy-momentum tensor is symmetric. He computes the angular momentum carried by the electromagnetic field of a charge–pole system, and independently of Poincaré observes that the sum of the electromagnetic and mechanical angular momenta is conserved. Thomson goes on to identify an electromagnetic contribution to the linear momentum of a current-loop dipole in the presence of an electric charge. He concludes that the net force on a magnet in an external electromagnetic field is the same whether the magnet is made from poles or from currents, and thus affirms and extends Ampère's principle of magnetic ambiguity. This important application of conservation laws is generally absent from textbooks on electromagnetism.

99. **"Electromagnetic Equations in Bivector Formalism (in German),"** L. Silberstein, Ann. Phys. Chem. **22**, 579–586 (1907) (I)

presents electrodynamics in complex vector notation, so that electric fields are real, and magnetic fields imaginary, parts of complex "bivectors." Multiplying by a phase factor $e^{i\theta}$ then corresponds to a duality rotation by angle θ of electric quantities into magnetic, and vice versa, making manifest the duality-rotation invariance of Maxwell's theory.

100. **"Electrodynamics and General Relativity,"** G. Y. Rainich, Trans. Am. Math. Soc. **27**, 106–136 (1925) (I,A)

introduces duality rotation in classical Einstein–Maxwell field theory (gravitoelectrodynamics) without assumed particles. This allows interpretation of Reissner–Nordström charged black holes (Refs. 59 and 60) as magnetically rather than electrically charged.

101. **"A New Law in Electrodynamics,"** O. Costa de Beauregard, Phys. Lett. A **24**, 177–178 (1967) (I)

102. **" 'Try Simplest Cases' Discovery of 'Hidden Momentum' Forces on 'Magnetic Currents,' "** W. Shockley and R. P. James, Phys. Rev. Lett. **18**, 876–879 (1967) (I)

103. **"Forces on a Current Loop,"** H. Haus and P. Penfield, Phys. Lett. A **26**, 412–413 (1968) (I)

104. **"Origin of 'Hidden Momentum Forces' on Magnets,"** S. Coleman and J. H. Van Vleck, Phys. Rev. **171**, 1370–1375 (1968) (I)

105. **"Examples of Momentum Distributions in the Electromagnetic Field and in Matter,"** W. H. Furry, Am. J. Phys. **37**, 621–636 (1969) (I)

106. **"Comment on 'Proposed Aharonov–Casher Effect: Another Example of an Aharonov–Bohm Effect Arising from a Classical Lag,' "** Y. Aharonov, P. Pearle, and L. Vaidman, Phys. Rev. A **37**, 4052–4055 (1988) (I)

study the effect of an electric field on a magnet, and agree on Ampère's principle. They work out the cunning manner in which Maxwell–Lorentz dynamics conspire to maintain the principle, making full use of the Thomson electromagnetic momentum density and deducing the presence of a "hidden" mechanical momentum inside a stationary magnet that sits in a static electric field. Taking account of this momentum, one finds that the electromagnetic force on a current loop dipole is *not* the same as its rate of change of momentum, but *is* the same as the force on an equivalent pole pair dipole.

B. Quantum electrodynamics

107. **"Generalized Spherical Harmonics and Wave Functions of an Electron in a Field of a Magnetic Pole (in German),"** I. E. Tamm, Z. Phys. **71**, 141–150 (1931) (I)

recognizes the possible angular wavefunctions for the motion of an electric charge about a monopole as what we now call rotation functions. Such a function, labeled by a total angular momentum, is defined as the projection of a state with angular momentum $-qg/c$ about the direction from the pole to the charge onto a state with angular momentum $m\hbar$ about some fixed direction. For $g = 0$ these are ordinary spherical harmonics. Tamm's realization is simply the expression in terms of quantum mechanical wavefunctions of the equivalence between the charge–pole interaction and the mechanical model of a spinning wheel with mass points at either end of its variable-length axis, mentioned in the discussion of Poincaré's work (Ref. 97). While Dirac emphasizes gauge invariance related to local conservation of electric charge, Tamm focuses on rotational invariance and the conservation of angular momentum, thus opening another way to understand the quantization condition.

***108.** **"Note on Dirac's Theory of Magnetic Poles,"** M. N. Saha, Phys. Rev. **75**, 1968 (1949) restating his argument in Ind. J. Phys. **10**, 141–153 (1936) (E)

109. **"Note on Dirac's Theory of Magnetic Poles,"** H. A. Wilson, Phys. Rev. **75**, 309 (1949) (E)

justify Dirac's quantization condition by requiring that the Poincarè–Thomson electromagnetic angular momentum be quantized. They assume rather than prove that the projected angular momentum along the charge–pole axis is a quantum eigenvalue, instead of an expectation value that need not be quantized.

110. **"On the Theory of Particles with Magnetic Charge (in German),"** M. Fierz, Helv. Phys. Acta **17**, 27–34 (1944) (A)

shows that if the quantization condition is not obeyed the wavefunctions are not representations of the rotation group.

111. **"Time Reversal, Charge Conjugation, Magnetic Pole Conjugation, and Parity,"** N. F. Ramsey, Phys. Rev. **109**, 225–236 (1958) (I)

points out that the discrete symmetries T and P, unlike continuous symmetries such as rotations, are valid in the simultaneous presence of charges and poles only if the corresponding symmetry operations include explicit reversal of all pole strengths.

112. **"Significance of Electromagnetic Potentials in the Quantum Theory,"** Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485–491 (1959) (I)

point out that electrons scattered around an impenetrable region containing magnetic flux will exhibit a diffraction pattern that is sensitive to the amount by which that flux differs from an integer multiple of the flux quantum hc/e. Thus, formally describing interactions of electrons with a monopole as if the pole were one end of an infinitely thin dipole "string" implies that the string could be unobservable only if it contained integer flux, but that is just Dirac's quantization condition for monopole charge.

113. **"Quantum Electrodynamics with Dirac Monopoles,"** N. Cabibbo and E. Ferrari, Nuovo Cimento **23**, 1147–1154 (1962) (I,A)

present monopole theory in terms of formally gauge-invariant but path-dependent fields. The quantization condition arises because the path-dependent phase is well defined for a closed path, and therefore unaffected by deformations of the surface bounded by that path. By Stokes' law, this phase is proportional to the magnetic flux through that surface. If the exponential of this phase must be surface independent, then the phase corresponding to the total flux out of any closed surface must be an integral multiple of 2π, and hence magnetic charge must be quantized.

114. **"Geometric Definition of Gauge Invariance,"** E. Lubkin, Ann. Phys. (NY) **23**, 233–283 (1963) (A)

explains, both for electrodynamics and for non-Abeliean gauge theories how a gauge interaction may be viewed geometrically as determining the parallel transport of vectors in an abstract space corresponding to any specified path in ordinary space-time. This approach allows definition of a topological or dual charge that in the simplest case is just that of a monopole.

115. **"Role of Spin in the Monopole Problem,"** A. S. Goldhaber, Phys. Rev. B **140**, 1407–1414 (1965) (I,A)

uses the correspondence principle to determine the S-matrix phase dependence on the azimuthal scattering angle of a charge deflected by a pole. Conservation of total angular momentum requires quantization of this phase variation, hence the Dirac quantization condition. Also, the classical and quantum dynamics of a charge–pole system are expressed in terms of the extra electromagnetic spin, which behaves like isospin.

116. **"Charge Quantization and Nonintegrable Lie Algebras,"** C. A. Hurst, Ann. Phys. (NY) **50**, 51–75 (1968) (A)

employs the requirement of Hamiltonian self-adjointness to show that a charge–pole system cannot be rotationally symmetric, and therefore cannot conserve total angular momentum unless Dirac's condition holds.

117. **"Magnetic Charge Quantization and Angular Momentum,"** H. J. Lipkin, W. I. Weisberger, and M. E. Peshkin, Ann. Phys. (NY) **53**, 203–214 (1969) (I,A)

describe the charge–pole interaction in terms of the position coordinates and the kinetic momenta, equivalent to gradients of Cabibbo–Ferrari path-dependent phases. For consistent commutation relations of these quantities, the quantization condition is forced. They make no explicit use of the vector potential, and like Zwanziger (Ref. 44) emphasizes that vanishing of the wavefunction at the coincidence point is a consistency requirement.

118. **"Concept of Nonintegrable Phase Factors and Global Formulation of Gauge Fields,"** T. T. Wu and C. N. Yang, Phy. Rev. D **12**, 3845–3857 (1975) (I,A)

bring out ideas nascent in the work of Dirac (Ref. 40) and of Cabibbo and Ferrari (Ref. 113), explored in Lubkin's work (Ref. 114), and illustrated by the Aharonov–Bohm effect (Ref. 112). The electromagnetic field is underdescribed by the field strength in accessible regions of space, overdescribed by the gauge-variant four-vector potential, but is properly specified by the phase factor obtained by exponentiating the vector potential line integral around any accessible closed circuit. The vector potential notion is generalized to reveal Dirac's string as a coordinate singularity with no observable consequences.

119. **"Interaction of a Magnetic Monopole with a Ferromagnetic Domain,"** C. Kittel and A. Manoliu, Phys. Rev. B **15**, 333–3365 (1977) (I)

use an energy conservation argument to show that the force on a monopole inside a ferromagnet is proportional to **H** rather than **B**.

120. **"Low-Energy Theorem for Electron Hyperfine Interactions and the Special Case of Magnetic Poles,"** A. S. Goldhaber, Phys. Rev. D **23**, 3071–3074 (1981) (I,A)

analyzes the Dirac equation for an electron in a magnetic field and shows that the hydrogen hyperfine interaction is unchanged if the proton magnetic moment is made of pole pairs instead of current loops, because the electron is smeared over its Compton wavelength, and so cannot probe the smaller proton.

121. **"Electromagnetic Force on a Magnetic Monopole,"** Y. Hara, Phys. Rev. A **32**, 1002–1006 (1985) (I)

argues that the velocity-dependent force on a pole is determined by **E**, not **D**, even though **H** gives the magnetic force.

122. **"Realizations of Magnetic-Monopole Gauge Fields: Diatoms and Spin Precession,"** J. Moody, A. Shapere, and F. Wilczek, Phys. Rev. Lett. **56**, 893–896 (1986) (I,A)

give a realizable example of a "charge–pole gyroscope," a diatomic molecule with unpaired electron angular momentum aligned along the internuclear axis. The equation of motion in the relative nuclear coordinate is the same as that for a charge around a pole, provided that $-qg$ is identified with the electron spin projection, and that the internuclear potential constrains the allowed range of radii for the equivalent charge–pole system.

C. Pedagogical materials

123. **Bibliographical History of Electricity and Magnetism,** P. F. Mottelay (Charles Griffin, London, 1922) (E)

is a fascinating compilation from ancient times through Faraday, best savored in small bits. It asserts that Peregrinus' paper on the magnet (Ref. 1) should be considered the first work of modern science.

124. **History of the Theories of Aether and Electricity,** E. T. Whittaker (Nelson, London, 1951, 1953) (E,I)

provides much valuable background as well as some prejudice; for example, the author does not recognize Einstein as the creator of relativity theory!

*125. **"Review of Particle Properties,"** G. P. Yost, R. M. Barnett, I. Hinchliffe, G. R. Lynch, A. Rittenberg, R. R. Ross, M. Suzuki, T. G. Trippe, C. G. Wohl, B. Armstrong, G. S. Wagman, F. C. Porter, L. Montanet, M. Aguilar-Benitez, J. J. Hernandez, G. Conforto, R. L. Crawford, K. R. Schubert, M. Ross, N. A. Törnqvist, G. Höhler, K. Hagiwara, S. Kawabate, D. M. Manley, K. A. Olive, K. G. Hayes, R. H. Schindler, B. Cabrera, R. E. Schrock, R. A. Eichler, L. D. Roper, and W. P. Trower, Phys. Lett. A **204**, 249–251 (1988) (E)

is the Particle Data Group's latest biannually published summary of magnetic monopole experiments and inferences.

126. **"A Complete Magnetic Monopole Bibliography: 1269–1986,"** S. Torres and W. P. Trower, Virginia Tech VPI-EPP-86-9 (1987) (E)

provide a relatively complete and up-to-date audit of the literature on magnetic poles.

127. **The Aharonov–Bohm Effect,** M. Peshkin and A. Tonomura (Springer-Verlag, New York, 1989) (I,A)

review developments since the effect was introduced (Ref. 112), including very beautiful recent experiments.

128. **Aspects of Symmetry,** S. Coleman (Cambridge U.P., New York, 1985) (I,A)

is a tour of topics in modern quantum physics, in which the chapter "Classical Lumps and their Quantum Descendants" is especially germane to monopoles.

129. **Theory and Detection of Magnetic Monopoles in Gauge Theories,** edited by N. S. Craigie (World Scientific, Singapore, 1986) (I,A)

is a collection of reviews and monographs ranging from algebraic geometry to experimental techniques.

130. **The Geometry and Dynamics of Magnetic Monopoles,** M. F. Atiyah and N. J. Hitchin (Princeton U.P., Princeton, 1988) (I,A)

is a description of the mathematical structure of systems of "floating" BPS monopoles, and their remarkable, radiation-free scattering at low relative velocities. This scattering is interpretable as a geodesic motion in a curved space which for overlapping monopoles links their three ordinary relative coordinates with an extra variable whose change is related to the flow of electric charge from one to the other.

131. **Monopoles in Quantum Field Theory,** edited by N. S. Craigie, P. Goddard, and W. Nahm (World Scientific, Singapore, 1982) (A)

132. **Magnetic Monopoles,** edited by R. A. Carrigan, Jr. and W. P. Trower (Plenum, New York, 1983) (I,A)

133. **Monopole '83,** edited by J. L. Stone (Plenum, New York, 1984) (I,A)

are reports on conferences whose subject was magnetic poles.

134. **"Magnetic Monopoles,"** J. P. Preskill, Ann. Rev. Nucl. Part. Sci. **34**, 461–530 (1984) (A)

135. **"In Search of the Supermassive Magnetic Monopole,"** D. E. Groom, Phys. Rep. **140**, 323–373 (1986) (I)

are, respectively, theoretically and experimentally oriented review articles on the status of magnetic poles.

136. **"On the Dirac Magnetic Poles,"** E. Amaldi, in *Old and New Problems in Elementary Particle Physics*, edited by G. Puppi (Academic, New York, 1968), pp. 1–61 (I)

complements the very recent review articles with a masterful survey of both theory and experiment just before the dawn of the structural era.

137. **"Fifty Years of the Magnetic Monopole,"** S. Coleman, in *The Unity of Fundamental Interactions*, edited by A. Zichichi (Plenum, New York, 1983), pp. 21–117 (I,A)

138. **"Vortices and Monopoles,"** J. P. Preskill, in *Architecture of Fundamental Interactions at Short Distances*, edited by P. Ramond and R. Stora (North-Holland, Amsterdam, 1987), pp. 238–337 (I)

are summer school lectures that explain many of the more recent developments in a more relaxed style than some journal articles.

*139. **"Proof Offered of Existence of Pure Magnetic Current,"** W. L. Lawrence, *New York Times*, 16 January 1944, pp. 1, 45 (E)

*140. **"Basic Unit of Magnetism Believed to be Detected."** W. Sullivan, *New York Times*, 15 August 1975, pp. 1, 33 (E)

*141. **"Unit of Magnetism Said to be Tested,"** W. Sullivan, *New York Times*, 28 April 1982, p. A9 (E) are press reports of monopole discovery claims. Like the ringing-down of a bell, each of these articles was followed by a sequence of smaller ones bringing out problems with the initial announcement.

142. **"Magnetic Monopoles,"** K. W. Ford, Sci. Am. **209**(6), 122–131 (1963) (E)

143. **"Quest for the Magnetic Monopole,"** R. A. Carrigan, Jr., Phys. Teach. **13**, 391–398 (1977) (E)

144. **"Superheavy Magnetic Monopoles,"** R. A. Carrigan, Jr., and W. P. Trower, Sci. Am. **246**(4), 106–118 (1982) (E)

provide a description for the educated lay person of the status of the monopole picture as seen at different times.

145. **"Fiber Bundles and Quantum Theory,"** H. J. Bernstein and A. V. Phillips, Sci. Am. **245**(1), 122–137 (1981) (E)

is perhaps the most accessible introduction to the geometrical view of electromagnetic and gauge interactions, illustrated with the Aharonov–Bohm effect and also neutron diffraction.

146. **"On the Question of Magnetic Monopoles and the Dirac Quantization Condition,"** in J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), pp. 251–261 (I)

presents a compact introdution to monopoles, including the neatest evaluation of the electromagnetic angular momentum associated with a charge–pole pair.

VI. EPILOGUE

The latest wave of monopole activity and excitement has crested, leaving much that is new at which to marvel but no monopoles found. At the very least black poles should exist, so Dirac's quantization condition still provides the nearest thing to a rigorous argument for quantization of electric charge. For this the possible presence of but one monopole in the observable Universe is sufficient. Other arguments, though appealing, depend on extrapolations to whole new theories, with many new particles and interactions for which we have no evidence.

The enormously strong magnetic field near the core of a pole opens the possibility of glimpsing dynamics at an energy scale otherwise inaccessible in the laboratory, so that the experimenter's incentive to seek out monopoles is stronger than ever, but so is the sober realization that monopoles are well hidden if accessible at all. A next wave of effort surely will depend on some combination of new conceptual realizations and new search techniques. All of that may require some time to contemplate and absorb what has already been learned about these magnificent if elusive creatures.

ACKNOWLEDGMENTS

In our adventures with monopoles, both of us have received inspiration and provocation from Luis Alvarez, a premier pole hunter of this era. We offer this collection as a tribute to his memory. We are grateful for the hospitality extended during the preparation of this work by the University of Cambridge (ASG) and the Lawrence Berkeley Laboratory (WPT). Further, we have benefited from comments by S. Coleman, D. E. Groom, E. W. Kolb, J. P. Preskill, E. M. Purcell, and R. R. Ross. Finally, we are indebted to Linda Mattox for her skill and glad grace in preparing the seemingly endless revisions of this manuscript and to Richard Robinson for his final check of the citations. This work has been supported in part by the National Science Foundation.

Canonical transformation in quantum mechanics

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(Received 23 November 1988; accepted for publication 22 June 1989)

The phase-space picture of quantum mechanics and some examples illustrating it are presented. Since the position and momentum are *c* numbers in this picture, it is possible to introduce the concept of phase space in quantum mechanics. The uncertainty relation is stated in terms of an area element in phase space, whose minimum size is Planck's constant. Area-preserving canonical transformations in phase space are therefore uncertainty-preserving transformations. The wave-packet spread, coherent-state representation, and squeezed states of light are discussed as illustrative examples.

I. INTRODUCTION

The present organization of the first-year graduate course in quantum mechanics is largely based on the Schrödinger picture and its applications to atomic and nuclear physics. The first widely accepted textbook on this subject was Schiff's book entitled *Quantum Mechanics*, whose first edition was published in 1949.[1] There are now many excellent textbooks, but their basic organization is not significantly different from that of Schiff's first edition. These days, due to many new physical applications, we are led to consider adding to the physics curriculum representations