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TIME IN PHILOSOPHY AND IN PHYSICS: FROM KANT AND EINSTEIN TO GÖDEL

> For the physical world, the four dimensions are natural. But for the mind, there is no such natural coordinate system; time is the only natural frame of reference.
>
> Gödel, conversation on 15.3.72

ABSTRACT. The essay centers on Gödel's views on the place of our intuitive concept of time in philosophy and in physics. It presents my interpretation of his work on the theory of relativity, his observations on the relationship between Einstein's theory and Kantian philosophy, as well as some of the scattered remarks in his conversations with me in the seventies – namely, those on the philosophies of Leibniz, Hegel and Husserl – as a successor of Kant – in relation to their conceptions of time.

Both physical and mental processes take place in time – 'that mysterious and seemingly contradictory being, which, on the other hand, seems to form the basis of the world's and our own existence' (Gödel 1990, p. 202). As Augustine puts it in his *Confessions* (Book 11, Ch. 14):

> For what is time? Who is able so much as in thought to comprehend it, so as to express himself concerning it? And yet what in our usual discourse do we more familiarly and knowingly make mention of than time?

Time plays an important part in everyone's life and there is a vast literature – in psychology, in physics, in biology, in philosophy, in history, in literature, and elsewhere – devoted to the various aspects of time. For instance, a survey of the philosophical issues and familiar studies of them is given under the entries on time (by J. J. C. Smart) and on time-consciousness (by C. W. K. Mundle) in Edwards' *Encyclopedia of Philosophy*. – The relation between time and its mathematization is a specialized concern of physics, which illustrates the applications of mathematics and the pervasive task of giving form to our experience.

The familiar representation of time by a directed line in Newtonian physics is based on an analogy with space. Time itself is in the first place a frame of our inner states; it has nothing to do with shape or position – in its literal spatial sense. Its spatial representation by a line, nonetheless, facilitates our thinking about it and, in particular, brings the organization of our operations with it into a mathematical frame. In Kant's words (*Pure Reason*, A33 or B50):

> We represent the time-sequence by a line progressing to infinity, in which the manifold constitutes a series of one dimension only; and we reason from the properties of this line to

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all the properties of time; with this one exception, that, while the parts of the line are simultaneous, the parts of time are already successive.

By way of this spatial analogue of time, we are able to represent space and time mathematically in such a manner that much of what is in our intuitive conception of space and time is preserved. Given the fact that time is primarily a frame of our inner sense, it is remarkable that this mathematization, through spatialization or externalization, of time – a highly precise but inflexible way of giving form to experience – has turned out to connect our inner and outer senses so well as to agree so completely with our observations of the external world for so long.

At the same time, it is not clear how we are to understand time as thus represented. On the one hand, it is generally believed that time and change are objective in the sense that the physical world 'consists of an infinity of layers of "now" which come into existence successively' (Gödel 1990, pp. 202–3). On the other hand, Kant and some other philosophers consider time and change as an appearance due to our special mode of perception. In Kant's philosophy, in particular, the structure of space and time for physical reality in itself may be totally different from that of the appearance, even though we have no way of knowing what it is like.

Eventually the increase of our physical experience and the refinement of its organization led to the introduction and the general acceptance of the theory of relativity in the early part of this century. – In this theory, the global organization of local observations reveals a more complex relation – than the Newtonian scheme – of the locally observed simultaneity and temporal succession to the assumption of a uniform lapse of time which sees the world as one infinite sequence of successive layers of 'now.'

There is a definite sense in which Einstein's theory is an improvement of Newton's: this sense could be extracted from the extended considerations which had led to the general acceptance of it. – Since Kant based his philosophy of our scientific knowledge on Newton's physics, a natural question is to determine the extent to which this progress in physics affects the relevant parts of Kant's philosophy. The usual emphasis is on the conflict between these parts and the philosophical implications of the new physics. In contrast, Gödel chooses to uncover and argue for a surprising similarity, in some respects, between relativity theory and Kant's doctrine about time and space.

In 1976 Gödel told me that his work on relativity theory had been caused by his interest in Kant's philosophy of space and time rather than by his frequent talks with Einstein. – Later in his 1949b, he stated more specifically that he had been struck by the agreement 'between Kant and relativistic physics insofar as in both theories the objective existence of a time in the Newtonian sense is denied.'

Gödel's interests in physics and in Kant's philosophy began early. At the age of 16 he read some of Kant's work; at about the same time, he

evaluated Goethe's dispute with Newton, and decided to side with Newton. The interest in Newton's work must have played a part in his choice to specialize in physics, from 1924 to 1926, when he was 18 to 20. An indication of his deep concern with the philosophical aspect of physics was the fact that he requested from the library, on 26.1.25, Kant's book of 1786 on the foundations of natural science. – His close contacts with Einstein started around 1942 and lasted till Einstein's death in 1955.

In June 1946 Schilpp invited Gödel to contribute an article to a volume to honor Einstein on his seventieth birthday in 1949. It appears that, for the next 4 or 5 years, Gödel concentrated on his 'Kant and Einstein' project. The fruits of this work included two published mathematical papers and a philosophical one which appeared in the Einstein volume (all reprinted in Gödel 1990). In addition, he wrote also three versions of a paper (1946–49) specifically on the relationship between theory of relativity and Kantian philosophy, and gave his lecture 1949b in May 1949 at the Institute for Advanced Study.

1. ON THE THREE PAPERS PUBLISHED BY GÖDEL HIMSELF

According to our intuitive concept of time, the world consists of an infinite succession of layers of world-states. Any two point events either belong to the same world-state or belong to two different world-states, of which one is earlier and the other is later. They are simultaneous in the first case, and one of them is later than the other in the second case. Temporal priority defines a complete linear ordering of all point-events, through their equivalence classes defined by the relation of simultaneity.

The content of Gödel's essay in the Einstein volume (Gödel 1990, pp. 202–7) and his two mathematical papers (pp. 190–8, 208–16) may be summarized as follows.

The theory of relativity reveals that, in a systematic and precise organization of all our physical experience, it is by no means obvious that there is a unique natural linear ordering of all world-points or point-events. In the first place, according to the special theory of relativity, each state of (uniform) motion determines a frame of reference and a temporal order. For two observers in different states of motion, the temporal orders are different, so that, for instance, it is possible to see A happening before B by one observer and B happening before A by the other. Since we can envisage observers in very different states of motion, there are, relative to such observers, very different temporal orders. None of these temporal orders, however, can claim, without arbitrariness, the prerogative of being the natural or the real one, which is to represent the true universal lapse of time.

In the general theory of relativity, the presence of matter introduces a new factor which helps to eliminate the equal claims of different observers and to single out the distinguished observers 'which follow in their motion

the mean motion of matter.' In fact, in all the solutions of Einstein's gravitational equations known before Gödel's new ones, 'the local times of all these observers fit together into one world time' (p. 204), which, moreover, may be said to be 'an "absolute" time coordinate' (p. 190). It seems, therefore, reasonable to single out this distinguished temporal order as the true one.

Gödel, however, questions this conclusion on two grounds: (1) he has found new solutions of Einstein's equations to which the procedure of defining the distinguished temporal order is not applicable; (2) his solutions show that a definition of true time of the type just suggested – even if it happens to be true of the actual world as we know it – is not satisfactory, because it is not determined entirely by the laws of nature but 'depends on the particular way in which matter and its motion are arranged in the world' (p. 207).

The new solutions found by Gödel are called *rotating universes* by him, because 'the compass of inertia in them everywhere rotates relative to matter, which in our world would mean that it rotates relative to the totality of galactic systems' (p. 204, note 10). It is not possible to define a distinguished world time in such universes because 'these worlds possess such properties of symmetry that for each possible concept of simultaneity and succession there exist others which cannot be distinguished from it by any intrinsic property, but only by reference to individual objects, e.g., a particular galactic system' (p. 204). – Indeed, the *non-existence* of a world time of the type – an 'absolute' time coordinate – defined by 'a one-parameter system of three-spaces everywhere orthogonal on the world lines of matter' is equivalent with 'a rotation of matter relative to the compass of inertia' (p. 190).

The rotating solutions given by Gödel's first mathematical paper (see his 1990, pp. 190–8) have the additional surprising feature of including closed time-like lines. 'In particular, if P, Q are any two points on a world line of matter, and P precedes Q on this line, there exists a time-like line connecting P and Q on which Q precedes P' (p. 191). Consequently, a sort of time-travel is possible in worlds of this kind. Elsewhere Gödel defends the non-absurdity of this consequence by establishing that 'time-travel in them would not be *practically* possible' (p. 205). I shall return to this complex issue later on.

The rotating solutions in his second paper (pp. 208–16) are expanding and include ones which do not have the possibility of travel into the past. Physicists seem to believe that these models are reasonable, although the rotations may be so slow that it will be hard to observe (see pp. 189–90). – According to Freeman Dyson, Gödel continued to be much interested in studying the new data from observations, presumably for the purpose of checking the possibility that the structure of the real world might conform to one of his solutions.

The rotating solutions of this paper include both ones with closed time-

like lines and ones without them. In this context, Gödel uses a more liberal concept of world-time than the one quoted above (from p. 190): 'For these solutions, also, *the nonexistence of closed time-like lines is equivalent with the existence of a "world-time,"* where by a world-time we mean an assignment of a real time t to every space-time point, which has the property that t always increases if one moves along a time-like line in its positive direction' (p. 213). Consequently, some of his solutions have these properties: (a) they are expanding, and (b) a 'world-time' in the specified sense can be defined in them. These rotating solutions are the models which 'could well be a reasonable description of the universe that we observe' (p. 189).

The above summary of Gödel's technical results is, I believe, sufficient for the purpose of interpreting and discussing his relevant philosophical ideas on several levels. – On the factual level, there is the question whether the world is indeed a rotating one, for which there exists no distinguished *true* world time. Apparently this possibility is, for the expanding solutions at least, as I just mentioned, not excluded by what we know at present. – The static solutions with closed time-like lines are not satisfactory because, as Gödel himself notes, they, being static, fail to account for the observed red-shifts of distant galaxies (p. 206).

On the theoretical level, Gödel makes a distinction between natural laws and contingent facts. Exactly how the distinction is to be drawn seems to me to depend on some more or less arbitrary decisions. As I see it, the distinction is one between laws of different degrees of generality. For instance, Gödel says that the cosmological constant 'evidently forms part of the laws of nature' (note 14, p. 206). On the other hand, he clearly sees the red-shifts as a contingent fact, since, in discussing his static solutions with closed time-like lines, he says in the same context that 'such conditions prevail in certain possible worlds' (p. 206).

Indeed, using the distinction between laws of nature and contingent facts, Gödel argues that, even if such solutions do not agree with the *actual* world, they do prevail in certain *possible* worlds, namely in worlds which obey the same laws of nature as our actual one but contain certain different contingent facts. In other words, even if a true world time can be defined for our actual world – but only with the help of contingent facts, this situation is not a sufficient reason for us to think of the world as possessing a true or objective or absolute time on a priori or conceptual grounds. Gödel seems to suggest: even if a physical theory is confirmed for which a true world time can be defined for the actual world, the possibility of a *better* physical theory incompatible with a true world time is not excluded.

The ordinary, intuitive concept of time includes two distinguishable components: the *content* of time and the *lapse* or *flow* of time or the *change* of the content in time. – When we say that the river flows, we have in mind the fact that the same water particles are over different parts

of the river bed at different times. When we think of time itself as flowing, we face the difficulty of finding something which could be said to occupy different positions at different times. I am conscious of the fact that I am writing now, with some memory of what I have just written and some anticipation of what I shall probably write soon. I experience a succession of my different states of mind, as though these states were flowing through my mind. This feeling seems to suggest the idea that time itself flows too.

When I project my consciousness of time onto the world, I get an idea of objective or absolute time. On the one hand, there is a continuing sequence of world-states which individually are at rest and collectively constitute the material content of time. On the other hand, there is a mysterious process of lapse or flow or change, by which every world-state travels through being future, present, and past. If we leave out my consciousness and that of other beings, then it is hard to see what is so special about Now, or indeed to make sense of the very distinction between past, present and future. Without this distinction, however, time would be like space in the sense that there is no flow and there is no distinguished direction or arrow of time.

When consciousness is made relevant to the study of physics by taking observations into consideration, the projection of the subjective consciousness of time onto the physical world is infected by the limited perspectives of the conscious observers due to their incapacity to observe more than a small part of the immense world. Consequently, there may be, on the observational level, different local schemes of space and time. There is no necessary reason why the intuitive concept of time, which is now being applied to physical events as observed from different local perspectives, would automatically also hold for the whole world and at the same time do justice to all local schemes of time. Indeed, as is shown by some of Gödel's solutions of Einstein's equations, it is not even excluded, by these equations alone, that, for every possible definition of world time which fits together all local schemes, there are events A and B such that both A precedes B and B precedes A along some time-like line.

Since physics, though it is developed by consciousness with the help of observations, aims at describing the world of physical objects as they are in themselves, the concept of Now is not part of physics. Even if physics does contain a world time as a linear ordering of world-states, it has no special place for Now. Shortly before his own death, Einstein wrote to the son and the sister of his friend Michele Besso, who had just died (Bernstein, p. 165): 'So in quitting this world he has once again preceded me by a little. That doesn't mean anything. For those of us who believe in physics, this separation between past, present, and future is only an illusion, however tenacious.'

A year or two before this, Einstein talked to Carnap about the relation between physics and the concept of Now (Carnap, pp. 37–8). He said that the experience of the Now, which means something special for human

beings, cannot be grasped by science, and that the important distinction of Now from the past and the future cannot occur within physics: 'there is something essential about Now [for our human needs] which is just outside the realm of science.' – It might be suggested that the experience of Now can be studied in psychology. Since, however, psychology as a science is commonly studied under the assumption of a psychophysical parallelism, it is hard to see how psychology, in its present form, could possibly capture what is essential for us about Now.

While it is clear that Now is important for us, there seem to me to be good reasons for not giving it a special place in our conception of the physical world and, generally, for not requiring physical time to possess all the properties of our intuitive concept of time. – There is for us an obvious sense of existence according to which only what is now exists: in this sense, the past has ceased to exist, the future does not yet exist. If we require physical time to have all the properties of intuitive time, it seems to follow that only the present world-state truly exists: but such a conclusion appears not to conform to our intuitive concept of physical reality.

It seems to me that this line of thought is implicit in Gödel's idea of an objective lapse of time and his apparent belief that it does not and should not exist. Thus, according to him, the existence of an objective lapse of time means 'that reality consists of an infinity of layers of "now" which come into [and go out of] existence successively' (pp. 202–3). On account of this association of intuitive time with existence, the lapse of time cannot be both relative and objective: 'A relative lapse of time, however, if any meaning at all can be given to this phrase, would certainly be something entirely different from the lapse of time in the ordinary sense, which means a change in the existing. The concept of existence, however, cannot be relativized without destroying its meaning completely' (note 5 on p. 203).

Moreover, it would not help to postulate – over and above the lapses for different observers – an intrinsic lapse of time that is detached from all experience: 'A lapse of time, however, which is not a lapse of time in some definite way seems to me as absurd as a colored object which has no definite colors.' – In any case, 'even if such a thing were conceivable, it would again be something totally different from the intuitive idea of the lapse of time' under consideration.

2. RELATIVITY THEORY AND KANTIAN PHILOSOPHY

Gödel's yet-unpublished manuscripts on relativity theory and Kantian philosophy have more to say about philosophy and its history – of course with special reference to Kant. – In the very first footnote, Gödel says explicitly that he is 'not an adherent to Kantian philosophy in general.' His principal interest in this context is to contrast intuitive time with

physical time by proposing and reflecting on some new interpretations – both of Kant's doctrine about time (and space) and of the theory of relativity. – I shall confine my attention to Manuscript C, which apparently was the latest of the three versions.

According to Gödel, Kant's main doctrine concerning our natural conception of the world is 'its largely subjectivistic character, even as to those concepts which seem to constitute the very backbone of reality.' Using his interpretation of relativity theory (including his own results about it), Gödel concludes that relativity theory confirms this doctrine of Kant's – in the sense that it supports Kant's belief in the subjectivistic character of this natural conception, by revealing that it is by no means fully true of the physical world as it is in itself. Specifically, our intuitive concept of time, which is an essential part of this natural conception, is more clearly seen – with the help of relativity theory, strengthened by Gödel's interpretation of it – to be not necessarily true of reality itself, which exists independently of us and our consciousness.

What has been refuted by modern physics is, Gödel says, 'Kant's view concerning the impossibility for theoretical science of stepping outside the limits of our natural conception of the world.' – If we accept modern physics as the best available organization of our physical knowledge, then Kant's 'conviction of the unknowability (at least by theoretical reason) of things in themselves,' Gödel says, 'should be modified' – 'it should be assumed that it is possible for scientific knowledge, at least partially and step by step, to go beyond appearances and approach the things in themselves.' – The theory of relativity may be viewed as an advance in our knowledge which supports this assumption.

According to Gödel's interpretation, both Kantian philosophy and relativity theory recognize that our intuitive time and its physical counterpart may differ. With respect to this similarity, Gödel observes: 'Kant doubtless held the difference between intuitive time and its objective correlate to be far greater than it is by relativity theory, even in the R-worlds [namely, Gödel's rotating universes]: in fact, so great that the latter cannot be described at all in concepts understandable for human beings.'

Kant's mistake, or at least the disagreement between his doctrines and modern physics, is, for Gödel, not that our intuitive conception cannot fully capture reality itself, but that his philosophy sets an eternal limit on our theoretical knowledge of things in themselves. – The fact that relativity theory is accepted as an improvement over Newton's scheme indicates that we are capable of knowing more about things in themselves than merely their Kantian appearances as determined by the Kantian frame of our natural conception of the world.

Once this possibility is acknowledged, it is rational to expect that we can in the future get to know more about the things in themselves and to discover better organizations, possibly in unexpected shapes, of our experience – than what is contained in physics now. Indeed, Gödel men-

tions (in his note 57), as an example, the fifth dimension introduced by Th. Kaluza in 1921 (*S. B. Preuss. Ak. Wiss.*, 1921, p. 966). – The idea is presumably that what is intended by the notion of a world-point might be more adequately represented by five, rather than only four, coordinates. – Of course there may also emerge other surprising advances in physics – such as, as Gödel also suggests, a better understanding of quantum mechanics through 'a satisfactory description of objective reality which would make the success of its rules of computation understandable.'

In terms of our intuitive concept of time, Kant's doctrine and relativity theory agree that it is a characteristic or ordering inherent – not in the things themselves but – only in the relation of them to our senses. The difference is that temporal properties of things are, for Kant, the same for all human beings, but they could, in relativity theory, be different for different observers.

The most familiar application of relativity theory to refute Kant's doctrine is of course to his claim that our picture of the physical space, which is determined by our a priori form of intuition, is necessarily Euclidean – in view of the fact that physical space is not Euclidean according to relativity theory. Regardless of whether we have an a priori spatial intuition, however, as Gödel observes, 'space and its properties express themselves also in the sensations, which we know only a posteriori.' It follows that the character of physical space as a part of physical theory may change as our experience increases. The successful inclusion of Euclidean space in Newtonian physics depends on the fact that, on the basis of experience available before the latter part of the last century, 'by projecting the sensations in a certain way on a three-dimensional Euclidean space, the laws connecting them can be expressed in a certain simple form.' Kant failed to notice this dependence of the spatial component of physical theory on our experience.

In this connection, Gödel wrote in a letter of 1973 (for source, see Wang 1987, p. 154):

> Geometrical intuition, strictly speaking, is not mathematical, but rather a priori physical intuition. In its purely mathematical aspect our Euclidean space intuition is perfectly correct, namely, it represents correctly a certain structure existing in the realm of mathematical objects. Even physically it is correct "in the small."

There is a fair amount of discussion about Gödel's work on time and relativity theory in the literature. Yourgrau 1991 is, in particular, an extended study, which considers – apart from Gödel's own writings – also much of the secondary literature, as well as related philosophical issues such as time and infinity, time and existence, what I have characterized as 'the dialectic of the formal and the intuitive,' etc. – I have derived much benefit from Yourgrau's writings and discussions with him on Gödel's treatment of time and relativity theory.

Within philosophy, the place of time in the physical world is but a

special issue about time, on which something more definite can be said than on some other issues, because of its close connection with physical theory. – Since philosophy attempts to trace things back to their beginnings, an obvious – and widely adopted – starting point is our own experience. Consider, for instance, the fundamental distinctions between subject and object, mind and matter, subjective and objective, universal and particular: all these pairs are ultimately divisions derived from experience – made within the mind. But time is, as Gödel says, the only natural coordinate system for the mind. It is, therefore, clear that the study of time, notoriously hard as it is, demands a very special place in philosophical investigations.

Gödel's discussion of the relationship between Kantian philosophy and relativity is a striking illustration of how philosophy and science may interplay in a fruitful manner. In my opinion, an especially illuminating observation is Gödel's use of the idea of steps or levels of *objectivation* proposed in Bollert 1921 (Chapter 7, pp. 46–57) – in the context of asserting the possibility of scientific knowledge 'to go beyond appearances [in the Kantian sense] and approach the things in themselves.'

In this context Gödel adds a note 42, which remains unchanged in all the three manuscripts of his yet unpublished essay:

> [Each step or level of objectivation] is obtained from the preceding one by the elimination of certain subjective elements. The "natural" world picture, i.e. Kant's world of appearances itself, also must of course be considered as one such level, in which a great many subjective elements of the "world of sensations" have already been eliminated. Unfortunately, whenever this fruitful viewpoint of a distinction between subjective and objective elements in our knowledge (which is so impressively suggested by Kant's comparison with the Copernican system) appears in the history of science, there is at once a tendency to exaggerate it into a boundless subjectivism, whereby its effect is annulled. Kant's thesis of the unknowability of the things in themselves is one example, another one is the prejudice that the positivistic interpretation of quantum mechanics must necessarily be the final stage of the theory.

As I understand this pregnant passage, the fruitful viewpoint of objectivation is to begin with the totality of all our (subjective) experiences at a given historical stage, to distinguish different kinds of component in these experiences, and to make a distinction between those components which vary from perspective to perspective, from person to person – the subjective elements – and those components which are invariant – the objective elements. In this way, even though we are never sure that we have fully captured objective reality as it transcendently is in itself, we have a powerful distinction between the subjective and the objective which serves as a fruitful basis of organizing all our experience: in such a way that the objective elements provide us with a frame of reference which resembles the natural frame of transcendent reality itself but has the advantage of being (more) accessible.

Kant's 'Copernican revolution' may be seen as a spectacularly successful objectivation of this type. His world of appearances is an attractive organi-

zation of the objective elements – in the sense of this objectivation – which provides a surprisingly satisfactory structure of our theoretical knowledge and of the interplay between mind and the world. – There is, however, a difficult choice to be made: how is this neat 'objective' world of appearances related to reality itself – the things in themselves? On the one hand, it is hard to envisage how theoretical knowledge of reality itself can go beyond (what is knowable of) this world of appearances, given its richness and its internal coherence. On the other hand, it seems dogmatic to assert that theoretical knowledge can never tell us anything more about reality itself than its effects on us which are captured in Kant's world of appearances.

As is well known, Kant chooses more or less the second alternative. When Gödel speaks of 'a tendency to exaggerate it [such as Kant's objectivation] into a boundless subjectivism,' he undoubtedly has in mind this choice by Kant and, even more unqualifiedly, by some of Kant's followers. Impressive as Kant's objectivation is, it is, according to what I take to be Gödel's own view, but one level of objectivation: 'it is possible for scientific knowledge, at least partially and step by step, to go beyond the [Kantian] appearances and approach the things in themselves.' One major purpose of his consideration of the relationship of Kantian philosophy to relativity theory is to see the latter as an evidence for this view of his, which is part of his version of 'rationalistic optimism' (see Wang 1974, p. 325).

For the same reason, Gödel considers 'the positivistic interpretation of quantum mechanics' to be another example of exaggerating a level of impressive objectivation into a boundless subjectivism. Since the rules of computation according to quantum mechanics are highly successful, the interpretation asserts that it embodies all we can know about reality itself, despite the necessity to assume, for instance, an action at distance with very strange properties (see note 57 of Gödel's manuscript C and its reference to Einstein–Podolsky–Rosen 1935). – The 'boundless subjectivism' in this case is again the position of taking one level of successful objectivation to be necessarily 'the final stage of the theory,' so that what are seen to be objective on this level are taken to be (parts of) reality itself or at least what we can ever know about it.

Gödel said that he had begun to study Husserl's work in 1959. The passage which I have just discussed had probably been written before 1950. It seems to me to express already a viewpoint which fits well into Husserl's outlook, at least to the extent of sharing a belief in the principle that our (direct) experiences in their full richness are the *ultimate* data for philosophy. In my opinion, the passage also gives some indication of the sense in which Gödel takes, as is clear from his observations from time to time, Husserl's phenomenology as a major development of – what is, to some extent, only implicit in – the best part of Kant's philosophy.

3. CLOSED TIME-LIKE LINES AND TIME TRAVEL

In connection with his rotating universes with closed time-like lines, Gödel asserts that 'it is possible in these worlds to travel into any region of the past, the present, and the future, and back again, exactly as it is possible in other worlds to travel to distant parts of space.' In reply to the standard objection – that one could then go back in time and undo the past, he said that it is practically impossible 'to complete the voyage in a reasonable length of time' (p. 205). – This complex argument seems to involve an interplay of the world as it really is with both human knowledge and human action.

It is hard for many of us to see either the force of the objection or the need for the defense Gödel offers. As Stein says in his 1990 – his introductory note to this essay: 'Objections of the type "What if I were to go back and, for example, murder my own younger self?" admit a perfectly straightforward answer: in cosmos of the sort in question, that act would simply not be possible' (p. 199).

If a description of the world, whether or not it includes closed time-like lines, is taken as given or perhaps also as true, then it depicts a fixed whole with all its parts *fixed*, so that there is no more change or travel in it, unless we envisage changing the original world into a different one. The idea of travelling to the past implicitly appeals to a perspective from outside of the whole world as it is: it is related to the difficulty of knowing a whole completely from the perspective of one who is entirely within it. In general, if we think of a world as a completed whole, then everything in it is already fixed. What we think of as changes in it belong to our interpretation of it for the purpose of gaining a grasp of how the parts of the whole fit together. On the other hand, if we knew the whole world – with or without closed time-like lines – as it is, and if we have free will, it would always be possible – theoretically at least – to make the world different from what it is.

In a draft of Manuscript C, Gödel did include in note 53 similar observations – which were, however, crossed out. If we restore these parts and put them in square brackets, note 53 includes:

> This and similar contradictions, however, [presuppose, not only the practical feasibility of the trip into the past (velocities very close to that of light would be necessary for it) but also certain decisions on the part of the traveller; whose possibility one concludes only from vague conviction of the freedom of the will. Practically the same inconsistencies (again by neglecting certain "practical" difficulties) can be derived from the assumption of strict causality and the freedom of the will in the sense just indicated. Hence, as far as the paradoxical situation under consideration is concerned, an R-world is not any more absurd than any world subject to strict causality.] in order to prove the impossibility of the R-worlds, presuppose at least the practical feasibility of the trip into the past, which may well be precluded by the velocities very close to that of light which would be necessary for it, or by other circumstances. [Therefore, this argument against the possibility of R-worlds is as little conclusive as that against strict causality which disregards the practical impossibility of

determining precisely the physical state of the world at a given moment and of carrying through the necessary computations for predicting the future. (Under these assumptions one would be in a position to do the opposite of what the theory predicts.)]

In Stein 1995 – his introductory note to this essay – Stein quotes the first deleted segment and says: 'This is in one respect very reassuring to the commentator; in another, disconcerting: Why did Gödel change his mind?' – The structure of the whole passage appears to suggest that Gödel at first deleted the first segment before continuing with the part about practical feasibility, and that he then deleted the second segment too.

If, following Gödel's notation in this context, we think of an R-world as a rotating universe with closed time-like lines, we may say that a theory of the cosmos which offers an R-world as its model is a special case of theories of 'strict causality.' When any such theory T is *proposed*, a natural question is whether it can possibly be true at all. The claim is not only that T is true but also that *we know* it to be true. If we begin with the premiss that T is known to be true by us now, then we may ask whether our other knowledge enables us to do something to falsify T. Gödel's argument from practical impossibility answers *this* question by replying that our other available knowledge indicates the practical impossibility of executing our proposed refutation of his particular theory T. In this sense, his defense is not pointless. It shows that his proposed theory T of R-worlds may not only be true but also be *known* to be true.

From this perspective, we may interpret Gödel's decision to cross out the comparison with other theories of strict causality as based on his wish to avoid getting involved in the thorny question of free will. – It is sufficient to disprove, merely by appealing to current knowledge, the objection that his theory of R-worlds is absurd. If so, why should he distract our attention by bringing in the difficulty shared by all theories of strict causality? – Moreover, once we bring in the shared difficulty, we face a more complex question: Can we know any cosmic theory with strict causality? – There are, as we know, familiar objections to an affirmative answer, which are hard to disentangle.

I am under the impression that Gödel wishes to believe it possible for us to know a cosmic theory with strict causality. If so, he seems to have to face the question of free will. To say that we have no free will would contradict our intuitive feeling and make moral responsibility dependent on a sort of hidden illusion. To admit free will would compel us to believe that we are either unwilling or unable to exercise our free will effectively to refute a cosmic theory with strict causality which is claimed to be known to be true. It is hard to see how either alternative could be made persuasive. – Gödel is, I believe, well aware of these obstacles to his desire to believe that we are capable of knowing a true cosmic theory with strict causality.

Rucker's quotations from what Gödel said in 1972 seem to reveal to us one of Gödel's attempts to deal with this dilemma (Rucker, p. 168):

It should be possible to form a complete theory of human behavior, i.e., to predict from the hereditary and environmental givens what a person will do. However, if a mischievous person learns of this theory, he can act in a way so as to negate it. Hence I conclude that such a theory exists, but that no mischievous person will learn of it. In the same way, time-travel is possible, but no person will ever manage to kill his past self.

There is no contradiction between free will and knowing in advance precisely what one will do. If one knows oneself completely, then this *is* the situation. One does not deliberately do the opposite of what one wants.

In his discussions with me in the seventies, Gödel occasionally hinted at views not unlike those expressed in these quotations – but always with a sort of apologetic smile, which seemed to me to indicate that he was aware of the fact that such a position was not convincing.

4. GÖDEL'S OBSERVATIONS IN HIS DISCUSSIONS WITH ME

In his discussions with me in the seventies Gödel made some scattered brief observations on the place of time in our experience – in its relation to the pursuit of philosophy. These observations are suggestive but by no means unambiguous for those, like me, who have only a very partial understanding of many of the subtleties of his thoughts.

Let me first give my reconstruction, from rough notes, of a list of Gödel's relevant observations.

Q1(15.3.72). The four dimensions of space-time are natural for the physical world. But there is no such natural coordinate system for the mind; time is the only natural frame of reference.

Q2(15.12.72). The Newtonian scheme was to a considerable extent obtained a priori. Proportionality, space, and time were a priori, while force, which produces acceleration, was empirical. Husserl believed that by his method, one can get Newton's scheme – even a better one – even without the scientific knowledge of Newton's time.

Q3(15.12.72). What remains in Husserl's approach is the observation of the working of the mind: this is the way to make the concept of time, etc. clear – not by studying how they work in science.

Q4(15.12.72). We forget how we arrived at the concept of time in our childhood and do not know how we use it. When we try to think about time, our reason is for making certain statements, yet our mind is working and working – on nothing at all. – For 25 years Husserl worked on just this one problem: the concept of time. [The period from 1893 to 1917 is indicated in volume 10 of *Husserliana*, which is devoted to Husserl's work on internal time-consciousness.]

Q5(25.11.75). Husserl's unpublished work does not contain more on time than his published work. – As we present time to ourselves it simply does not agree with fact. To call time subjective is just a euphemism for this failure. Problems remain. One problem is to describe how we arrive at time.

Q6(25.11.75). Another problem is the relation of our concept of time to real time. The real idea behind time is causation: the time structure of the world is just its causal structure. Causation in mathematics, in the sense of, say, a fundamental theorem causing its consequences, is not in time, but we take it as a scheme in time.

Q7(25.11.75). In terms of time, there are different moments and different worlds. [One interpretation of this remark is to take it as a reference to the different worlds determined by the spatiotemporal schemes of different observers.]

Q8(26.11.75). In sense perception what is originally given is not lost, but in our experience having to do with time and mathematical objects we lose a large part of what is originally given.

Q9(27.11.75). Causation is unchanging in time and does not imply change. It is an empirical – but not a priori – fact that causation is always accompanied by change. Change is subjective in the Einstein universe. For Kant, change is the essence of time.

Q10(8.12.75). Independently of Hegel's primitive terms [which begin with being, nothing and becoming], the process is not in time nor an analogy with history. It is right to begin with being, because we have to have something to talk about. But becoming should not come immediately after being and nothing: this is taking time too seriously. It is very clear that possibility is the synthesis between being and nothing. It is an essential and natural definition of possibility to take it as the synthesis of being and nothing. – Possibility is a weakened form of being.

Q11(29.11.75). Synthesis is always a reinforcement of thesis, i.e. possibility, i.e. force; the antithesis is in empirical fact. Being in time is too special and should not appear so early as in Hegel's scheme. A complete understanding should reduce everything to these elements [something like Hegel's categories or ultimately the initial categories]. How you go on may be different from Hegel.

[Q11 is related to Gödel's idea that the meaning of the world – undoubtedly understood by him in the sense of the reason why it is what it is – is the separation of force – or wish if we assume, as Gödel does, that every substance is a monad which is always conscious to some extent – and fact. As I understand his observation, being as thesis is wish or force which encounters its antithesis in fact, which modifies the wish to arrive at a synthesis. Synthesis is force or wish to realize some new possibility which reinforces the original thesis by taking into consideration the resistance from its antithesis which is in empirical fact.]

Q12(8.12.75). Time is no specific character of being. In relativity theory the temporal relation is like far and near in space. I do not believe in the objectivity of time. The concept of Now never occurs in science itself and science is supposed to be concerned with the objective [all that is objective]. Kant was before Hegel. [I take the last observation to mean that, even though Hegel was later, he regressed from Kant's right view of time.]

Q13(23.1.76). [Once I asked Gödel to tell me some specific impressive results which had been obtained by using Husserl's phenomenological method, so that I could learn the method by studying such examples. In reply, he mentioned Husserl's work on time, but added that the important part had been lost. – Even though Gödel usually praised Husserl's work, he did occasionally express his frustration in studying it. I have a record of what he said on one of these occasions:] I don't like particularly Husserl's way: long and difficult. He tells us no detailed way about how to do it. His work on time has been lost from the manuscripts.

It is clear from the above list that I have not been able to obtain a satisfactory reconstruction of Gödel's pregnant but fragmentary observations on time. Two basic points in Gödel's view on time are, however, clear. (1) Time is subjective, at least when it is understood in the sense of our intuitive concept of it (Q9 and Q12); it is to be clarified by observing the working of the mind (Q3). (2) Clarification of the concept of time is fundamental to the study of philosophy which depends centrally on clarifying how the mind works (Q1 and the several references to Husserl's approach); this task is very difficult (Q4, Q5, Q9, Q13).

There are in these quotations some terminological difficulties which typically are hard to avoid in observations which deal with fundamental situations but are stated outside of a comprehensive context. – One difficulty is Gödel's calling it a *euphemism* to call time subjective (Q5). He may be objecting to the idea that, being subjective, the concept of time is to be studied in (empirical) psychology, as it is commonly pursued today. – The two remaining problems suggested in Q5 and Q6 are an indication of his belief that there are in any case specific difficulties to be overcome before we can reach a clear understanding of our concept of time. In other words, he is objecting to those who give up the attempt to clarify our intuitive concept of time, by using as an excuse the euphemism that it is *subjective*. – In any case, while acknowledging that we have so far failed to attain a clear understanding of the intuitive concept of time, he believes that it is possible – and indeed extremely important for the advance of philosophy – to reach such an understanding.

Indeed, according to Gödel's general philosophical position, objective reality includes both the physical and the conceptual worlds, of which we can know better and better. In particular, he believes, I think, that there is a sharp concept which corresponds to our vague intuitive concept of time – only we have so far not yet found the right perspective to perceive it clearly. (Compare what he says about the concept of mechanical procedure on pp. 84–5 Wang 1974.)

In Q6, Gödel contrasts our intuitive concept of time with 'real time,' and says that the real idea behind time is causation. I take him to be saying that, even though our concept of time is not objective in the sense of being inherent in physical reality, there is an objective relation – that

of causation – which lies behind our idea of a real or objective temporal structure of reality and may, somewhat misleadingly, be called the *real time*. Under this interpretation of Q6, our natural tendency to think of the physical world as the spatio-temporal reality is a result of our habit of associating causation with time and change.

Observation Q9 suggests that Gödel wishes to dissociate causation from time and change, which, according to his views discussed before, are not objective. When we, however, try to capture the causal structure of the physical world without appealing to the concept of time, we still seem to need something like matter or physical objects to serve as the bearers of causes and effects.

If we begin with one of Gödel's rotating universes, with or without closed time-like lines – as a representation of the causal structure of the whole, completed physical world – as something fixed, we may, theoretically, make do without applying the concept of change and the concept of time linked to it. But we would still be thinking in terms of four-dimensional world-points which involve a residue of our intuitive conception of space and time, as it is embodied in Newton's and Kant's schemes. – This situation may be the reason why Gödel continues to speak of 'real time,' while asserting at the same time that causation is the real idea behind time.

The concept of causation does involve the concept of succession and its iteration, whether or not they are temporal. Gödel's example of causation in mathematics, mentioned in Q6, is probably intended as an illustration of the fact that not all successions are in time. Once we remove the restriction to the temporal, the order of causal succession need no longer possess all the properties of temporal order as required by our concept of time. It may be a partial ordering or also a relation which is symmetric or circular so that, within what is ordered by the causal structure, it is possible for A to both precede B and succeed B in the relation. – Clearly, causal dependence in general may involve more complex relations than linearly ordered causal chains. – Whether or not effect can precede cause is a controversial issue, which is discussed in Charles Taylor's comprehensive entry on causation in Edwards 1967.

In his discussions with me, Gödel indicated clearly that he was aiming at using Husserl's method to arrive at something like the monadology of Leibniz. It seems that he was also interested in projects like Hegel's system of categories or 'science of logic.' The observations Q10 and Q11 appear to be hinting at a way to modify Hegel's scheme to arrive at a satisfactory theory of the conceptual world. One of Gödel's main criticisms of Hegel's approach is that he is taking time too seriously.

On the whole, Gödel seems to favor the fundamental perspective of seeing objective reality, both the physical and the conceptual, as eternal, timeless, and fixed. At the same time, he believes that it is possible for

us, at least partially and step by step, to go beyond every seemingly natural stopping point, such as the Kantian realm of phenomenon or appearance, and approach closer to objective reality itself.

On the other hand, our internal consciousness of time is an essential ingredient of our experience, since, as Gödel asserts in Q1, it is the only natural coordinate system for the mind. Gödel's repeated mention of Husserl's lost work on time suggests a belief on his part that a satisfactory understanding of the working of our time-consciousness would be a decisive advance for philosophy. It would be of interest to ask the related question: what would follow if we had such an understanding?

Gödel's observation Q2 illustrates his belief in the important part which a priori philosophical reflections can play in the study of fundamental science. In particular, even though our intuitive concept of time is not objective, by being clear about it and about the other concepts mentioned in Q2, we are supposed to be able to arrive at something like Newton's fruitful scheme, or even a better one, on the basis of everyday experience alone.

In his discussions with me, Gödel made a number of observations which are relevant to the ideas expressed in the statements Q1 to Q13. In particular, he gave a formulation of his view on the character and the value of Husserl's work, mentioned a Leibnizian contrast between science and philosophy, and made several statements on the different merits and defects of the approaches of Leibniz, Hegel and Husserl to philosophy – directly or indirectly related to their attitudes toward time.

Q14(24.11.71). Husserl's is a very important method as an entrance into philosophy – finally to arrive at some metaphysics. Transcendental phenomenology with bracketing as its methodology is the investigation – without knowledge of scientific facts – of the cognitive process, to find out what really appears to be: to find the objective concepts.

Q15(22.3.76). According to a Leibnizian idea, science only *combines* concepts, it does not *analyze* concepts. For instance, from this Leibnizian perspective, Einstein's theory of relativity in itself is not an analysis of concepts, but it is stimulating for real analysis. It deals with observations and does not penetrate into the last analysis, because it presupposes certain metaphysics, which is distinct from the true metaphysics of the Leibnizian science. On the other hand, real analysis strives to find the correct metaphysics.

Q16(18.10.72). Husserl also thinks that mathematical logic should not be made the basis of philosophical investigations – not the chief tool but the basic tool, insofar as it clarifies the foundation for all conceptual thinking that reveals the fundamental structure of rational speech. – The basis of everything is meaningful predication: x is P, x belongs to A, $x\,R\,y$, etc. There is no other basis. Husserl had this. Hegel did not have this: that is why his philosophy lacks clarity. – Mathematical logic is important for carrying out ideas, not for finding the right ideas. Idealistic philoso-

phers are not able to make good ideas precise and into a science. – Husserl introduced a method: clearly every mathematician had that in his head before mathematical logic was formulated – it is just the axiomatic method. [This defect of Hegel's approach may, I believe, be seen as related to his taking time too seriously.]

Q17(8.11.72). Husserl had in mind something like intuitive knowledge in the sense of Leibniz. Even Schelling adhered to this ideal; but Hegel moved away from it. – Kant was a skeptic or at least believed that skepticism is necessary for the transition to true philosophy. [The reference to Leibniz here is, I believe, to his concept of intuitive knowledge as set out in his paper of 1684 (Loemker, pp. 291–5).]

Q18(19.10.75). With regard to the structure of the real world, Leibniz did not go nearly as far as Hegel, but merely gave some preliminary polemics.

Q19(29.11.75). Leibniz put more emphasis than Hegel on real definitions – to get higher level concepts from lower level ones.

On several occasions Gödel said that his philosophy is, in its general outline, like the monadology of Leibniz. It seems to me that his idea of the interplay of force or wish with fact corresponds to the interplay of appetition with perception in the Leibnizian monadology. The central place of causation in Gödel's scheme undoubtedly is associated with force or wish or appetition. The derivative character of space and time in ontology is also a part of the metaphysics of Leibniz. – The observations just quoted suggest that Gödel felt it possible to make good use of, not only the ideas of Leibniz and Husserl, but also, to some extent, those of Hegel.

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