Spin-orbit coupled Fermi liquid theory of ultracold magnetic dipolar fermions

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We investigate Fermi liquid states of the ultracold magnetic dipolar Fermi gases in the simplest two-component case including both thermodynamic instabilities and collective excitations. The magnetic dipolar interaction is invariant under the simultaneous spin-orbit rotation but not under either the spin or the orbit one. Therefore, the corresponding Fermi liquid theory is intrinsically spin-orbit coupled. This is a fundamental feature of magnetic dipolar Fermi gases different from electric dipolar ones. The Landau interaction matrix is calculated and is diagonalized in terms of the spin-orbit coupled partial-wave channels of the total angular momentum J. The leading thermodynamic instabilities lie in the channels of ferromagnetism hybridized with the ferronematic order with $J = 1^+$ and the spin-current mode with $J = 1^-$, where $+$ and $-$ represent even and odd parities, respectively. An exotic propagating collective mode is identified as spin-orbit coupled Fermi surface oscillations in which spin distribution on the Fermi surface exhibits a topologically nontrivial hedgehog configuration.

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I. INTRODUCTION

Recent experimental progress of ultracold electric dipolar heteronuclear molecules has become a major focus of ultracold atom physics.[1–3] Electric dipole moments are essentially classic polarization vectors induced by the external electric field. When they are aligned along the z axis, the electric dipolar interaction becomes anisotropic exhibiting the $d_{r^2-3z^2}$-type anisotropy. In Fermi systems, this anisotropy has important effects on many-body physics including both single-particle and collective properties.[4–14] Fermi surfaces of polarized electric dipolar fermions exhibit quadrupolar distortion elongated along the z axis.[4,5,7,13] Various Fermi surface instabilities have been investigated including the Pomeranchuk-type nematic distortions[6,7] and stripelike orderings.[10,14] The collective excitations of the zero-sound mode exhibit anisotropic dispersions: The sound velocity is largest if the propagation wave vector $\vec{q}$ is along the z axis, and the sound is damped if $\vec{q}$ lies in the xy plane.[7,8] Under the dipolar anisotropy, the phenomenological Landau interaction parameters become tridiagonal matrices, which are calculated at the Hartree-Fock level,[6,7] and the anisotropic Fermi liquid theory for such systems has been systematically studied.[7]

The magnetic dipolar gases are another type of dipolar system. Compared to the extensive research on electric dipolar Fermi systems, the study on magnetic dipolar ones is a new direction of research. On the experimental side, laser cooling and trapping Fermi atoms with large magnetic dipole moments (e.g., ^{161}Dy and ^{163}Dy with $\mu = 10\mu_B$)[15–17] have been achieved, which provides a new opportunity to study exotic many-body physics with magnetic dipolar interactions. There has also been a great amount of progress for realizing Bose-Einstein condensations of magnetic dipolar atoms.[17–21]

Although the energy scale of the magnetic dipolar interaction is much weaker than that of the electric one, it is conceptually more interesting if magnetic dipoles are not aligned by external fields. Magnetic dipole moments are proportional to the hyperfine spin up to a Lande factor, thus, they are quantum-mechanical operators rather than the nonquantized classic vectors as electric dipole moments are. Furthermore, there is no need to use external fields to induce magnetic dipole moments. In fact, the unpolarized magnetic dipolar systems are isotropic. The dipolar interaction does not conserve spin nor orbit angular momentum but is invariant under simultaneous spin-orbit (SO) rotation. This is essentially a spin-orbit coupled interaction. Different from the usual spin-orbit coupling of electrons in solids, this coupling appears at the interaction level but not at the kinetic-energy level.

The study of many-body physics of magnetic dipolar Fermi gases is just at the beginning. For the Fermi liquid properties, although magnetic dipolar Fermi gases were studied early in Refs. 6 and 22, the magnetic dipoles are frozen, thus, their behavior is not much different from the electric ones. It is the spin-orbit coupled nature that distinguishes nonpolarized magnetic dipolar Fermi gases from polarized electric ones. The study along this line was was pioneered by Fregoso and Fradkin.[23,24] They studied the coupling between ferromagnetic and ferronematic orders, thus, spin polarization distorts the spherical Fermi surfaces and leads to a spin-orbit coupling in the single-particle spectrum.

Since Cooper pairing superfluidity is another important aspect of the many-body phase, we also briefly summarize the current progress in electric and magnetic dipolar systems. For the single-component electric dipolar gases, the simplest possible pairing lies in the p-wave channel because s-wave pairing is not allowed by the Pauli exclusion principle. The dipolar anisotropy selects the p_z-channel pairing.[25–32] Interestingly, for the two-component case, the dipolar interaction still favors the triplet pairing in the p_z channel, even though the s wave is also allowed. It provides a robust mechanism for the triplet pairing to the first order in the interaction strength.[33–36] The mixing between the singlet and the triplet pairings is with a relative phase $\pm\frac{\pi}{2}$, which leads to a novel time-reversal symmetry-breaking pairing state.[33] The investigation of the unconventional Cooper pairing symmetry in magnetic dipolar systems was studied by the authors.[37] We have found that it provides a robust mechanism for a novel p-wave ($L = 1$) spin triplet ($S = 1$) Cooper pairing to the first order in interaction strength. It comes directly from the attractive part of the magnetic dipolar interaction. In comparison, the triplet Cooper pairings in ^{3}He and solid-state systems come from

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spin fluctuations, which is a second-order effect in interaction strength.[38,39] Furthermore, that pairing symmetry was not studied in ^{3}He systems before in which orbital and spin angular momenta of the Cooper pair are entangled into the total angular momentum $J = 1$. In contrast, in the ^{3}He-B phase,[40] L and S are combined as $J = 0$, and in the ^{3}He-A phase, L and S are decoupled, and J is not well defined.[41,42]

Fermi liquid theory is one of the most important paradigms in condensed-matter physics on interacting fermions.[38,43] Despite the pioneering papers,[6,22–24] a systematic study of the Fermi liquid properties of magnetic dipolar fermions is still lacking in the literature. In particular, Landau interaction matrices have not been calculated, and a systematic analysis of the renormalizations from magnetic dipolar interactions to thermodynamic quantities has not been performed. Moreover, collective excitations in magnetic dipolar ultracold fermions have not been studied before. All these are essential parts of Fermi liquid theory. The experimental systems of ^{161}Dy and ^{163}Dy are with a very large hyperfine spin of $F = \frac{21}{2}$, thus, the Fermi liquid theory, taking into account of all the complicated spin structure, should be very challenging. We take the first step by considering the simplest case of spin-$\frac{1}{2}$ magnetic dipolar fermions, which preserve the essential features of spin-orbit physics and address the above questions.

In this paper, we systematically investigate the Fermi liquid theory of the magnetic dipolar systems including both the thermodynamic properties and the collective excitations, focusing on the spin-orbit coupled effect. The Landau interaction functions are calculated and are diagonalized in the spin-orbit coupled basis. Renormalizations for thermodynamic quantities and the Pomeranchuk-type Fermi surface instabilities are studied. Furthermore, the collective modes are also spin-orbit coupled with a topologically nontrivial configuration of the spin distribution in momentum space. Their dispersion relation and configurations are analyzed.

Upon the completion of this paper, we became aware of the nice work by Sogo *et al.*[44] Reference 44 constructed the Landau interaction matrix for dipolar fermions with a general value of spin. The Pomeranchuk instabilities were analyzed for the special case of spin $\frac{1}{2}$, and collective excitations were discussed. Our paper has some overlaps on the above topics with Ref. 44 but with a significant difference, including the physical interpretation of the Pomeranchuk instability in the $J = 1^-$ channel and our discovery of an exotic propagating spin-orbit sound mode.

The remaining part of this paper is organized as follows. The magnetic dipolar interaction is introduced in Sec. II. The Landau interaction matrix is constructed at the Hartree-Fock level and is diagonalized in Sec. III. In Sec. IV, we present the study of the Fermi liquid renormalization to thermodynamic properties from the magnetic dipolar interaction. The leading Pomeranchuk instabilities are analyzed. In Sec. V, the spin-orbit coupled Boltzmann equation is constructed. We further perform the calculation of propagating spin-orbit coupled collective modes. We summarize the paper in Sec. VI.

II. MAGNETIC DIPOLAR HAMILTONIAN

We introduce the magnetic dipolar interaction and the subtlety of its Fourier transform in this section.

The magnetic dipolar interaction between two spin-$\frac{1}{2}$ particles located at $\vec{r}_{1,2}$ reads

$$V_{\alpha\beta;\beta'\alpha'}(\vec{r}) = \frac{\mu^2}{r^3}[\vec{S}_{\alpha\alpha'} \cdot \vec{S}_{\beta\beta'} - 3(\vec{S}_{\alpha\alpha'} \cdot \hat{r})(\vec{S}_{\beta\beta'} \cdot \hat{r})], \quad (1)$$

where $\vec{S} = \frac{1}{2}\vec{\sigma}, \alpha, \alpha', \beta, \beta'$ take values of $\uparrow$ and $\downarrow$, $\vec{r} = \vec{r}_1 - \vec{r}_2$, and $\hat{r} = \vec{r}/r$ is the unit vector along $\vec{r}$.

The Fourier transform of Eq. (1) is

$$V_{\alpha\beta;\beta'\alpha'}(\vec{q}) = \frac{4\pi\mu^2}{3}[3(\vec{S}_{\alpha\alpha'} \cdot \hat{q})(\vec{S}_{\beta\beta'} \cdot \hat{q}) - \vec{S}_{\alpha\alpha'} \cdot \vec{S}_{\beta\beta'}], \quad (2)$$

which depends on the direction along the momentum transfer but not its magnitude. It is singular as $\vec{q} \to 0$. More rigorously, $V_{\alpha\beta,\beta'\alpha'}(\vec{q})$ should be further multiplied by a numeric factor[7] as

$$g(q) = 3\left(\frac{j_1(q\epsilon)}{q\epsilon} - \frac{j_1(qL)}{qL}\right), \quad (3)$$

where ϵ is a short-range scale cutoff and L is the long-distance cutoff at the scale of sample size. The spherical Bessel function $j_1(x)$ shows the asymptotic behavior $j_1(x) \to \frac{x}{3}$ at $x \to 0$, and $j_1(x) \to \frac{1}{x}\sin(x - \frac{\pi}{2})$ as $x \to \infty$. In the long wavelength limit satisfying $q\epsilon \to 0$ and $qL \to \infty$, $g(q) \to 1$, and we recover Eq. (2). If $\vec{q}$ is exactly zero, $V_{\alpha\beta;\beta'\alpha'} = 0$ because the dipolar interaction is neither purely repulsive nor attractive, and its spatial average is zero.

The second quantization form for the magnetic dipolar interaction is expressed as

$$H_{\text{int}} = \frac{1}{2V}\sum_{\vec{k},\vec{k}',\vec{q}} \psi^\dagger_\alpha(\vec{k}+\vec{q})\psi^\dagger_\beta(\vec{k}')V_{\alpha\beta;\beta'\alpha'}(\vec{q}) \times \psi_{\beta'}(\vec{k}'+\vec{q})\psi_{\alpha'}(\vec{k}), \quad (4)$$

where V is the volume of the system. The density of states of two-component Fermi gases at the Fermi energy is $N_0 = \frac{mk_f}{\pi^2\hbar^2}$, and we define a dimensionless parameter $\lambda = N_0\mu^2$. λ describes the interaction strength, which equals the ratio between the average interaction energy and the Fermi energy up to a factor on the order of 1.

III. SPIN-ORBIT COUPLED LANDAU INTERACTION

In this section, we present the Landau interaction functions of the magnetic dipolar Fermi liquid and perform the spin-orbit coupled partial-wave decomposition.

A. The Landau interaction function

Interaction effects in the Fermi liquid theory are captured by the Landau interaction function. It describes the particle-hole channel forward-scattering amplitudes among quasiparticles on the Fermi surface. At the Hartree-Fock level, the Landau function is expressed as

$$f_{\alpha\alpha',\beta\beta'}(\hat{k},\hat{k}') = f^H_{\alpha\alpha',\beta\beta'}(\hat{q}) + f^F_{\alpha\alpha',\beta\beta'}(\hat{k},\hat{k}'), \quad (5)$$

where $\vec{k}$ and $\vec{k}'$ are at the Fermi surface with the magnitude of k_f and $\vec{q}$ is the small momentum transfer in the forward-scattering process in the particle-hole channel. $f^H_{\alpha\alpha',\beta\beta'}(\vec{q}) = V_{\alpha\beta,\beta'\alpha'}(\hat{q})$ is the direct Hartree interaction, and

$f^F_{\alpha\alpha',\beta\beta'}(\vec{k};\vec{k}') = -V_{\alpha\beta,\alpha'\beta'}(\vec{k}-\vec{k}')$ is the exchange Fock interaction. As $\vec{q}\to 0$, f^H is singular, thus, we need to keep its dependence on the direction of $\hat{q}$. More explicitly,

$$f^H_{\alpha\alpha',\beta\beta'}(\hat{q}) = \frac{\pi\mu^2}{3} M_{\alpha\alpha',\beta\beta'}(\hat{q}), \tag{6}$$

$$f^F_{\alpha\alpha',\beta\beta'}(\hat{k};\hat{k}') = -\frac{\pi\mu^2}{3} M_{\alpha\alpha',\beta\beta'}(\hat{m}), \tag{7}$$

where the tensor is defined as $M_{\alpha\alpha',\beta\beta'}(\hat{q}) = 3(\vec{\sigma}_{\alpha\alpha'}\cdot\hat{q})(\vec{\sigma}_{\beta\beta'}\cdot\hat{q}) - \vec{\sigma}_{\alpha\alpha'}\cdot\vec{\sigma}_{\beta\beta'}$ and $\hat{m}$ is the unit vector along the direction of the momentum transfer $\hat{m} = \frac{\vec{k}-\vec{k}'}{|\vec{k}-\vec{k}'|}$. We have used the following identity:

$$\begin{aligned} &3(\vec{\sigma}_{\alpha\beta'}\cdot\hat{m})(\vec{\sigma}_{\beta\alpha'}\cdot\hat{m}) - \vec{\sigma}_{\alpha\beta'}\cdot\vec{\sigma}_{\beta\alpha'} \\ &\quad = 3(\vec{\sigma}_{\alpha\alpha'}\cdot\hat{m})(\vec{\sigma}_{\beta\beta'}\cdot\hat{m}) - \vec{\sigma}_{\alpha\alpha'}\cdot\vec{\sigma}_{\beta\beta'} \end{aligned} \tag{8}$$

to obtain Eq. (7).

B. The spin-orbit coupled basis

Due to the spin-orbit nature of the magnetic dipolar interaction, we introduce the spin-orbit coupled partial-wave basis for the quasiparticle distribution over the Fermi surface following the steps below.

The $\delta n_{\alpha\alpha'}(\vec{k})$ is defined as

$$\delta n_{\alpha\alpha'}(\vec{k}) = n_{\alpha\alpha'}(\vec{k}) - \delta_{\alpha\alpha'} n_0(\vec{k}), \tag{9}$$

where $n_{\alpha\alpha'}(\vec{k}) = \langle\psi^\dagger_\alpha(\vec{k})\psi_{\alpha'}(\vec{k})\rangle$ is the Hermitian single-particle density matrix with momentum $\vec{k}$ and satisfies $n_{\alpha\alpha'} = n^*_{\alpha'\alpha}$ and $n_0(\vec{k})$ is the zero-temperature equilibrium Fermi distribution function $n_0(\vec{k}) = 1-\theta(k-k_f)$. $\delta n_{\alpha\alpha'}(\vec{k})$ is expanded in terms of the particle-hole angular momentum basis as

$$\begin{aligned} \delta n_{\alpha\alpha'}(\vec{k}) &= \sum_{Ss_z} \delta n_{Ss_z}(\vec{k})\chi_{Ss_z,\alpha\alpha'} \\ &= \sum_{Ss_z} \delta n^*_{Ss_z}(\vec{k})\chi^\dagger_{Ss_z,\alpha\alpha'}, \end{aligned} \tag{10}$$

where $\chi_{Ss_z,\alpha\alpha'}$ are the bases for the particle-hole singlet (density) channel with $S=0$ and triplet (spin) channel with $S=1$, respectively. They are defined as

$$\begin{aligned} \chi_{00,\alpha\alpha'} &= \delta_{\alpha\alpha'}, \quad \chi_{10,\alpha\alpha'} = \sigma_{z,\alpha\alpha'}, \\ \chi_{1\pm1,\alpha\alpha'} &= \frac{\mp 1}{\sqrt{2}}(\sigma_{x,\alpha\alpha'} \pm i\sigma_{y,\alpha\alpha'}), \end{aligned} \tag{11}$$

which satisfy the orthonormal condition $\mathrm{tr}(\chi^\dagger_{Ss_z}\chi_{S's'_z}) = 2\delta_{SS'}\delta_{s_zs'_z}$.

Since quasiparticles are only well defined around the Fermi surface, we integrate out the radial direction and arrive at the angular distribution,

$$\delta n_{\alpha\alpha'}(\hat{k}) = \int \frac{k^2dk}{(2\pi)^3}\delta n_{\alpha\alpha'}(\vec{k}). \tag{12}$$

Please note that angular integration is not performed in Eq. (12). We expand $\delta n_{\alpha\alpha'}(\hat{k})$ in the spin-orbit decoupled bases as

$$\begin{aligned} \delta n_{\alpha\alpha'}(\hat{k}) &= \sum_{LmSs_z} \delta n_{LmSs_z} Y_{Lm}(\hat{k})\chi_{Ss_z,\alpha\alpha'}, \\ &= \sum_{LmSs_z} \delta n^*_{LmSs_z} Y^*_{Lm}(\hat{k})\chi^\dagger_{Ss_z,\alpha\alpha'}, \end{aligned} \tag{13}$$

where $Y_{Lm}(\hat{k})$ is the spherical harmonics satisfying the normalization condition $\int d\hat{k}\, Y^*_{Lm}(\hat{k})Y_{Lm}(\hat{k}) = 1$.

We can also define the spin-orbit coupled basis as

$$\begin{aligned} \mathcal{Y}_{JJz;LS}(\hat{k},\alpha\alpha') &= \sum_{ms_z}\langle LmSs_z|JJ_z\rangle Y_{Lm}(\hat{k})\chi_{Ss_z,\alpha\alpha'}, \\ \mathcal{Y}^\dagger_{JJz;LS}(\hat{k},\alpha\alpha') &= \sum_{ms_z}\langle LmSs_z|JJ_z\rangle Y^*_{Lm}(\hat{k})\chi^\dagger_{Ss_z,\alpha\alpha'}, \end{aligned} \tag{14}$$

where $\langle LmSs_z|JJ_z\rangle$ is the Clebsch-Gordon coefficient and $\mathcal{Y}_{JJz;LS}$ satisfies the orthonormal condition of

$$\int d\hat{k}\, \mathrm{tr}[\mathcal{Y}^\dagger_{JJ_z;LS}(\hat{k})\mathcal{Y}_{J'J'_z;L'S'}(\hat{k})] = 2\delta_{JJ'}\delta_{J_zJ'_z}\delta_{LL'}\delta_{SS'}. \tag{15}$$

Using the spin-orbit coupled basis, $\delta n_{\alpha\alpha'}(\hat{k})$ is expanded as

$$\begin{aligned} \delta n_{\alpha\alpha'}(\hat{k}) &= \sum_{JJ_z;LS}\delta n_{JJ_z;LS}\mathcal{Y}_{JJ_z;LS}(\hat{k},\alpha\alpha') \\ &= \sum_{JJ_z;LS}\delta n^*_{JJ_z;LS}\mathcal{Y}^\dagger_{JJ_z;LS}(\hat{k},\alpha\alpha'), \end{aligned} \tag{16}$$

where $\delta n_{JJz;LS} = \sum_{ms_z}\langle LmSs_z|JJ_z\rangle\delta n_{LmSs_z}$.

C. Partial-wave decomposition of the Landau function

We are ready to perform the partial-wave decomposition for Landau interaction functions. The tensor structures in Eqs. (6) and (7) only depend on $\vec{\sigma}_{\alpha\alpha'}$ and $\vec{\sigma}_{\beta\beta'}$, thus, the magnetic dipolar interaction only contributes to the spin-channel Landau parameters, i.e., $S=1$. In the spin-orbit decoupled basis, the Landau functions of the Hartree and Fock channels are expanded, respectively, as

$$\begin{aligned} \frac{N_0}{4\pi} f^{H,F}_{\alpha\alpha';\beta\beta'}(\hat{k},\hat{k}') = &\sum_{Lms_z;L'm's'_z} Y_{Lm}(\hat{k})\chi_{1s_z}(\alpha\alpha') \\ &\times T^{H,F}_{Lm1s_z;L'm'1s'_z} Y^*_{L'm'}(\hat{k}')\chi^\dagger_{1s'_z}(\beta\beta'). \end{aligned} \tag{17}$$

For later convenience, we have multiplied the density of states N_0 and the factor of $1/4\pi$ such that $T^{H,F}$ are dimensionless matrices. Without loss of generality, in the Hartree channel, we choose $\hat{q} = \hat{z}$.

The matrix elements in Eq. (17) are presented below. In the Hartree channel,

$$T^H_{Lm1s_z;L'm'1s'_z} = \frac{\pi\lambda}{3}\left(2\delta_{s_z,0} - \delta_{s_z,\pm1}\right)\delta_{L,0}\delta_{L',0}\delta_{m,0}\delta_{m',0}\delta_{s_zs'_z}, \tag{18}$$

and in the Fock channel,

$$\begin{aligned} &T^F_{Lm1s_z;L'm'1s'_z} \\ &= -\frac{\pi\lambda}{2}\left(\frac{\delta_{LL'}}{L(L+1)} - \frac{\delta_{L+2,L'}}{3(L+1)(L+2)} - \frac{\delta_{L-2,L'}}{3(L-1)L}\right) \\ &\quad\times\int d\Omega_r\left[\delta_{s_zs'_z} - 4\pi Y_{1s_z}(\Omega_r)Y^*_{1s'_z}(\Omega_r)\right]Y_{Lm}(\Omega_r)Y^*_{L'm'}(\Omega_r). \end{aligned} \tag{19}$$

The magnetic dipolar interaction is isotropic, thus, the spin-orbit coupled basis is the most convenient. In these bases, the Landau matrix is diagonal with respect to the total angular momentum J and its z component J_z as

$$\frac{N_0}{4\pi} f_{\alpha\alpha';\beta\beta'}(\hat{k},\hat{k}') = \sum_{JJ_zLL'} \mathcal{Y}_{JJz;L1}(\hat{k},\alpha\alpha') \times F_{JJ_zL1;JJ_zL'1} \mathcal{Y}^{\dagger}_{JJz;L'1}(\hat{k},\beta\beta'). \quad (20)$$

The matrix kernel $F_{JJ_zL1;JJ_zL'1}$ reads as

$$F_{JJ_zL1;JJ_zL'1} = \frac{\pi\lambda}{3}\delta_{J,1}\delta_{L,0}\delta_{L',0}(2\delta_{J_z,0} - \delta_{J_z,\pm1}) + \sum_{ms_z;m's_z'} \langle Lm1s_z|JJ_z\rangle\langle L'm'1s_z'|JJ_z\rangle \times T^F_{Lm1s_z;L'm'1s_z'}. \quad (21)$$

We found that, up to a positive numeric factor, the second term in Eq. (21) is the same as the partial-wave matrices in the particle-particle pairing channel, which was derived for the analysis of the Cooper pairing instability in magnetic dipolar systems.[37]

However, the above matrix kernel $F_{JJ_zL1;JJ_zL'1}$ is not diagonal for channels with the same values of JJ_z but different orbital angular momentum indices L and L'. Moreover, the conservation of parity requires that even and odd values of L do not mix. Consequently, $F_{JJ_zL1;JJ_zL'1}$ is either diagonalized or reduced into a small size of just 2×2. For later convenience of studying collective modes and thermodynamic instabilities, we present below the prominent Landau parameters in some low partial-wave channels. Below, we use $(J^{\pm}J_zLS)$ to represent these channels in which $\pm$ represents even and odd parities, respectively.

The parity odd channel of $J = 0^-$ only has one possibility of (0^-011) in which

$$F_{0^-011;0^-011} = \frac{\pi}{2}\lambda. \quad (22)$$

There is another even-parity density channel with $J = 0^+$, i.e., (0^+000), which receives contribution from short-range s-wave interaction but no contribution from the magnetic dipolar interaction at the Hartree-Fock level. The parity odd channel of $J = 1^-$ only comes from (1^-J_z11) in which

$$F_{1^-J_z11;1^-J_z11} = -\frac{\pi}{4}\lambda. \quad (23)$$

Another channel of $J = 1^-$, i.e., (1^-J_z10), channel from the p-wave channel density interactions, which again receives no contribution from magnetic dipolar interaction at the Hartree-Fock level. These two $J = 1^-$ modes are spin- and charge-current modes, respectively, and thus, do not mix due to their opposite symmetry properties under time-reversal transformation.

We next consider the even-parity channels. The $J = 1^+$ channels include two possibilities of $(JJ_zLS) = (1^+J_z01),(1^+J_z21)$. The former is the ferromagnetism channel, and the latter is denoted as the ferronematic channel in Refs. 6 and 24. Due to the spin-orbit nature of the magnetic dipolar interaction, these two channels are no longer independent but are coupled to each other. Because the Hartree term breaks the rotational symmetry, the hybridization matrices for $J_z = 0, \pm1$ are different. For the case of $J_z = 0$, it is

$$F_{1^+0} = \begin{pmatrix} F_{1001;1001} & F_{1001;1021} \\ F_{1021;1001} & F_{1021;1021} \end{pmatrix} = \frac{\pi\lambda}{12}\begin{pmatrix} 8 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}, \quad (24)$$

whose two eigenvalues and their associated eigenvectors are

$$w_1^{1^+0} = 0.69\pi\lambda, \quad \psi_1^{1^+0} = (0.98,0.19)^T, \qquad w_2^{1^+0} = 0.06\pi\lambda, \quad \psi_2^{1^+0} = (-0.19,0.98)^T. \quad (25)$$

The hybridization is small. For the case of $J_z = \pm1$, the Landau matrices are the same as

$$F_{1^+1} = \begin{pmatrix} F_{1101;1101} & F_{1101;1121} \\ F_{1121;1101} & F_{1121;1121} \end{pmatrix} = \begin{pmatrix} -4 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}\frac{\pi\lambda}{12}. \quad (26)$$

Again, the hybridization is small as shown in the eigenvalues and their associated eigenvectors,

$$w_1^{1^+1} = -0.37\pi\lambda, \quad \psi_1^{1^+1} = (0.97,-0.25)^T, \qquad w_2^{1^+1} = 0.12\pi\lambda, \quad \psi_2^{1^+1} = (0.25,0.97)^T. \quad (27)$$

Landau parameters, or matrices, in other high partial-wave channels are neglected because their magnitudes are significantly smaller than those above.

We need to be cautious on using Eqs. (24) and (26) in which the Hartree contribution of Eq. (6) is taken. However, Eq. (6) is valid in the limit $q \ll k_f$ but should be much larger than the inverse of sample size $1/L$. It is valid to use Eqs. (24) and (26) when studying the collective spin excitations in Sec. V below. However, when studying thermodynamic properties, say, magnetic susceptibility, under the external magnetic-field uniform at the scale of L, the induced magnetization is also uniform. In this case, the Hartree contribution is suppressed to zero, thus, the Landau matrices in the $J = 1^+$ channel are the same for all the values of J_z as

$$F_{1^+,thm}(\lambda) = \begin{pmatrix} F_{1J_z01;1J_z01} & F_{1J_z01;1J_z21} \\ F_{1J_z21;1J_z01} & F_{1J_z21;1J_z21} \end{pmatrix}_{thm} = \frac{\pi\lambda}{12}\begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}. \quad (28)$$

In this case, the hybridization between these two channels is quite significant. The two eigenvalues and their associated eigenvectors are

$$w_1^{1^+} = -\frac{\pi}{12}\lambda, \quad \psi_1^{1^+} = \left(\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}\right)^T, \qquad w_2^{1^+} = \frac{\pi}{6}\lambda, \quad \psi_2^{1^+} = \left(\sqrt{\frac{1}{3}},\sqrt{\frac{2}{3}}\right)^T. \quad (29)$$

IV. THERMODYNAMIC QUANTITIES

In this section, we study the renormalizations for thermodynamic properties by the magnetic dipolar interaction and investigate the Pomeranchuk-type Fermi surface instabilities.

A. Thermodynamics susceptibilities

The change in the ground-state energy with respect to the variation in the Fermi distribution density matrix include the kinetic and interaction parts as

$$\frac{\delta E}{V} = \frac{\delta E_{\text{kin}}}{V} + \frac{\delta E_{\text{int}}}{V}. \tag{30}$$

The kinetic-energy variation is expressed in terms of the angular distribution of $\delta n_{\alpha\alpha'}(\hat{k})$ as

$$\begin{aligned}\frac{\delta E_{\text{kin}}}{V} &= \frac{4\pi}{N_0}\sum_{\alpha\alpha'}\int d\hat{k}\,\delta n_{\alpha\alpha'}(\hat{k})\delta n_{\alpha'\alpha}(\hat{k}) \\ &= \frac{8\pi}{N_0}\sum_{LmSS_z}\delta n^*_{LmSS_z}\delta n_{LmSS_z},\end{aligned} \tag{31}$$

where the units of $\delta n_{SS_z}(\hat{k})$ and δn_{LmSS_z} are the same as the inverse of the volume. The variation in the interaction energy is

$$\begin{aligned}\frac{\delta E_{\text{int}}}{V} &= \frac{1}{2}\sum_{\alpha\alpha'\beta\beta'}\iint d\hat{k}\,d\hat{k}'\,f_{\alpha\alpha',\beta\beta'}(\hat{k},\hat{k}')\delta n_{\alpha'\alpha}(\hat{k})\delta n_{\beta'\beta}(\hat{k}') \\ &= 2\sum_{Lms_zL'm's'_z;S}\delta n^*_{LmSs_z}f_{LmSs_z,L'm'Ss'_z}\delta n^*_{L'm'Ss'_z}.\end{aligned} \tag{32}$$

Adding them together and changing to the spin-orbit coupled basis, we arrive at

$$\frac{\delta E}{V} = \frac{8\pi}{N_0}\sum_{JJ_z;LL';S}\delta n^*_{JJ_z;LS}M_{JJ_zLS;JJ_zL'S}\delta n_{JJ_z;L'S}, \tag{33}$$

where the matrix elements are

$$M_{JJ_zLS;JJ_zL'S} = \delta_{LL'} + F_{JJ_zLS;JJ_zL'S}. \tag{34}$$

In the presence of the external field h_{JJ_zLS}, the ground-state energy becomes

$$\begin{aligned}\frac{\delta E}{V} = 16\pi\Bigg\{&\frac{1}{2\chi_0}\sum_{JJ_zLL'S}\delta n^*_{JJ_z;LS}M_{JJ_zLS;JJ_zL'S}\delta n_{JJ_z;L'S} \\ &-\sum_{JJ_zLS}h_{JJ_zLS}\delta n_{JJ_z;LS}\Bigg\},\end{aligned} \tag{35}$$

where $\chi_0 = N_0$ is the Fermi liquid density of states. At the Hartree-Fock level, N_0 receives no renormalization from the magnetic dipolar interaction. The expectation value of δn_{JJ_zLS} is calculated as

$$\delta n_{JJ_zLS} = \chi_0\sum_{L'}(M)^{-1}_{JJ_zLS;JJ_zL'S}h_{JJ_zL'S}. \tag{36}$$

For the $J = 1^+$ channel, $M^{-1} \approx I - F_{1^+,thm}(\lambda)$ up to first order of λ in the case of $\lambda \ll 1$. As a result, the external magnetic field $\vec{h}$ along the z axis not only induces the z-component spin polarization, but also induces a spin-nematic order in the channel of $(J^+J_zLS) = (1^+021)$, which is an effective spin-orbit coupling term as

$$\begin{aligned}\delta H = \frac{\sqrt{2}}{12}\pi\lambda h\sum_k\psi^\dagger_\alpha(\vec{k})\big\{\big[(k^2-3k_z^2)\sigma_z \\ -3k_z(k_x\sigma_x+k_y\sigma_y)\big]\big\}\psi_\beta(\vec{k}).\end{aligned} \tag{37}$$

Apparently, this term breaks time-reversal symmetry and, thus, cannot be induced by the relativistic spin-orbit coupling in solid states. This magnetic-field-induced spin-orbit coupling in magnetic dipolar systems was studied by Fregoso *et al.*[6] and Fregoso and Fradkin.[24]

B. Pomeranchuk instabilities

Even in the absence of external fields, Fermi surfaces can be distorted spontaneously known as Pomeranchuk instabilities.[45] Intuitively, we can imagine the Fermi surface as the elastic membrane in momentum space. The instabilities occur if the surface tension in any of its partial-wave channels becomes negative. In the magnetic dipolar Fermi liquid, the thermodynamic stability condition is equivalent to the fact that all the eigenvalues of the matrix $M_{JJ_zLS;JJ_zL'S}$ are positive.

We next check the negative eigenvalues of the Landau matrix in each partial-wave channel. Due to the absence of external fields, the Pomeranchuk instabilities are allowed to occur as a density wave state with a long wavelength $q \to 0$. For the case of $J = 1^+$, it is clear that, in the channel of $J_z = \pm 1$, the eigenvalue $w_1^{1^+1}$ in Eq. (27) is negative and the largest among all the channels. Thus, the leading channel instability is in the $(JJ_z) = (1^+ \pm 1)$ channel, which occurs at $w_1^{1^+1} < -1$, or, equivalently, $\lambda > \lambda^c_{1^+1} = 0.86$. The corresponding eigenvector shows that it is mostly a ferromagnetism order parameter with small hybridization with the ferronematic channel. A repulsive short-range s-wave scattering, which we neglected above will enhance ferromagnetism and, thus, will drive $\lambda^c_{1^+1}$ to a smaller value. The wave vector $\vec{q}$ of the spin polarization should be on the order of $1/L$ to minimize the energy cost of twisting spin, thus, essentially exhibiting a domain structure. The spatial configuration of the spin distribution should be complicated by actual boundary conditions. In particular, the three-vector nature of spins implies the rich configurations of spin textures. An interesting result is that the external magnetic field actually weakens the ferromagnetism instability. If the spin polarization is aligned by the external field, the Landau interaction matrix changes to Eq. (28). The magnitude of the negative eigenvalue is significantly smaller than that of Eq. (26). As a result, an infinitesimal external field cannot align the spin polarization to be uniform, but a finite amplitude is needed.

For simplicity, we only consider ferromagnetism with a single plane-wave vector $\vec{q}$ along the z axis, then the spin polarization spirals in the xy plane. Since $q \sim 1/L$, we can still treat a uniform spin polarization over a distance large comparable to the microscopic length scale. Without loss of generality, we set the spin polarization along the x axis. As shown in Ref. 24, ferromagnetism induces ferronematic ordering. The induced ferronematic ordering is also along the x axis, whose spin-orbit coupling can be obtained based on Eq. (37) by a permutation among components of $\vec{k}$ as $H'_{so}(\vec{k}) \propto (k^2 - 3k_x^2)\sigma_x - 3k_x(k_y\sigma_y + k_z\sigma_z)$. According to Eq. (27), ferromagnetism and ferronematic orders are not strongly hybridized, the energy scale of the ferronematic SO coupling is about 1 order smaller than that of ferromagnetism. An interesting point of this ferromagnetism is that it distorts the spherical shape of the Fermi surface as pointed by Fregoso and Fradkin.[24] This anisotropy will also affect the propagation of Goldstone modes. Furthermore, spin waves couple to the oscillation of the shape of Fermi surfaces bringing Landau damping to spin waves. This may result in non-Fermi liquid behavior for fermion excitations and will be studied in a later paper. This effect in the nematic symmetry-breaking Fermi liquid state has been extensively studied before in the literature.[46–51]

The next subleading instability is in the $J = 1^-$ channel with $L = 1$ and $S = 1$ as shown in Eq. (23), which is a spin-current channel. The generated order parameters are spin-orbit coupled. For the channel of $J_z = 0$, the generated SO coupling at the single-particle level exhibits the three-dimensional (3D) Rashba type as

$$H_{so,1^-} = |n_z| \sum_k \psi^\dagger_\alpha(\vec{k})(k_x\sigma_y - k_y\sigma_x)_{\alpha\beta}\psi_\beta(\vec{k}), \quad (38)$$

where $|n_z|$ is the magnitude of the order parameter. The same result was also obtained recently in Ref. 44. In the absence of spin-orbit coupling, the $L = S = 1$ channel Pomeranchuk instability was studied in Refs. 52 and 53, which exhibits the unconventional magnetism with both isotropic and anisotropic versions. They are particle-hole channel analogies of the p-wave triplet Cooper pairings of ^{3}He isotropic B and anisotropic A phases, respectively. In the isotropic unconventional magnetic state, the total angular momentum of the order parameter is $J = 0$, which exhibits the $\vec{k}\cdot\vec{\sigma}$-type spin-orbit coupling. This spin-orbit coupling is generated from interactions through a phase transition and, thus, was denoted as the spontaneous generation of spin-orbit coupling. In Eq. (38), the spin-orbit coupling that appears at the mean-field single-particle level cannot be denoted as spontaneous because the magnetic dipolar interaction possesses the spin-orbit nature. Interestingly, in the particle-particle channel, the dominant Cooper pairing channel has the same partial-wave property of $L = S = J = 1$.[37]

The instability in the $J = 1^-$ (spin-current) channel is weaker than that in the 1^+ (ferromagnetism) channel because the magnitude of Landau parameters is larger in the former case. The 1^- channel instability should occur after the appearance of ferromagnetism. Since spin-current instability breaks parity, whereas, ferromagnetism does not, this transition is a genuine phase transition. For simplicity, we consider applying an external magnetic field along the z axis in the ferromagnetic state to remove the spin texture structure. Even though the $J = 1^+$ and 1^- channels share the same property under rotation transformation, they do not couple at the quadratic level because of their different parity properties. The leading-order coupling occurs at the quartic order as

$$\delta F = \beta_1(\vec{n}\cdot\vec{n})(\vec{S}\cdot\vec{S}) + \beta_2|\vec{n}\times\vec{S}|^2, \quad (39)$$

where $\vec{n}$ and $\vec{S}$ represent the order parameters in the $J = 1^-$ and 1^+ channels, respectively. β_1 needs to be positive to keep the system stable. The sign of β_2 determines the relative orientation between $\vec{n}$ and $\vec{S}$. It cannot be determined purely from the symmetry analysis but depends on microscopic energetics. If $\beta_2 > 0$, it favors $\vec{n} \parallel \vec{S}$, and $\vec{n}\perp\vec{S}$ is favored at $\beta_2 < 0$.

V. THE SPIN-ORBIT COUPLED COLLECTIVE MODES

In this section, we investigate another important feature of the Fermi liquid, the collective modes, which again exhibit the spin-orbit coupled nature.

A. Spin-orbit coupled Boltzmann equation

We employ the Boltzmann equation to investigate the collective modes in the Fermi liquid state,[43]

$$\begin{aligned} &\frac{\partial}{\partial t}n(\vec{r},\vec{k},t) - \frac{i}{\hbar}[\epsilon(\vec{r},\vec{k},t),n(\vec{r},\vec{k},t)] \\ &\quad + \frac{1}{2}\sum_i \left\{\frac{\partial\epsilon(\vec{r},\vec{k},t)}{\partial k_i},\frac{\partial n(\vec{r},\vec{k},t)}{\partial r_i}\right\} \\ &\quad - \frac{1}{2}\sum_i \left\{\frac{\partial\epsilon(\vec{r},\vec{k},t)}{\partial r_i},\frac{\partial n(\vec{r},\vec{k},t)}{\partial k_i}\right\} = 0, \end{aligned} \quad (40)$$

where $n_{\alpha\alpha'}(\vec{r},\vec{k},t)$ and $\epsilon_{\alpha\alpha'}(\vec{r},\vec{k},t)$ are the density and energy matrices for the coordinate $(\vec{r},\vec{k})$ in the phase spaceand [,] and {,} mean the commutator and anticommutator, respectively. Under small variations in $n_{\alpha\alpha'}(\vec{r},\vec{k},t)$ and $\epsilon_{\alpha\alpha'}(\vec{r},\vec{k},t)$,

$$\begin{aligned} n_{\alpha\alpha'}(\vec{r},\vec{k},t) &= n_0(k)\delta_{\alpha\alpha'} + \delta n_{\alpha\alpha'}(\vec{r},\vec{k},t), \\ \epsilon_{\alpha\alpha'}(\vec{r},\vec{k},t) &= \epsilon(k)\delta_{\alpha\alpha'} + \int \frac{d^3k'}{(2\pi)^3} f_{\alpha\alpha',\beta\beta'}(\hat{k},\hat{k}')\delta n_{\beta\beta'}(\hat{k}'), \end{aligned} \quad (41)$$

the above Boltzmann equation can be linearized. Plugging the plane-wave solution of

$$\delta n_{\alpha\alpha'}(\vec{r},\vec{k},t) = \sum_q \delta n_{\alpha\alpha'}(\vec{k})e^{i(\vec{q}\cdot\vec{r}-\omega t)}, \quad (42)$$

we arrive at

$$\begin{aligned} &\delta n_{\alpha\alpha'}(\hat{k}) - \frac{1}{2}\frac{\cos\theta_k}{s-\cos\theta_k}\sum_{\beta\beta'}\int d\Omega_{k'} \\ &\quad \times \frac{N_0}{4\pi} f_{\alpha\alpha',\beta\beta'}(\hat{k},\hat{k}')\delta n_{\beta\beta'}(\hat{k}') = 0, \end{aligned} \quad (43)$$

where s is the dimensionless parameter $\omega/(v_f q)$. The propagation direction of the wave vector $\vec{q}$ is defined along the z direction.

In the spin-orbit decoupled basis defined as δn_{LmSs_z} in Sec. III B, the linearized Boltzmann equation becomes

$$\delta n_{LmSs_z} + \Omega_{LL';m}(s)F_{L'm'Ss_z;L''m''Ss''_z}\delta n_{L''m''Ss''_z} = 0, \quad (44)$$

where $\Omega_{LL'}(s)$ is equivalent to the particle-hole channel Fermi bubble in the diagrammatic method as

$$\Omega_{LL';m}(s) = -\int d\Omega_{\hat{k}} Y^*_{Lm}(\hat{k})Y_{L'm}(\hat{k})\frac{\cos\theta_k}{s-\cos\theta_k}. \quad (45)$$

For later convenience, we present $\Omega_{LL';m}$ in several channels of LL' and m as follows:

$$\begin{aligned} \Omega_{00;0}(s) &= 1 - \frac{s}{2}\ln\left|\frac{1+s}{1-s}\right| + i\frac{\pi}{2}s\Theta(s<1), \\ \Omega_{10;0}(s) &= \Omega_{01;0} = \sqrt{3}s\Omega_{00;0}(s), \\ \Omega_{11;0}(s) &= 1 + 3s^2\Omega_{00;0}(s), \\ \Omega_{11;1}(s) &= \Omega_{11;-1}(s) = -\frac{1}{2}[1 - 3(1-s^2)\Omega_{00;0}(s)]. \end{aligned} \quad (46)$$

Equation (44) can be further simplified by using the spin-orbit coupled basis $\delta n_{JJ_z;LS}$ defined in Sec. III B,

$$\delta n_{JJ_z;LS}+\sum_{J';LL'} K_{JJ_zLS;J'J_zL'S}(s)F_{J'J_zL'S;J'J_zL''S}\delta n_{J'J_zL''S}=0, \tag{47}$$

where the matrix kernel $K_{JJ_zLS;J'J_zL'S}$ reads

$$K_{JJ_zLS;J'J_zL'S}(s) = \sum_{ms_z}\langle LmSs_z|JJ_z\rangle\langle L'mSs_z|J'J_z\rangle\Omega_{LL';m}(s). \tag{48}$$

B. The spin-orbit coupled sound modes

Propagating collective modes exist if Landau parameters are positive. In these collective modes, interactions among quasiparticles rather than the hydrodynamic collisions provide the restoring force. Because only the spin channel receives renormalization from the magnetic dipolar interaction, we only consider spin-channel collective modes. The largest Landau parameter is in the (1^+001) channel in which the spin oscillates along the direction of $\vec{q}$. The mode in this channel is the longitudinal spin zero sound. On the other hand, due to the spin-orbit coupled nature, the Landau parameters are negative in the transverse spin channels of $(1^+\pm 1\ 0\ \pm 1)$, and thus, no propagating collective modes exist in these channels. The hybridization between (1^+001) and (1^+021) is small as shown in Eq. (25), and the Landau parameter in the (1^+021) channel is small, thus, this channel also is neglected below for simplicity.

Because the propagation wave vector $\vec{q}$ breaks the parity and 3D rotation symmetries, the (1^+001) channel couples to other channels with the same J_z. As shown in Eq. (47), the coupling strengths depend on the magnitudes of Landau parameters. We truncate Eq. (47) by keeping the orbital partial-wave channels of $L=0$ and $L=1$ because Landau parameters with orbital-partial waves $L\geqslant 2$ are negligible. There are three channels with $L=S=1$ as (0^-011), (1^-011), and (2^-011). We further check the symmetry properties of these four modes under the reflection with respect to any plane containing $\vec{q}$. The mode of (1^-011) is even, and the other three are odd, thus, it does not mix with them. The Landau parameter in the (2^-011) channel is calculated as $\frac{\pi}{20}\lambda$, which is 1 order smaller than those in (1^+001) and (1^-001), thus, this channel is also neglected. We only keep these two coupled channels (1^+001) and (1^-001) in the study of collective spin excitations.

The solution of the two coupled modes reduces to a 2×2 matrix linear equation as

$$\begin{pmatrix} 1+\Omega_{00;0}(s)F_{1001;1001} & s\Omega_{00;0}(s)F_{0011;0011} \\ s\Omega_{00;0}(s)F_{1001;1001} & 1+\Omega_{00;0}(s)F_{0011;0011}\end{pmatrix} \times \begin{pmatrix}\delta n_{1001}\\ \delta n_{0011}\end{pmatrix}=0, \tag{49}$$

where the following relations are used:

$$\begin{aligned} K_{1001;1001}(s) &= \Omega_{00;0}(s),\\ K_{1001;0011}(s) &= K_{0011;1001}(s)\\ &= \langle 0010|10\rangle\langle 1010|00\rangle\Omega_{01;0}(s)\\ &= s\Omega_{00;0}(s),\\ K_{0011,0011}(s) &= \sum_m |\langle 1m1-m|00\rangle|^2\Omega_{11;m}(s)\\ &= \frac{1}{3}\Omega_{11;0}(s)+\frac{2}{3}\Omega_{11;1}(s)\\ &= \Omega_{00;0}(s). \end{aligned} \tag{50}$$

The condition of the existence of nonzero solutions of Eq. (49) becomes

$$(1-s^2)\Omega^2_{00;0}(s)+2\Omega_{00;0}(s)\frac{F_+}{F_\times^2}+\frac{1}{F_\times^2}=0, \tag{51}$$

where $F_+=(F_{1001:1001}+F_{0011;0011})/2$ and $F_\times=\sqrt{F_{1001:1001}F_{0011;0011}}$.

Let us discuss several important analytical properties of its solutions. In order for collective modes to propagate in Fermi liquids, its sound velocity must satisfy $s>1$, otherwise it enters the particle-hole continuum and is damped, a mechanism called Landau damping. We can solve Eq. (51) as

$$\Omega^{\pm}_{00;0}(s)=\frac{F_+\pm\sqrt{F_+^2+(s^2-1)F_\times^2}}{(s^2-1)F_\times^2}. \tag{52}$$

Only the expression of the $\Omega^-_{00;0}(s)$ is consistent with $s>1$ and is kept. The other branch has no solution of the propagating collective modes.

Let us analytically check two limits with large and small values of λ, respectively. In the case of $0<\lambda\ll 1$ such that $s\to 1+0^+$, Eq. (51) reduces to

$$\Omega_{00;0}(s_{\lambda\ll 1})\approx 1-\frac{1}{2}\ln 2+\frac{1}{2}\ln(s-1)=-\frac{1}{2F_+}. \tag{53}$$

Its sound velocity solution is

$$s_{\lambda\ll 1}\approx 1+2e^{-2(1+1/2F_+)}=1+2e^{-2-12/7\pi\lambda}. \tag{54}$$

The eigenvector can be easily obtained as $\frac{1}{\sqrt{2}}(1,1)^T$, which is an equal mixing between these two modes. On the other hand, in the case of $\lambda\gg 1$, we also expect $s\gg 1$, and thus, Eq. (51) reduces to

$$\Omega_{00;0}(s_{\lambda\gg 1})\approx -\frac{1}{sF_\times}=-\frac{1}{3s^2}, \tag{55}$$

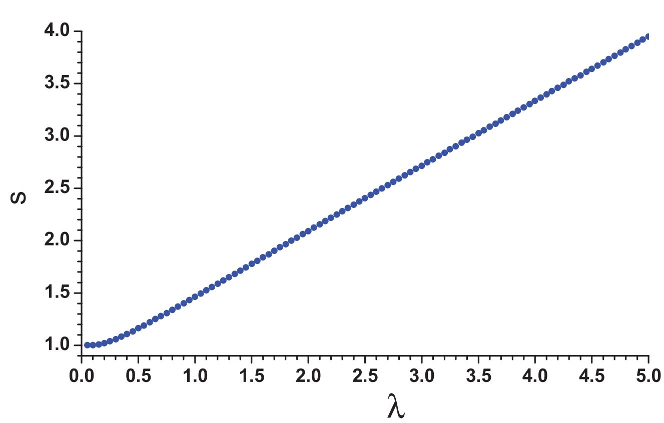

FIG. 1. (Color online) The sound velocity s in the unit of v_f vs the dipolar coupling strength λ. At $0<\lambda\ll 1$, $s(\lambda)\approx 1+0^+$. On the order of $\lambda\gg 1$, $s(\lambda)$ becomes linear with the slope indicated in Eq. (56).

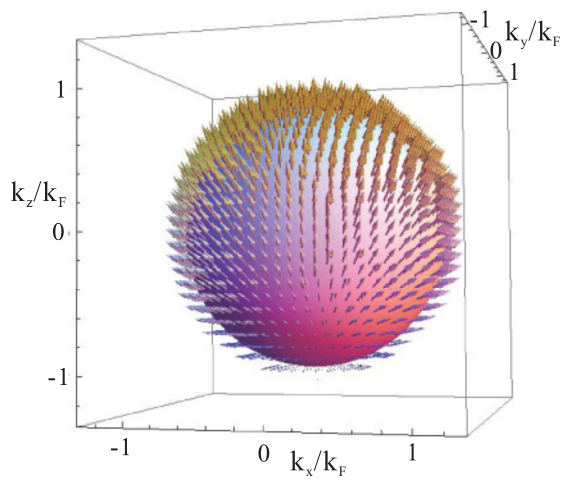

FIG. 2. (Color online) The spin configuration [Eq. (57)] of the zero-sound mode over the Fermi surface shows hedgehog-type topology at $\lambda = 10$. The common sign of u_1 and u_2 is chosen to be positive, which gives rise to the Pontryagin index +1. Although the hedgehog configuration is distorted in the z component, its topology does not change for any values of λ describing the interaction strength.

whose solution becomes

$$s_{\lambda\gg 1} \approx \frac{F_\times}{3} = \frac{\pi}{3\sqrt{3}}\lambda. \tag{56}$$

In our case, F_{1001} is larger than F_{0011} but is on the same order. The eigenvector can be solved as $\frac{1}{\sqrt{2F_+}}(\sqrt{F_{0011}},\sqrt{F_{1001}})^T$ in which the weight of the (0011) channel is larger.

The dispersion of the sound velocity s with respect to the dipolar interaction strength λ is solved numerically as presented in Fig. 1. Collective sound excitations exist for all the interaction strengths with $s > 1$. In both limits of $0 \ll \lambda \ll 1$ and $\lambda \gg 1$, the numerical solutions agree with the above asymptotic analysis of Eqs. (54) and (56). In fact, the linear behavior of $s(\lambda)$ already appears at $\lambda \sim 1$, and the slope is around 0.6. For all the interaction strengths, the (1^+001) and (0^-011) modes are strongly hybridized.

This mode is an oscillation of spin-orbit coupled Fermi surface distortions. The configuration of the (0^-011) mode exhibits an oscillating spin-orbit coupling of the $\vec{k}\cdot\vec{\sigma}$ type. This is the counterpart of the isotropic unconventional magnetism, which spontaneously generates the $\vec{k}\cdot\vec{\sigma}$-type coupling.[52,53] The difference is that, here, it is a collective excitation rather than an instability. It strongly hybridizes with the longitudinal spin mode. The spin configuration over the Fermi surface can be represented as

$$\vec{s}(\vec{r},\vec{k},t) = \begin{pmatrix} u_2 \sin\theta_{\vec{k}} \cos\phi_{\vec{k}} \\ u_2 \sin\theta_{\vec{k}} \sin\phi_{\vec{k}} \\ u_2 \cos\phi_{\vec{k}} + u_1 \end{pmatrix} e^{i(\vec{q}\cdot\vec{r}-sqv_f t)}, \tag{57}$$

where $(u_1,u_2)^T$ is the eigenvector for the collective mode. We have checked that, for all the values of λ, $|u_2| > |u_1|$ is satisfied with no change in their relative sign, thus, the spin configuration as shown in Fig. 2 is topologically nontrivial with the Pontryagin index ± 1, which periodically flips the sign with time and the spatial coordinate along the propagating direction. It can be considered as a topological zero sound.

VI. CONCLUSIONS

To summarize, we have presented a systematic study on the Fermi liquid theory with the magnetic dipolar interaction, emphasizing its intrinsic spin-orbit coupled nature. Although this spin-orbit coupling does not exhibit at the single-particle level, it manifests in various interaction properties. The Landau interaction function is calculated at the Hartree-Fock level and is diagonalized by the total angular momentum and parity quantum numbers. The Pomeranchuk instabilities occur at the strong magnetic dipolar interaction strength generating effective spin-orbit coupling in the single-particle spectrum.

We have also investigated novel collective excitations in the magnetic dipolar Fermi liquid theory. The Boltzmann transport equations are decoupled in the spin-orbit coupled channels. We have found an exotic collective excitation, which exhibits spin-orbit coupled Fermi surface oscillations with a topologically nontrivial spin configuration, which can be considered as a topological zero-sound-like mode.

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