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Mixed triplet and singlet pairing in ultracold multicomponent fermion systems with dipolar interactions

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The symmetry properties of the Cooper pairing problem for multicomponent ultracold dipolar molecular systems are investigated. The dipolar anisotropy provides a natural and robust mechanism for both triplet and singlet Cooper pairing to first order in the interaction strength. With a purely dipolar interaction, the triplet p_z-like polar pairing is the most dominant. A short-range attractive interaction can enhance the singlet pairing to be nearly degenerate with the triplet pairing. We point out that these two pairing channels can mix by developing a relative phase of $\pm\frac{\pi}{2}$, thus spontaneously breaking time-reversal symmetry. We also suggest the possibility of such mixing of triplet and singlet pairing in other systems.

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The study of ultracold dipolar molecules has recently become a research focus of cold atom physics.[1–4] The prominent feature of the dipolar interaction is its $d_{r^2-3z^2}$-type anisotropy when the dipolar moments are aligned by an external electric field. Considerable progress has been made in studying anisotropic condensation of dipolar bosons.[5–8] Furthermore, dipolar fermionic systems provide an exciting opportunity to study exotic anisotropic many-body physics of fermions. Experimentally, a near quantum-degenerate gas of the dipolar fermion ^{40}K-^{87}Rb has been achieved.[1] A number of theoretical works have been done for the anisotropic Fermi-liquid properties of dipolar Fermi gases,[9–13] including both singlet particle and collective excitations.

The dipolar interaction also has important effects in the Cooper pairing symmetry as studied in Refs. 14–18. In the single component case, the only possible pairing channels are of odd parity. Assuming dipole moments along the z axis, the pairing symmetry is mainly of p_z with slight hybridization with other odd partial wave components. Dipolar molecules can have an internal degree of freedom arising from the hyperfine configurations of the constituent atoms. The electric dipolar interaction is independent of these internal components which will be denoted as spin below. The intercomponent interaction opens up the possibility of both spin singlet and triplet pairings for the simplest two-component case. It would be interesting to study even richer Cooper pairing patterns and the competition among them.

In this Rapid Communication, we show that the dipolar interaction favors Cooper pairing in the triplet channel over the singlet channel. This is an effect directly arising from the anisotropy of the dipolar interaction, and it occurs to first order in the interaction strength. In contrast, it does not appear in the usual condensed-matter triplet pairing systems such as superfluid ^{3}He (Refs. 19–21): the spin-fluctuation mechanism based on the strong ferromagnetic tendency in ^{3}He arises from the repulsive part of the ^{3}He-^{3}He interaction at second order. For a two-component dipolar fermion system, we find the dominate pairing in the spin triplet p_z-like channel with the purely dipolar interaction. It can mix with the singlet $s+d_{r^2-3z^2}$ pairing whose pairing strength is tunable through the short-range nondipolar s-wave scattering. The mixing occurs with a relative phase of $\pm\frac{\pi}{2}$ which breaks time-reversal (TR) symmetry spontaneously. Pairing in dipolar Fermi gases with more than two components is also discussed.

Samokhin *et al.* studied nonuniform mixed parity superfluid states in the presence of dipolar interactions.[22] They considered coupling between singlet and triplet channels with zero relative phase only. Kabanov[23] also considered recently mixture of singlet and triplet pairing with zero relative phase. While this work was being completed, a study of the competition between triplet and singlet pairing in dipolar fermionic systems appeared that analyzed some of the cases considered here.[24]

We begin with the two-component dipolar fermionic system with the electric dipole moments aligned along the z axis. The dipolar interaction reads $V_{3D}(\vec{r}_1-\vec{r}_2)=-2d^2/|\vec{r}_1-\vec{r}_2|^3 P_2(\cos\theta_{\vec{r}_1-\vec{r}_2})$, where $\theta_{\vec{r}_1-\vec{r}_2}$ is the angle between $(\vec{r}_1-\vec{r}_2)$ and the electric field $\vec{E}$; d is the electric dipole moment. The anisotropy is manifested in the angular dependence of the V_{3D} with the form of the second Legendre polynomial. The Fourier transform of the three-dimensional (3D) dipolar interaction, $V(\vec{k})=\frac{8\pi d^2}{3}P_2(\cos\theta_k)$, only depends on the polar angle of $\vec{k}$. The Hamiltonian is written as

$$H=\sum_{k,\alpha}[\epsilon(\vec{k})-\mu]c^\dagger_\alpha(\vec{k})c_\alpha(\vec{k})+\frac{1}{2V}\sum_{k,k',q}V(\vec{k}-\vec{k}')\times P^\dagger_{\beta\alpha}(\vec{k};\vec{q})P_{\alpha\beta}(\vec{k}';\vec{q}), \quad (1)$$

where $\epsilon(\vec{k})=\hbar k^2/(2m)$; μ is the chemical potential; $P_{\alpha\beta}(\vec{k};\vec{q})=c_\alpha(-\vec{k}+\vec{q})c_\beta(\vec{k}+\vec{q})$ is the pairing operator; α,β refer to $\uparrow$ and $\downarrow$. Please note that $V(\vec{k}-\vec{k}')$ depends on the polar angle of the vector $\vec{k}-\vec{k}'$, not the relative angle between $\vec{k}$ and $\vec{k}'$. We define a dimensionless parameter to describe the interaction strength as the ratio between the characteristic interaction energy and the Fermi energy: $\lambda\equiv E_{int}/E_F=\frac{2}{3}\frac{d^2mk_f}{\pi^2\hbar^2}$.

We only consider uniform pairing states at the mean-field level, thus set $\vec{q}=0$ in the pairing interaction in Eq. (1). We define $P_{\alpha\beta}(\vec{k})=P_{\alpha\beta}(\vec{k};\vec{q}=0)$ which satisfies $P_{\alpha\beta}(\vec{k})=-P_{\beta\alpha}(-\vec{k})$. The pairing operators can be decomposed into the spin singlet P_{si} and triplet channels $P_{tri}^{x,y,z}$: $P_{si}(\vec{k})=\frac{1}{\sqrt{2}}[P_{\uparrow\downarrow}(\vec{k})-P_{\downarrow\uparrow}(\vec{k})]$, $P_{tri}^{z}(\vec{k})=\frac{1}{\sqrt{2}}[P_{\uparrow\downarrow}(\vec{k})+P_{\downarrow\uparrow}(\vec{k})]$, $P_{tri}^{x}(\vec{k})=-\frac{1}{\sqrt{2}}[P_{\uparrow\uparrow}(\vec{k})-P_{\downarrow\downarrow}(\vec{k})]$, and $P_{tri}^{y}(\vec{k})=-\frac{i}{\sqrt{2}}[P_{\uparrow\uparrow}(\vec{k})+P_{\downarrow\downarrow}(\vec{k})]$. $P_{si}(\vec{k})$ and $P_{tri}^{\mu}(\vec{k})$ are even and odd functions of $\vec{k}$, respectively; P_{tri}^{μ} describes the triplet pairing operators whose total spin is the eigenstate of $\hat{e}_{\mu}\cdot\vec{S}_{pair}$ with zero eigenvalue. Using these operators, the pairing interaction of Eq. (1) with $\vec{q}=0$ can be rewritten as

$$H_{pair}=\frac{1}{2V}\sum_{k,k',}\{V_{tri}(\vec{k};\vec{k}')\Big[\sum_{\mu=x,y,z}P_{tri}^{\dagger,\mu}(\vec{k})P_{tri}^{\mu}(\vec{k}')\Big] + V_{si}(\vec{k};\vec{k}')P_{si}^{\dagger}(\vec{k})P_{si}(\vec{k}')\}, \quad (2)$$

where $V_{tri,si}(\vec{k};\vec{k}')=\frac{1}{2}\{V(\vec{k}-\vec{k}')\mp V(\vec{k}+\vec{k}')\}$. $V_{si}(\vec{k};\vec{k}')$ is an even function of both arguments $\vec{k}$ and $\vec{k}'$, while $V_{tri}(\vec{k};\vec{k}')$ is an odd function of both.

The decoupled mean-field Hamiltonian reads

$$H_{mf}=\sum_{\vec{k}}{}' \Psi^{\dagger}(\vec{k})\begin{pmatrix}\xi(\vec{k})I & \Delta_{\alpha\beta}(\vec{k})\\ \Delta^{*}_{\beta\alpha}(\vec{k}) & -\xi(\vec{k})I\end{pmatrix}\Psi(\vec{k}), \quad (3)$$

where Σ_k' means summation over half of momentum space; $\xi(\vec{k})=\epsilon(\vec{k})-\mu$; $\Psi(\vec{k})=[c_{\uparrow}(\vec{k}),c_{\downarrow}(\vec{k}),c_{\uparrow}^{\dagger}(-\vec{k}),c_{\downarrow}^{\dagger}(-\vec{k})]^{T}$. The mean-field gap function is defined as $\Delta_{\alpha\beta}(\vec{k})=\frac{1}{V}\Sigma_{\vec{k}'}V(\vec{k}-\vec{k}')\langle|P_{\alpha\beta}(\vec{k}')|\rangle$. $\Delta_{\alpha\beta}$ can be decomposed into singlet and triplet channels as $\Delta_{\alpha\beta}(\vec{k})=\Delta_{si}(\vec{k})i\sigma^{y}_{\alpha\beta}+\Delta_{tri,\mu}(\vec{k})(i\sigma^{\mu}\sigma^{y})_{\alpha\beta}$. The Bogoliubov quasiparticle spectra become $E_{1,2}(\vec{k})=\sqrt{\xi_k^2+\lambda_{1,2}^2(\vec{k})}$, where $\lambda_{1,2}^2(\vec{k})$ are the eigenvalues of the positive-definite Hermitian matrix $\Delta^{\dagger}(\vec{k})\Delta(\vec{k})$. Its trace satisfies $\lambda_1^2(\vec{k})+\lambda_2^2(\vec{k})=|\Delta_{si}(\vec{k})|^2+\Sigma_{\mu}|\Delta_{tri,\mu}(\vec{k})|^2$. The free energy becomes

$$F=-\frac{2}{\beta}\sum_{k,i=1,2}\ln\Big[2\cosh\frac{\beta E_{\vec{k},i}}{2}\Big]-\frac{1}{2V}\sum_{\vec{k},\vec{k}',a=si,(tri,\mu)}\times[\Delta_a^{*}(\vec{k})V_a^{-1}(\vec{k};\vec{k}')\Delta_a(\vec{k})], \quad (4)$$

where $V_{si,tri}^{-1}(\vec{k};\vec{k}')$ is the inverse of the interaction matrix defined as $\frac{1}{V}\Sigma_{k'}V_{si,tri}(\vec{k},\vec{k}')V_{si,tri}^{-1}(\vec{k}',\vec{k}'')=\delta_{\vec{k},\vec{k}''}$.

The gap equations are expressed as

$$\Delta_{tri,\mu}(\vec{k})=-\int\frac{d^3k'}{(2\pi)^3}V_{tri}(\vec{k};\vec{k}')K(\vec{k}')\Delta_{tri,\mu}(\vec{k}'),$$

$$\Delta_{si}(\vec{k})=-\int\frac{d^3k'}{(2\pi)^3}V_{si}(\vec{k};\vec{k}')K(\vec{k}')\Delta_{si}(\vec{k}'), \quad (5)$$

where $K(\vec{k}')=\tanh[\frac{\beta}{2}E_i(\vec{k}')]/[2E_i(\vec{k}')]$. Equation (5) formally diverges. It can be regularized following the standard procedure explained in Ref. 14 and 25 by replacing the bare interaction $V_{tri,si}$ with the renormalized zero energy vertex functions $\Gamma_{tri,si}$. At the level of the Born approximation, this regularization is equivalent to just introducing an energy cutoff of $\pm\bar{\omega}$ for ξ_k' in Eq. (5), where $\bar{\omega}$ is at the scale of the Fermi energy.

To analyze the dominant pairing instability around T_c, Eq. (5) is linearized. Considering that the strongest pairing occurs at the Fermi surface and following the standard procedure in Ref. 14, we define the eigengap functions $\phi^{j}_{tri,si}(\vec{k})$ satisfying

$$N_0\int\frac{d\Omega_{k'}}{4\pi}V_{tri,si}(\vec{k};\vec{k}')\phi^{j}_{tri,si}(\vec{k}')=w^{j}_{tri,si}\phi^{j}_{tri,si}(\vec{k}), \quad (6)$$

where $N_0=mk_f/\pi^2\hbar^2$ is the density of state at the Fermi surface; $w^{j}_{tri,si}$ are dimensionless eigenvalues; $\vec{k},\vec{k}'$ are at the Fermi surface. We neglect the effect of Fermi surface distortion on pairing which is a higher order effect in the interaction strength λ. Then Eq. (5) is linearized into a set of decoupled equations

$$\phi^{j}_{tri,si}\{1+w_{tri,si}\ln[(2e^{\gamma}\bar{\omega})/(\pi k_B T)]\}=0. \quad (7)$$

The spherical harmonics decomposition of $V_{tri,si}(\vec{k};\vec{k}')$ reads

$$\frac{N_0}{4\pi}V_{tri,si}(\vec{k};\vec{k}')=\sum_{l,l';m}V_{ll';m}Y^{*}_{lm}(\Omega_k)Y_{l'm}(\Omega_{\vec{k}'}), \quad (8)$$

where $V_{ll';m}$ remains diagonal for m but couples partial wave channels with $l'=l,l\pm 2$. $V_{tri,si}$ only have matrix elements in odd and even partial wave channels, respectively. $V_{ll;m}$ has the same expressions of the Landau parameters of the dipolar Fermi gases given by Fregoso *et al.*[13] except for an overall minus sign due to pairing and a trivial overall numerical factor difference. For $l=l'=m=0$, $V_{00;0}=0$ because the average value of the dipolar interaction is zero.

We diagonalize the matrix $V_{ll';m}$ to find the dominant negative eigenvalues which determine the dominant pairing channels. Two eigenvectors are found with eigenvalues much more negative than other channels. One lies in the triplet odd-parity sector with dominant p_z-wave character with slight hybridization with other odd-parity channels as same as in the single component case:[14] $\phi^{z}(\Omega_k)$ with the most negative eigenvalue $w^{z}_{tri}=-3.820\lambda$, whose eigenvector is $\phi^{z}(\Omega_k)\approx 0.993Y_{10}-0.120Y_{30}$. The other one lies in the even-parity spin singlet channel. For the purely dipolar interaction, its eigenvalue is $w^{s+d}_{si}=-1.935\lambda$ and the eigenvector lies in the mixed channel of $s+d_{k^2-3k_z^2}$ as $\phi^{s+d}(\Omega_k)\approx 0.6Y_{00}-0.8Y_{20}$ with nodes. However, this channel is sensitive to the strength of the short-range s-wave scattering, which contributes only to the matrix element of $V_{00;0}$ as depicted in Fig. 1. Experimentally, this scattering can be tuned to the scale of the Fermi energy through Feshbach resonance, i.e., $V_{00;0}$ can become of order 1. As for λ, as estimated in Ref. 13, it could reach $0.1\sim0.2$. Thus the competition between ϕ^{s+d} and ϕ^{s} can be studied experimentally in the future. When they become degenerate at $V_{00;0}/\lambda\approx-3.15$, ϕ^{s+d} becomes mostly of s-wave character as $\phi^{s+d}(\Omega_k)\approx 0.901Y_{00}-0.434Y_{20}$.

We first consider the case of the dominant triplet pairing, whose critical temperature is determined from Eq. (7) as $T_{p_z}\approx(2e^{\gamma}\bar{\omega}/\pi)\exp(-1/|w^{z}_{tri}|)$. Its order-parameter configuration is $\Delta_{tri,\mu}(\vec{k})=\Delta_{tri,\phi^z}(\Omega_k)\hat{d}_{\mu}$. $\hat{d}$ is a spin space unit vector. Without losing generality, it is taken along the z axis as $\hat{d}=\hat{e}_z$. $\Delta_{tri,\phi^z}(\Omega_k)=\Delta_{tri}e^{i\gamma}\phi^{z}(\Omega_k)$, where γ is the pairing phase. This phase has line nodes on the equator. It breaks the $U_c(1)$ gauge, and the spin $SU_s(2)$ symmetries but maintains TR,

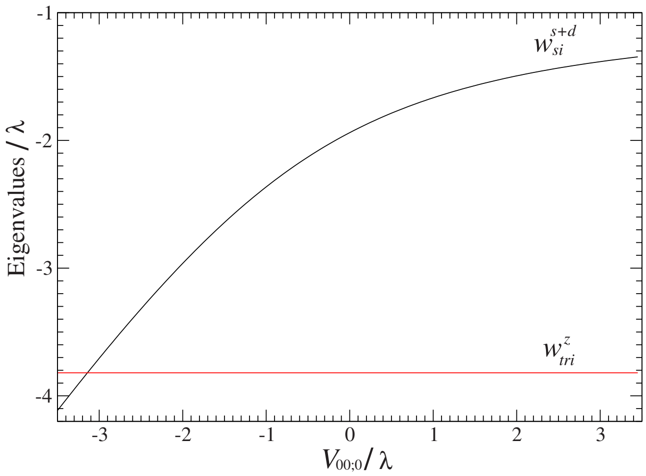

FIG. 1. (Color online) The eigenvalues w_{tri}^z for the spin triplet ϕ^z channel and w_{si}^{s+d} for the singlet ϕ^{s+d} channel. The latter depends on the short-range s-wave interaction $V_{00;0}$.

parity, and a Z_2 symmetry of the combined operation of $\hat{d}_\mu \to -\hat{d}_\mu$ and $\gamma \to \gamma + \pi$.[26,27] Its Goldstone manifold is $G = [\mathrm{U}_c(1) \otimes \mathrm{SU}_s(2)]/[\mathrm{U}_s(1) \otimes Z_2] = [\mathrm{U}_c(1) \otimes S_s^2]/Z_2$. The corresponding low-energy excitations include the phonon and the spin-wave modes. It supports two different classes of vortices: the usual integer vortex of superfluidity and the half-integer quantum vortex of superfluidity combined with a π disclination of the d vector.

Next we consider the coexistence of the singlet ϕ^{s+d} pairing and the triplet ϕ^z pairing when they become nearly degenerate. The order parameter can be chosen as $\Delta_{si}(\Omega_k) = \Delta_0 \phi^{s+d}(\Omega_k)$ and $\Delta_{tri,\mu}(\Omega_k) = \Delta_z \phi^z(\Omega_k) \delta_{\mu,z}$. An important observation is that a relative $\pm\frac{\pi}{2}$-phase difference between Δ_{si} and Δ_{tri} is favored, thus spontaneously breaking TR symmetry. This can be proved as follows: the quasiparticle spectra reads $E_i = \sqrt{\xi^2 + \lambda_i^2}$, with $\lambda_{1,2}^2 = |\Delta_0 \phi^{s+d}(\Omega_k)|^2 + |\Delta_z \phi^z(\Omega_k)|^2 \pm 2\,\mathrm{Re}(\Delta_z^* \Delta_0) \phi^{s+d}(\Omega_k) \phi^z(\Omega_k)$. The last term vanishes for relative phase $\pm\frac{\pi}{2}$ between Δ_0 and Δ_z. In other words, $\phi^z + i\phi^{s+d}$ is unitary pairing, i.e., $\Delta^\dagger(\vec{k})\Delta(\vec{k})$ is an identity matrix up to a factor, and $\lambda_1^2 = \lambda_2^2 = |\Delta_0 \phi^{s+d}(\Omega_k)|^2 + |\Delta_z \phi^z(\Omega_k)|^2|$. All other phase differences give $\lambda_1 \neq \lambda_2$, thus are nonunitary pairing. To show that unitary pairing is optimal, we follow the method presented in Ref. 28 to define the function $f(x) = -\frac{2}{\beta} \ln[2\cosh\frac{\beta}{2}\sqrt{\xi_k^2 + x}]$, which satisfies $d^2/dx^2 f(x) > 0$, thus $f(\lambda_1^2) + f(\lambda_2^2) \geq 2f(\frac{\lambda_1^2 + \lambda_2^2}{2})$. Then the first term in Eq. (4) is minimized by the unitary pairing, and the second term is degenerate for unitary and nonunitary pairings. Therefore, $\phi^z + i\phi^{s+d}$ is favored. This is an exotic fully gapped TR breaking pairing state because the nodes of ϕ^{s+d} and ϕ^s do not coincide. It also breaks parity but is invariant under the combined parity and TR operation. Its Goldstone manifold for the continuous symmetry breaking is the same as in the purely triplet-ϕ^z pairing phase.

The above analysis can be recaptured in the Ginzburg-Landau (GL) framework. The bulk pairing order parameters are defined as $\Delta_\mu^z = \Sigma_k \phi^z(\vec{k}) \Delta_{tri,\mu}(\vec{k})$ and $\Delta_\mu^{s+d} = \Sigma_k \phi^{s+d}(\vec{k}) \Delta_{si}(\vec{k})$. The GL free energy is constructed as

$$F = \alpha_z(T) \sum_\mu |\Delta_\mu^z|^2 + \alpha_{s+d}(T) |\Delta^{s+d}|^2 + \beta_z |\Delta_\mu^z|^4 + \beta_{s+d} |\Delta^{s+d}|^4 + \gamma_1 \sum_\mu |\Delta_\mu^z|^2 |\Delta^{s+d}|^2 + \gamma_2 \sum_\mu \{\Delta_\mu^{z*} \Delta_\mu^{z*} \Delta^{s+d} \Delta^{s+d} + \text{c.c.}\}, \tag{9}$$

where $\alpha_z = N_0 \ln(T/T_{p_z})$, $\alpha_{s+d} = N_0 \ln(T/T_{s+d})$, and T_{s+d} is defined as $T_{s+d} \approx (2e^\gamma \bar{\omega}/\pi) \exp(-1/|w_{si}^{s+d}|)$; β_z and β_{s+d} terms are the standard quartic terms for the triplet ϕ^z and singlet ϕ^{s+d} channels, respectively; $\gamma_{1,2}$ describe the coupling between the ϕ^z and ϕ^{s+d} channels. Following the analysis on a similar problem in Ref. 29, we consider the situation where the two channels are nearly degenerate and T_{p_z} is slightly larger than T_{s+d}, then the triplet pairing develops first at $T_c = T_{p_z}$. Defining $r = (\gamma_1 - 2|\gamma_2|)/(2\beta_z)$, the condition for the second instability to occur is that there exists a lower temperature T' below which $|\alpha_{s+d}(T')| > r|\alpha_{p_z}(T')|$. This can be satisfied for $r < 1$, which results in $T'/T_c = (T_{s+d}/T_c)^{1/(1-r)}$. The $\pm\frac{\pi}{2}$-phase difference between the triplet and singlet channel pairing requires that $\gamma_2 > 0$. In Refs. 22 and 23, coupling between the singlet and triplet pairings through a linear spatial derivative is considered, which leads to spatially nonuniform states. Due to spin conservation, such a term is not allowed here.

It is natural to further consider competing pairings for even larger number of components represented by the internal hyperfine spin degrees of freedom, which is an even number. The dipolar interaction is independent of them, thus the system has an SU($2N$) symmetry. The $2N \times 2N$ pairing matrix $\Delta_{\alpha\beta}(\vec{k})$ can be classified as $N(2N+1)$-component symmetric (odd-parity) pairing and $N(2N-1)$-component antisymmetric (even-parity) pairing, which are generalizations of the triplet and singlet channel pairings, respectively. They can be explicitly constructed as follows. We define the charge conjugation matrix R as $R_{ij} = (-)^i \delta_{i,2N-i+1}$ and the time-reversal operators $T = RC$, where C is complex conjugation. For $2N = 2$, R reduces to the familiar $-i\sigma_y$. R satisfies $R^T = -R$ and $R^2 = -1$. On the other hand, any $2N \times 2N$ Hermitian matrix can be expanded in the basis of the identity matrix and $4N^2 - 1$ generators of the SU($2N$) group. They can be classified as even and odd under TR transformation. $N(2N+1)$ of them are TR odd which can be constructed as spin-tensor matrices P_μ with odd rank numbers (e.g., spin, spin octupoles, etc.), and $N(2N-1)$ of them are TR even which can be constructed as spin-tensor matrices Q_ν with even rank numbers (e.g., the identity matrix, spin quadrupole, etc.). Using $TP_\mu T^{-1} = -P_\mu$, $TQ_\nu T^{-}1 = Q_\nu$, and $R^T = -R$, it can be shown that the matrices of $P_\mu R$ are symmetric and $Q_\nu R$ are antisymmetric, respectively. Thus we can decompose $\Delta_{\alpha\beta}(\Omega_k) = \Delta_{asy,\mu}(\Omega_k)(Q_\mu R)_{\alpha\beta} + \Delta_{sym,\nu}(\Omega_k)(P_\nu R)_{\alpha\beta}$, where $\Delta_{asy,\mu}(\Omega_k)[\mu = 1 \sim N(2N-1)]$ and $\Delta_{sym,\nu}(\Omega_k)[\nu = 1 \sim N(2N+1)]$ are even and odd functions of Ω_k, respectively.

The eigenvalue analysis for competing pairing channels is the same as in Eq. (8) by replacing the triplet (singlet) pairing with the spin-symmetric (antisymmetric) pairing, respectively. We next consider the unitary pairing in both spin-symmetric and spin asymmetric channels, respectively. A convenient choice for the matrix kernels is that $P_z = \sigma_z \otimes I_N$

for the spin-symmetric channel, and $Q=I_{2N}$ where I_N and I_{2N} are the identity matrices with N and $2N$ dimensions. The first one corresponds to the pairing of $\Sigma_{i=1\sim 2N} c_i^\dagger(\vec{k}) c_{2N-i}^\dagger(-\vec{k})$, while the second corresponds to $\Sigma_{i=1\sim 2N}(-)^{i-1} c_i^\dagger(\vec{k}) c_{2N-i}^\dagger(-\vec{k})$. In the ϕ^z-channel pairing states, the spin-SU($2N$) symmetry is broken down into $SU(N)\times SU(N)\times U(1)$, thus it has $2N^2$ branches of spin-wave Goldstone modes. The vortex configuration is similar to the $N=1$ case including the usual integer vortex and the half-quantum vortex combined with a π disclination of spin texture. Again for the mixing between pairing in the ϕ^{s+d} (spin antisymmetric) and ϕ^z (spin symmetric) channels, a relative phase $\pm\frac{\pi}{2}$ is needed to maintain the unitary pairing.

In summary, we have investigated the competing pairing symmetries in ultracold multicomponent dipolar molecular systems, which provides a wonderful opportunity to investigated exotic pairings. We predict that the anisotropy of the dipolar interaction provides a well-defined pairing mechanism to the spin triplet or, more generally, the spin-symmetric channel Cooper pairing. The spin singlet even-parity channel pairing in the ϕ^{s+d} channel is tunable by the short-range s-wave scattering. It mixes with the spin triplet odd-parity channel pairing by developing a relative phase $\pm\frac{\pi}{2}$ to maintain the unitary pairing. This is another type of unconventional Cooper pairing breaking TR symmetry.

We point out that our mechanism of TR breaking mixing between triplet and singlet pairings is very general. For example, in superfluid ^{3}He it was originally proposed that pairing would occur in the singlet d-wave channel,[30] induced by the attractive part of the van der Waals interaction. Later, attention focused exclusively on pairing in the triplet p-wave channel induced by the short-range repulsive interaction.[30,31] It is natural to expect that both channels could contribute to pairing at sufficiently low temperatures, leading to coupled Balian-Werthamer (BW) triplet pairing and singlet channel pairing with a relative phase $\pm\frac{\pi}{2}$. In metallic superconductors, coupling of an isotropic s-wave state with a BW triplet state will lead to a single isotropic gap only for the particular case where the relative phase is $\pm\frac{\pi}{2}$. These possibilities will be discussed separately.

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