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Does gravity come from quantum information?

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Reconciling quantum mechanics with gravity has long posed a challenge for physicists. Recent developments have seen concepts originally developed in quantum information theory, such as entanglement and quantum error correction, come to play a fundamental role in understanding quantum gravity.

More than a century after Einstein proposed the general theory of relativity, it remains a mystery how to describe gravity in a theory that is compatible with quantum mechanics. Why is it so difficult to 'quantize' the theory of gravity? There is evidence that the problem is not merely due to technical difficulties, but instead has a deep physical origin. As Bekenstein and Hawking[1–4] pointed out, to preserve the second law of thermal dynamics in a gravitational system with black holes, a black hole should have an entropy $S=\frac{A}{4G}$, where A is its horizon area and G is Newton's gravitational constant. The second law of thermodynamics implies that the area law of the black hole entropy is an upper bound to that of all possible states of matter in a region of the same size[5,6]. On the other hand, entropy is a measure of the number of independent degrees of freedom. Therefore, any theory with local degrees of freedom—such as quantum field theory—implies that the maximal entropy should follow a volume law. Clearly, there is a sharp contradiction here.

The holographic principle

Motivated by this problem, 't Hooft[7] and Susskind[5] formulated the holographic principle, which suggests that the seemingly three-dimensional space is actually two-dimensional. In other words, the three-dimensional world is like the virtual reality we see from virtual reality (VR) glasses. In effect, it is as if the VR glasses coupled with our vision system reconstruct the (3+1)-dimensional reality from a (2+1)-dimensional video played on the screen—which means the information one can obtain from the virtual reality is upper-bounded by an area law—that is, the number of pixels on the screen. From this point of view, our three-dimensional space is like the shadow in Plato's cave, while the video is the 'reality'. Interestingly, the shadow is in higher dimension than the reality that creates it.

It is difficult to build a theory of quantum gravity that satisfies the holographic principle. A breakthrough occurred in 1997, following Maldacena's discovery of AdS (anti-de Sitter)/CFT (conformal field theory) correspondence[8]. AdS/CFT is a conjectured duality between gravity in asymptotically AdS space and a quantum field theory living at the spatial infinity of AdS space[8–10]. In physics, duality refers to the situation when two theories appear to be very different, but there is a hidden one-to-one correspondence between their properties once we know how to compare them. This is similar to two languages that sound totally different but can be translated between each other once we have a dictionary in our hands. Roughly speaking, one can start from the boundary and think of the extra dimension in the bulk representing the length scale in the boundary. Dynamics with lower energy and longer wavelength live deeper in the bulk, as illustrated in Fig. 1a. Assuming that the duality is exact, all properties of the boundary should have a bulk explanation. In particular, Ryu and Takayanagi (RT)[11] proposed that the entanglement entropy of a boundary region A is determined by $S=\frac{|\gamma_A|}{4G}$, the same formula as the black hole entropy, but with the black hole horizon area replaced by an extremal surface area $|\gamma_A|$ bounding region A (Fig. 1b).

In this boundary theory, states with different entanglement structures correspond to different bulk geometries. For example, the ground state of a conformal field theory corresponds to an AdS space, for which the RT surface area grows more slowly than the volume of the boundary region. In comparison, when the boundary is in thermal equilibrium at finite temperature, the dual geometry gives rise to a black hole with the horizon parallel to the boundary. The RT surface is restricted between the boundary and the black hole horizon (Fig. 1c), so that its area is also proportional to the volume of the boundary region. Increasing the temperature therefore corresponds to increasing the size of the black hole in the bulk, which pushes the RT formula further towards the boundary, leading to a larger entropy.

The RT formula and its generalizations[12,13] suggest that the spacetime geometry is constructed from quantum entanglement[14,15]. To build a holographic quantum gravity theory, the first step is to look for quantum states satisfying the RT formula (for some geometry). However, this is a non-trivial task. For example, the RT formula requires that the correlation between a region A with two regions B and C together, measured by mutual information, is always bigger than the sum of that with each region separately[16], a condition that is not true in general.

Locality from quantum error correction

If we believe that the holographic duality holds exactly, each state in the bulk gravity theory corresponds to a unique state of the boundary theory. In particular, we can consider an electron wavepacket localized around a bulk position **x**, far from the boundary (Fig. 2a). The electron can have its spin along a generic direction, carrying one qubit of information. The duality tells us that one should be able to access the spin qubit of this electron from the boundary. For example, the spin z component should correspond to an operator on the boundary that, in principle at least, could be measured. However, this seems to be inconsistent with locality. Since the bulk theory is relativistic, no signal can travel faster than speed of light. So how can someone standing at the boundary measure or rotate the spin in the centre instantaneously?

A resolution of this apparent contradiction was proposed in a beautiful paper of Almheiri, Dong and Harlow[17]. Based on previous works on how to reconstruct bulk operators from a subset of the boundary[18,19]—known as local reconstruction—they pointed out that the bulk spin is only accessible by boundary operators supported by a sufficiently big boundary region, such as B in Fig. 2a, while it is not accessible from any smaller boundary regions, such as A in the same figure. As a consequence, although there exists a way to control the bulk spin instantaneously from the boundary, this action would require simultaneous access to a large portion of the boundary. In other words, a local observer living in a small region

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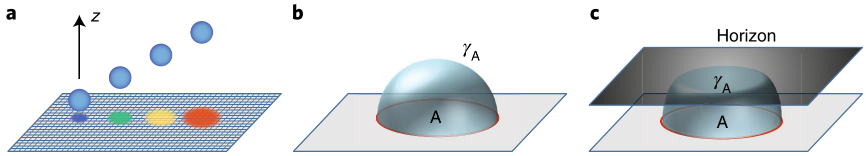

Fig. 1 | Schematic illustration of holographic duality and RT formula. a, The bulk theory is a hologram that emerges from the boundary theory. The total number of degrees of freedom is determined by the number of 'pixels' in the boundary theory. The emergent direction corresponds to length scale, such that an object deeper in the bulk corresponds to a boundary feature with larger size and lower energy. **b**, The entanglement entropy of region A is determined by the area of the extremal surface γ_A, which is obtained by deforming A into the bulk and searching for the saddle point of area. **c**, For a black hole geometry, the RT surface is restricted between the boundary (lower plane) and the horizon (upper plane), and thus has an area that is proportional to the volume of region A.

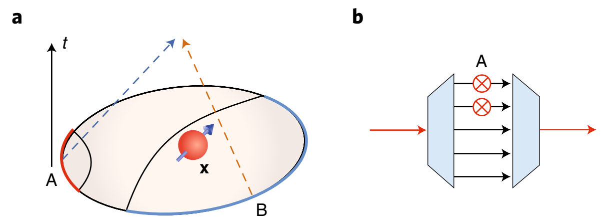

Fig. 2 | Analogy between accessing bulk qubit from the boundary and quantum error correction. a, A qubit in the bulk position **x** is not immediately accessible from any small region on the boundary, such as A, since the signal takes finite time to propagate. This translates to the quantum error correction state that the bulk qubit is protected against local errors around the boundary. However, the information is accessible for a big region like B. **b**, An example of a quantum error correction code that encodes one qubit into five qubits. If an error occurs in a subsystem A with two qubits, the encoded information can still be recovered.

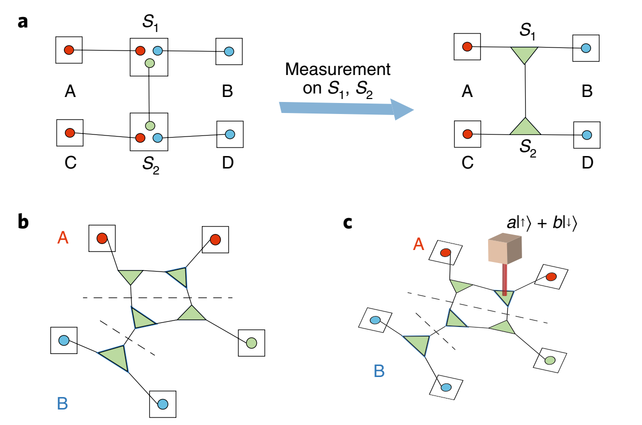

Fig. 3 | Tensor network state and the encoded bulk qubit. a, Tensor network state, built by preparing EPR pairs (coloured balls connected by a line), and then carrying an entangled measurement on intermediate server nodes S_1 and S_2. **b**, The two regions A (red dots) and B (blue dots) have the same size but different maximal entropy. A can have more entropy than B because its entanglement with the complement is mediated by two qubits rather than one. **c**, A bulk qubit (orange cube) is encoded by a tensor network with a bulk leg (red vertical link) into boundary quantum states.

of the boundary has no way to access the bulk spin, so that the bulk spin looks far away for her. Locality and upper limit of speed—the speed of light—are therefore not exactly true in such a world, but it effectively holds for all local observers that do not have the ability of simultaneously measure a big part of the Universe. Intriguingly, this mechanism is the same as quantum error correction code (QECC) in quantum information theory. The idea of error correction is to store the information redundantly, so that if part of the storage device is corrupted, the information can still be restored from the remaining part (Fig. 2b).

With this analogy, two boundary states corresponding to the two bulk spin states, $|\uparrow\rangle$ and $|\downarrow\rangle$, can be regarded as an encoding of these bulk qubits into the boundary system. The fact that the qubit cannot be accessed from a small region of the boundary guarantees that the stored information is robust against errors in the boundary system, as long as the corrupted region is not too big. What looks like locality in the bulk actually follows from the quantum error correction property of the duality as an encoding map. The QECC properties are also closely related to the RT formula[20]: roughly speaking, the RT entropy is a 'resource of entanglement' that allows QECC to occur.

Tensor networks

How do we make further progress based on this new understanding of locality? Are properties like the RT formula and QECC suggesting that we are looking for some very special theory, or are they generic properties satisfied by a large class of models? What are the key ingredients for achieving such properties? To gain more understanding, it is helpful to construct toy models with similar properties. Starting from Swingle's work[21], a family of such toy models known as tensor networks have been devised. Historically, tensor networks have been examined in the context of condensed matter physics as variational wavefunctions for strongly correlated systems[22–24]. The network that is particularly relevant to Swingle's work is known as the multiscale entanglement renormalization ansatz (MERA) proposed by Vidal[24].

A tensor network is a many-body wavefunction obtained by 'gluing' few-body quantum states, which are the tensors. Building a tensor network is similar to connecting computers to the Internet. When we communicate by e-mail or instant messenger, our computers are not directly sending signals to each other. Instead, we send signals to some e-mail servers, which mediate the communication. Similarly, a tensor network state was built by first preparing some Einstein–Podolsky–Rosen (EPR) entangled pairs of qubits, and then measuring some qubits in an entangled basis (Fig. 3a). The measured qubits are now in some entangled pure state, and they 'glue' the remaining qubits into a more complicated entangled state, just like how e-mail servers connect us to a communication network. In such a way, one can build complicated quantum entanglement even if each node only entangles a few qubits[25].

It is natural to relate tensor networks to holographic duality because the entanglement entropy of a tensor network is controlled by its graph geometry. For example, Fig. 3b shows a tensor network in which region A and B have the same size, but A can have a higher entanglement entropy than B since there are more EPR pairs mediating the entanglement of A with its complement. In general, if each EPR pair maximally entangles D states of the two qubits, the entanglement entropy of a region A is bounded from above by the minimal number of cuts that separate A and its complement, times

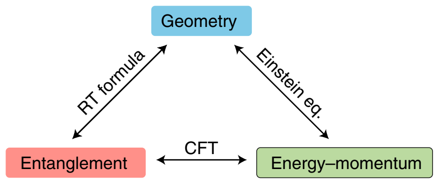

Fig. 4 | The relation between entanglement, bulk geometry and energy–momentum of the boundary. The entropy of states is related to the area of an extremal surface through the RT formula, while the linearized Einstein equation relates the extremal surface to the energy–momentum distribution at the boundary. On the other hand, the conformal symmetry allows one to relate the energy–momentum and the entanglement entropy for small perturbations around the vacuum.

log D. If the actual entropy is proportional to this upper bound, the RT formula applies, which is not true for all tensor networks.

Special tensor networks with RT entanglement entropy and quantum error correction properties have been constructed using stabilizer codes[26,27] and random tensors with large bond dimension[28]. Roughly speaking, a tensor network with a random tensor on each node can be thought as a random state with the restriction given by the network geometry. For the same reason that a random state in the Hilbert state is almost maximally entangled[29], the entanglement entropy of a random tensor network is closed to the maximal allowed value, which is given by the RT formula. A bulk qubit can then be introduced by a small perturbation in the bulk, such as changing one of the tensors (Fig. 3c). Again the randomness helps in making sure that the qubit is encoded in a highly entangled way so that no information can be revealed within small regions of the boundary.

The random tensor networks provide a large family of states that are interesting toy models for exploring holographic duality. One interesting feature is that the RT formula holds for random tensor networks with large bond dimension, even if the geometry is not hyperbolic. This provides some hope that even when we go beyond AdS space and think about quantum gravity in more general geometries[30–32], the random tensor networks may still be useful. However, there are many aspects of quantum gravity that tensor networks have not captured so far. A major open question is how to describe the gravitational dynamics. Ideally, one would like to introduce a Hamiltonian to describe the dynamics of tensor network states, and map that to some dynamics of the bulk geometry. However, the randomness in choice of tensors makes that difficult. In three dimensions, random tensor networks have been related to gravitational action through Regge calculus[33].

Dynamics and chaos

Since geometry is a characterization of the entanglement structure, it is natural to expect that the dynamics of geometry, described by Einstein's equation, should be related to the dynamics of entanglement. Indeed, in the special case of small perturbations around the ground state of conformal field theory, Van Raamsdonk and collaborators[34,35] have derived the linearized Einstein's equation from the RT formula. The idea of this proof is summarized in the triangle in Fig. 4. The RT formula relates the entropy of states to the area of an extremal surface. On the other hand, the conformal symmetry allows one to relate the energy momentum and the entanglement entropy for small perturbations around the vacuum. Therefore, the area of the extremal surface is related to the energy–momentum distribution at the boundary, which turns out to be equivalent to the linearized Einstein equation. One expects the derivation of the Einstein equation to hold for other background geometries, which corresponds to states far from the CFT vacuum, but the proof has not been generalized to that extent.

On general geometry, we do not understand the dynamics from the boundary point of view, but certain aspects of the bulk gravitational dynamics have an interesting interpretation on the boundary. Consider a black hole geometry in the bulk, which corresponds to a thermal state on the boundary. Consider two particles a and b, which scatter near the black hole, with particle b moving towards the boundary (Fig. 5). To escape to the boundary with a definite energy, b has to have a much higher energy near the horizon. The closer the scattering is to the horizon, the higher is b's energy as it approaches the horizon, and the later will b reach the boundary. From the boundary point of view, if we write down the annihilation operators of these two particles, $\hat{a}(0)$ and $\hat{b}(0)$, the near-horizon scattering effect translates to a commutator that grows in time t. In fact, $[\hat{b}(t), \hat{a}^\dagger(0)] \propto e^{2\pi T t}$ grows exponentially with a universal exponent $2\pi T$ (where T is the temperature). This is a signature that the boundary dynamics are chaotic, such that a single particle operator 'scrambles'[36] and evolves into a complicated multi-particle operator, which thus can have a non-trivial commutator with more and more single particle operators. This commutator growth is characterized by out-of-time-ordered correlation (OTOC) functions[36–38]. The exponential growth with exponent $2\pi T$ is actually maximal among all quantum systems[39], so that holographic theories not only have to be chaotic, but maximally chaotic. An explicit model with maximal chaos has been recently put forward and studied, known as the Sachdev–Ye–Kitaev model[40–42].

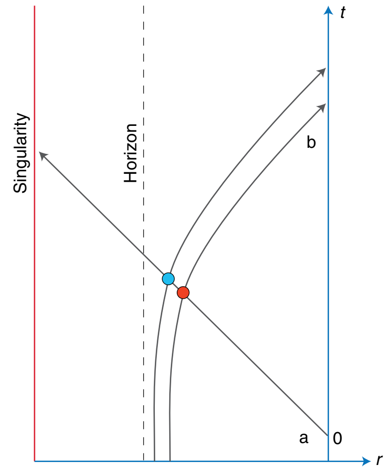

Fig. 5 | The scattering between two particles, a and b, near the black hole horizon. The scattering amplitude grows exponentially with the arrival time of the particle b.

The operator scrambling is also related to the QECC property discussed above. When a qubit is thrown towards the black hole horizon, it becomes more and more difficult to access, because the corresponding boundary perturbation becomes more and more non-local. In this sense, one can say that the chaotic dynamics of the boundary provides the QECC that protects quantum information in the bulk, and the information hiding behind the horizon is protected best. This naturally leads to many further questions: what happens when the qubit hits the singularity? Is information behind the horizon accessible from the boundary, if we can carry non-local measurement on the boundary? If yes, how do we make this point of view consistent with the causal structure of the bulk and a smooth geometry across the horizon? There are a lot of open questions related to various forms of black hole information paradox, such as the firewall paradox[43].

Towards quantum gravity

It is clear that there is a long way to go before we have a complete theory of quantum gravity. However, it is fair to say that the recent developments have already significantly changed our view of gravity and spacetime. Concepts developed in quantum information theory, such as entanglement entropy and quantum error correction, enter the description of spacetime in a fundamental way. Other fundamental aspects of quantum mechanics, such as the complexity of quantum circuits, have also been proposed to have gravity counterparts in the dynamics of black holes[44,45]. The goal is to describe precisely how spacetime geometry and gravity emerge from quantum information characteristics of many-body states. If we speculate how the quantum gravity theory will look, there remain two different possibilities: gravity may be entirely an emergent phenomenon in quantum mechanics, which means quantum mechanics is the most fundamental theory of our world[46]; or perhaps we find that to describe gravity beyond the AdS background we have to go beyond quantum mechanics, such that gravity and quantum mechanics are different approximations of a more fundamental theory.

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