Hidden symmetry and exotic quantum magnetism with large-spin alkali and alkaline-earth fermions

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Ref: C. Wu, J. P. Hu, and S. C. Zhang, Phys. Rev. Lett. 91, 186402 (2003).

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Outline

• Why large spin (multi-component) cold fermions are interesting?

• Large spin ultra-cold fermions are quantum-like NOT semi-classical.

• The simplest case of spin-3/2 fermions are characterized by a generic Sp(4) (SO(5)) symmetry without fine tuning.

• Spin-3/2 Hubbard model unifies antiferromagnetism, superconductivity, and charge-density-wave phases with exact symmetries.

• Exotic "color magnetism" exhibits dominant N-particle correlations (N>=3) --- a feature of QCD.

Why large spin physics with cold atoms is interesting?

- Novel physics **inaccessible** in usual solid state systems.

- Bosons. spin-1: ^{23}Na, ^{87}Rb; spin-2: ^{87}Rb; spin-3 ^{52}Cr.

  Spinor cond. : Ho and Yip (1998), K. Machida (1998), Ueda, Diener and Ho (2006);

  Topological properties: Zhou (2001--), Demler(2001--), ……

- Large spin fermions with alkaline-earth and alkali atoms.

  Fermi liquid and Cooper pairing: Ho and Yip (1999);

  Large symmetries of Sp(2N)/SU(2N): Wu, Hu, Zhang, Chen, Wang (2003 ---); Azaria and Lecheminant (2006 ---);

  V. Guriare, M. Hermele, A. Rey et al. (2010 ---).

Experiment progress of multi-component fermions

Selected for a Viewpoint in *Physics*

90401 (2010) PHYSICAL REVIEW LETTERS wee 5 NOVE

PRL 105, 190401 (2010)

Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]

)2 (2010) PHYSICAL REVIEW LETTERS

Degenerate Fermi Gas of ^{87}Sr

PRL 105, 030402 (2010)

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian

Viewpoint

Exotic many-body physics with large-spin Fermi gases

Physics 3, 92 (2010)

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The experimental realization of quantum degenerate cold Fermi gases with large hyperfine spins opens up a new opportunity for exotic many-body physics.

Classical (large S): large-spin solid state systems

- Hund's rule coupled electrons → large onsite spin.

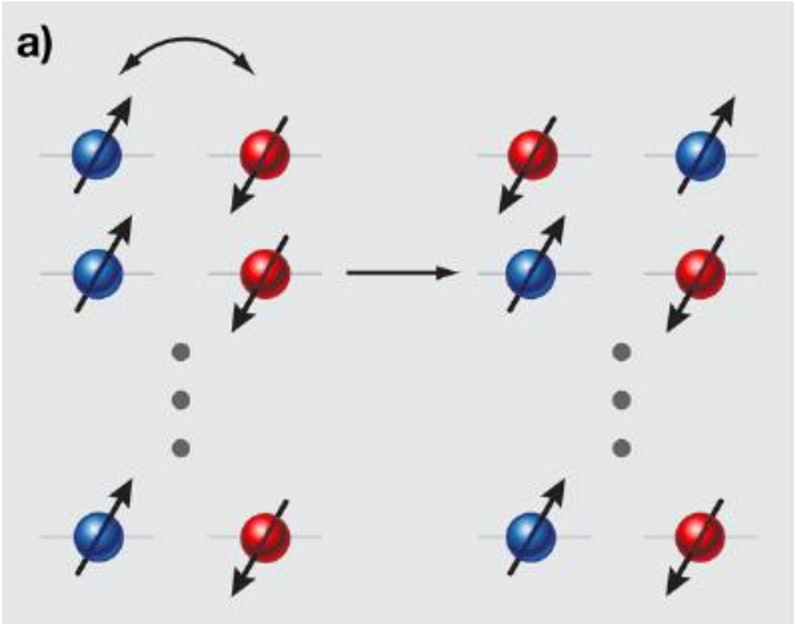
- Inter-site coupling is dominated by exchanging a single pair of electrons.

- ΔS_z only +1 or -1. Quantum spin-fluctuations are suppressed by 1/S.

- In solid state systems, the larger the spin is, the more classical the physics is.

- Bilinear exchange dominates

$$\frac{t^2}{U}\vec{S}_i\cdot\vec{S}_j+\frac{t^3}{U^2}(\vec{S}_i\cdot\vec{S}_j)^2+\ldots$$



C. Wu, Mod. Phys. Lett. (2006); Physics 3, 92 (2010).

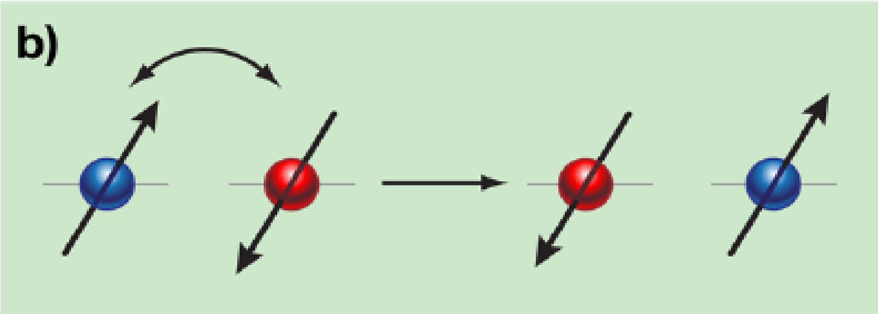
Not classical but quantum!: large-spin cold atoms

- Large-spin cold fermion moves as a whole object. The exchange of a pair of fermions can completely flip spin-configuration.

$$\Delta S_z = \pm 1, \pm 2, \ldots \pm S$$

- Quantum fluctuations are enhanced by the large number of spin components, just opposite to the large-S limit.

- Bilinear, bi-qudratic, bi-cubic terms, etc., are all at equal importance.



$$\vec{S}_i \cdot \vec{S}_j, (\vec{S}_i \cdot \vec{S}_j)^2, (\vec{S}_i \cdot \vec{S}_j)^3$$

- Large N instead of large S: SU(2N), Sp(2N); 2N=2S+1.

C. Wu, Mod. Phys. Lett. (2006); Physics 3, 92 (2010).

The simplest case spin-3/2: **Hidden symmetry**!

- Spin 3/2 atoms: ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba, ^{201}Hg.

C. Wu et al. Phys. Rev. Lett. 91, 186402 (2003).

- **Sp(4) (SO(5))** symmetry without fine tuning regardless of dimensionality, particle density, and lattice geometry!

Sp(4) in spin 3/2 systems ←→ SU(2) in spin ½ systems

- SU(4) symmetry is realized iff the interaction is spin-independent.

- Importance of high symmetries: unification of competing orders, description of strong spin fluctuations, etc.

Spin-3/2 Hubbard model in optical lattices

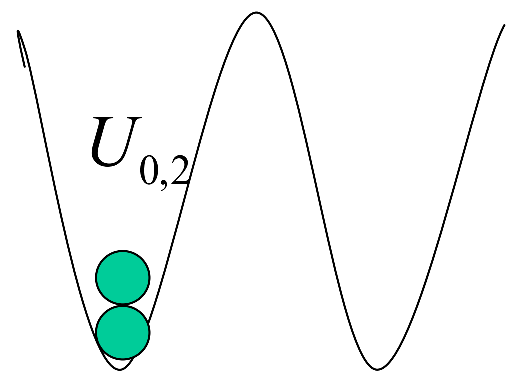
$$H = \sum_{\langle ij \rangle, \alpha} -t\{c^+_{i,\alpha} c_{j,\alpha} + h.c.\} - \mu \sum_i c^+_{i,\alpha} c_{i,\alpha}$$
$$+ U_0 \sum_i \eta^+(i)\eta(i) + U_2 \sum_{a=1\sim5} \chi^+_a(i)\chi_a(i)$$

$$\Uparrow \left|\tfrac{3}{2}\right\rangle \quad \uparrow \left|\tfrac{1}{2}\right\rangle \quad \downarrow \left|-\tfrac{1}{2}\right\rangle \quad \Downarrow \left|-\tfrac{3}{2}\right\rangle$$

• Fermi statistics: only F_{tot}=0, 2 are allowed; F_{tot}=1, 3 are forbidden.

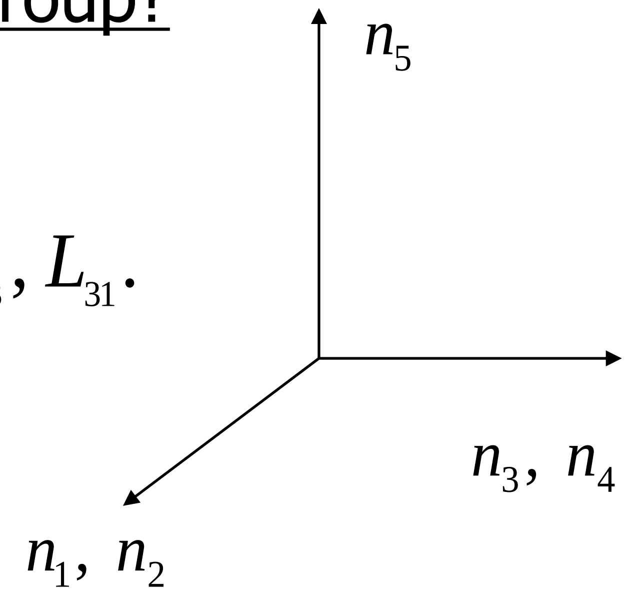
singlet: $\eta^+(i) = \sum_{\alpha\beta} \langle 00 | \tfrac{3}{2}\tfrac{3}{2}; \alpha\beta \rangle c^+_\alpha(i) c^+_\beta(i)$

quintet: $\chi^+_a(i) = \sum_{\alpha\beta} \langle 2a | \tfrac{3}{2}\tfrac{3}{2}; \alpha\beta \rangle c^+_\alpha(i) c^+_\beta(i)$



• For arbitrary values of t, μ, U_0, U_2 and lattice geometry, there is an **exact** Sp(4), or SO(5) symmetry.

What is Sp(4)(SO(5)) group?



- SU(2) (SO(3)) group.

  3-vector: x, y, z; 3-generator: L_{12}, L_{23}, L_{31}.

  2-spinor: $|\uparrow\rangle, \quad |\downarrow\rangle$

- Sp(4)(SO(5)) group.

  5-vector: n_1, n_2, n_3, n_4, n_5

  10-generator: $L_{ab}\ (1 \le a < b \le 5)$

  4-spinor: $\Uparrow \left|\frac{3}{2}\right\rangle \ \uparrow \left|\frac{1}{2}\right\rangle \ \downarrow \left|-\frac{1}{2}\right\rangle \ \Downarrow \left|-\frac{3}{2}\right\rangle$

- For spin-3/2 Hubbard model, 3-spin are obviously conserved. what are the other 7-**hidden** conserved quantities?

spin-3/2 algebra $\psi_\alpha^+ M_{\alpha\beta} \psi_\beta$

- Total degrees of freedom: 4^2=16=1+3+5+7.

1 density operator and 3 spin operators are far from complete.

	rank:		
	0	$1,$	
	1	F_x, F_y, F_z	
$M_{\alpha\beta}$	2	$\xi^a_{ij} F_i F_j \ (a = 1 \sim 5):$ ⟷	$F_x^2 - F_y^2, F_z^2 - \frac{5}{4},$ $\{F_x, F_y\}, \{F_y, F_z\}, \{F_z, F_x\}$
	3	$\xi^a_{ijk} F_i F_j F_k \ (a = 1 \sim 7)$	

- **Spin-quadrupole matrices** (rank-2 tensors) form five-Γ matrices (SO(5) vector) --- the same Γ-matrices in Dirac equation.

$$\Gamma^a = \xi^a_{ij} F_i F_j , \quad \{\Gamma^a, \Gamma^b\} = 2\delta_{ab}, \quad (1 \le a, b \le 5)$$

Hidden conserved quantities: **spin-octupoles**

- Both $F_{x,y,z}$ and $\xi^{a}_{ijk} F_i F_j F_k$ commute with Hamiltonian. 10 SO(5) generators: 10=3+7.

- **7 spin-octupole operators** are the hidden conserved quantities.

$$\Gamma^{ab} = \tfrac{i}{2}[\Gamma^{a}, \Gamma^{b}] \;\; (1 \le a < b \le 5)$$

- **SO(5): 1 scalar + 5 vectors + 10 generators = 16**

		Time Reversal
1 density:	$n = \psi^{+}\psi;$	even
5 spin-quadrupole:	$n_a = \frac{1}{2}\psi^{+}\Gamma^{a}\psi;$	even
3 spins + 7 spin-octupole:	$L_{ab} = \frac{1}{2}\psi^{+}\Gamma^{ab}\psi;$	odd

digression: spin-1/2 Hubbard model (2D square lattice)

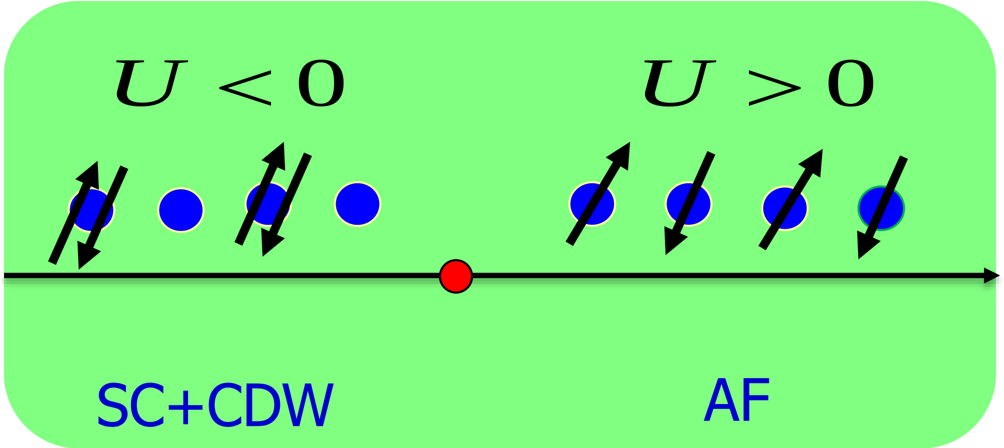
- Half-filling – well-known

U>0: spin-SU(2); long range order (LRO) 3-antiferromagntism (AF)

U<0: pseudo-spin SU(2);

charge density wave (CDW)
+ superconductivity (SC)

--- C. N. Yang's eta-pairing

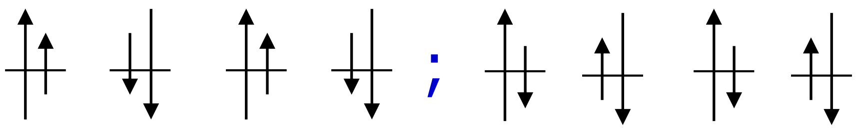


- Away from half-filling of U>0 – mostly unknown

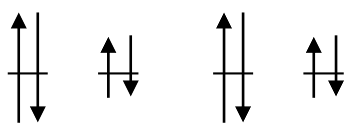
Competing orders at 2/4-filling (two particles per site)

- Two types of AF: Sp(4)-adjoint and vector Reps.

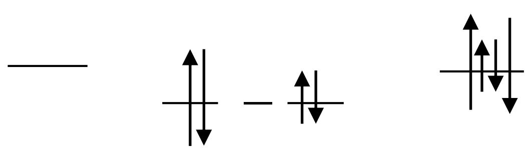
A) 10-AF (spin + spin octupole).

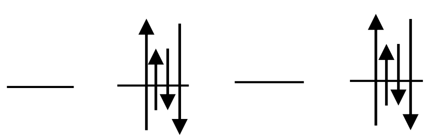


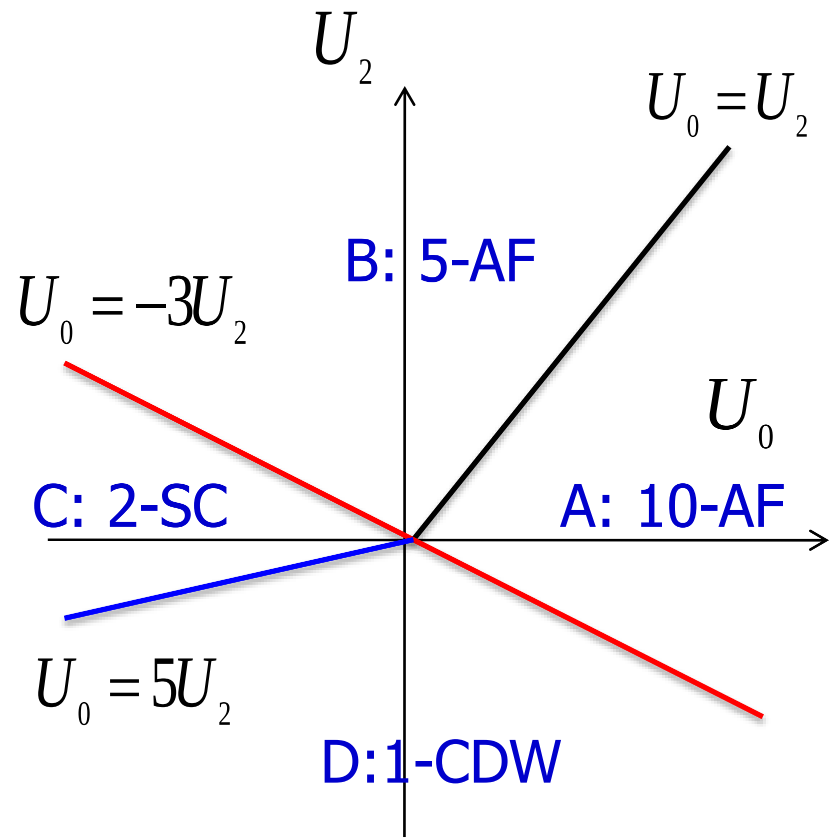
B) 5-AF (spin quadrupole).



- Two types of Sp(4) singlet states.

C) SC 

D) CDW. 



mean-field phase diagram at half-filling (square lattice)

Unifying AF, SC, CDW with even higher exact symmetries!

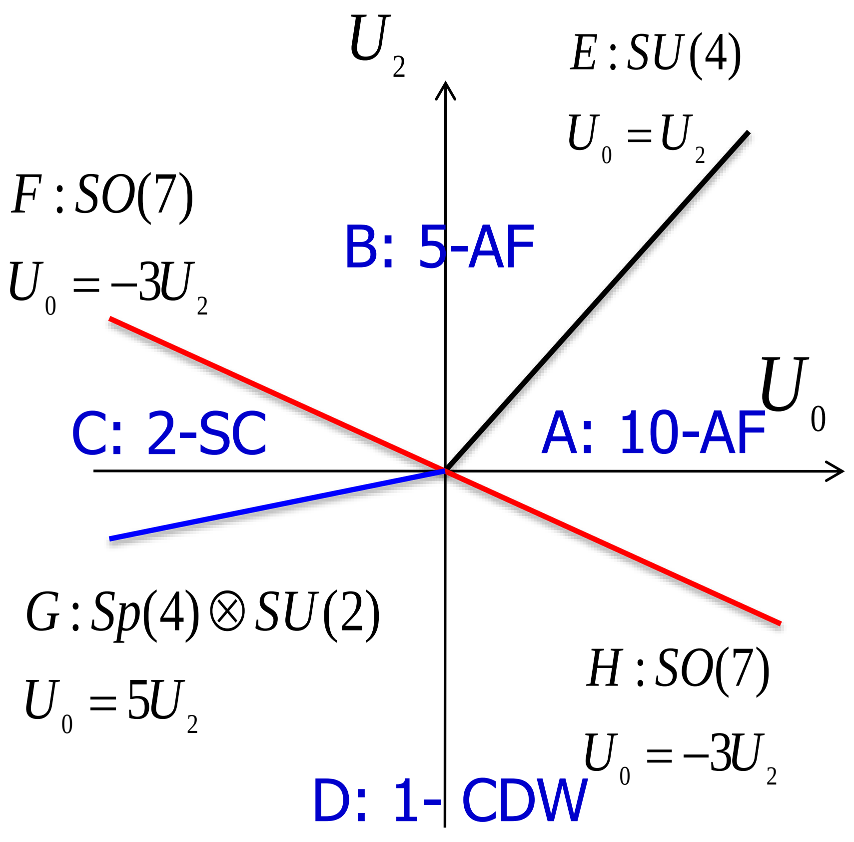
- E: SU(4) line. 15-AF (spin+spin quadrapole+spin octupole).

- F: **Exact** SO(7) line. 5-AF + 2-SC=7.

c.f. SO(5) theory of high Tc: 3-AF + 2 SC=5.

- G: Sp(4)*SU(2): CDW+SC; generalization of eta-pairing

- H: Exact SO(7) line adjoint Rep. 10-AF + 10-quintet SC + 1-CDW =21 dim

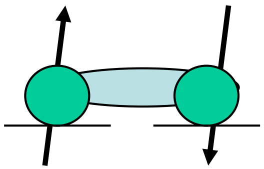


¼-filling (one particle per site) -- "color magnetism"

- Strong spin fluctuations: N=4.

- When the onsite Neel ordering is suppressed, multi-site correlations develop.

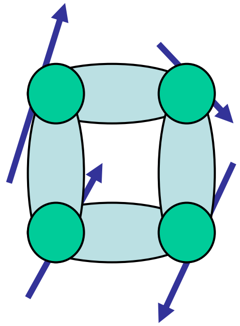
- spin-1/2: 2 sites to form an SU(2) singlet.



- 4 sites to form an SU(4) singlet. Each site belongs to the fundamental Rep.

baryon-like $$\frac{\varepsilon_{\alpha\beta\gamma\delta}}{4!}\psi_\alpha^+(1)\psi_\beta^+(2)\psi_\gamma^+(3)\psi_\delta^+(4)\left|0\right\rangle$$

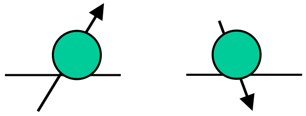
Bossche et. al., Eur. Phys. J. B 17, 367 (2000).



- c. f. QCD. At least three quarks form an SU(3) color singlet: baryons; multi-particle color/magnetic correlations.

Sp(4) (SO(5)) Heisenberg model at ¼-filling

- Spin exchange: bond singlet (J_0), quintet (J_2). No exchange in the triplet and septet channels.



$$H_{ex} = \sum_{\langle ij \rangle} - J_0 Q_0(ij) - J_2 Q_2(ij)$$

$$J_0 = 4t^2/U_0, J_2 = 4t^2/U_2, J_1 = J_3 = 0$$

$$\frac{3}{2} \times \frac{3}{2} = 0+2+1+3$$

- Heisenberg model with bi-linear, bi-quadratic, bi-cubic terms.

- SO(5) or Sp(4) explicitly invariant form:

$$H_{ex} = \sum_{ij} \frac{J_0 + J_2}{4} L_{ab}(i) L_{ab}(j) + \frac{-J_0 + 3J_2}{4} n_a(i) n_b(j) \qquad a,b = 1 \sim 5$$

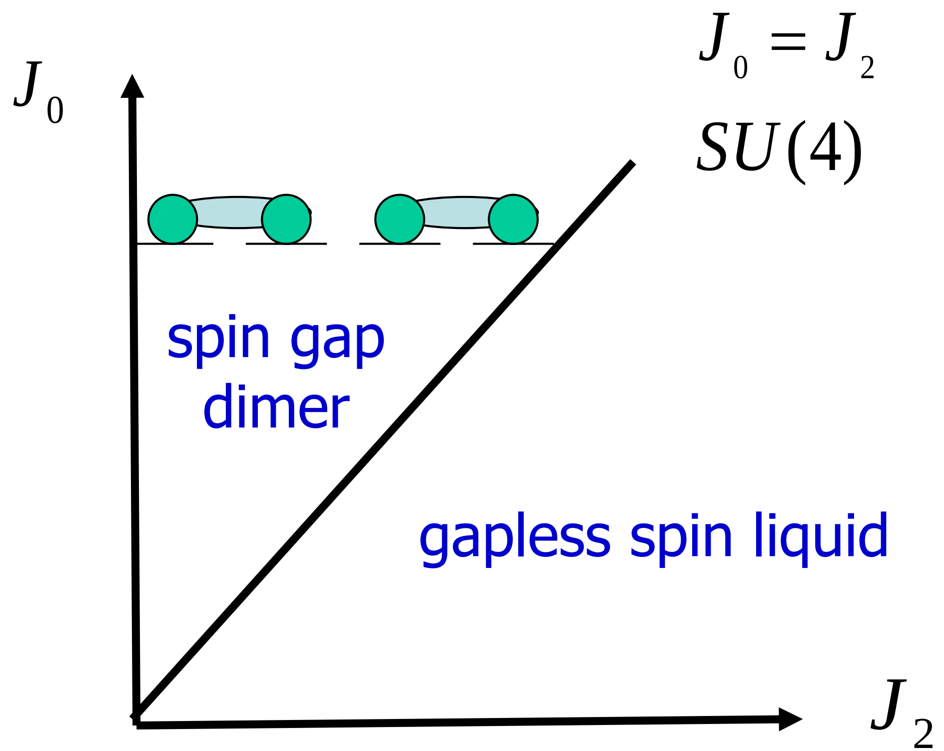
L_{ab}: 3 spins + 7 spin-octupole tensors; n_a: spin-quadrapole operators; L_{ab} and n_a together form the 15 SU(4) generators.

1D lattice (one particle per site)

- Phase diagram is obtained from bosonization analysis and confirmed from DMRG calculations.

- Gapped spin dimer phase at $J_0 > J_2$; bond spin singlet.

- Gapless spin liquid phase at $J_0 \leq J_2$. Spin correlation exhibits 4-site periodicity of oscillations.



C. Wu, Phys. Rev. Lett. 95, 266404 (2005); Hung, Wang, and Wu, PRB 05446, (2011)

Unsolved difficulty: 2D phase diagram

- J_2=0, Neel ordering obtained by QMC.

K. Harada et. al. PRL 90, 117203, (2003).

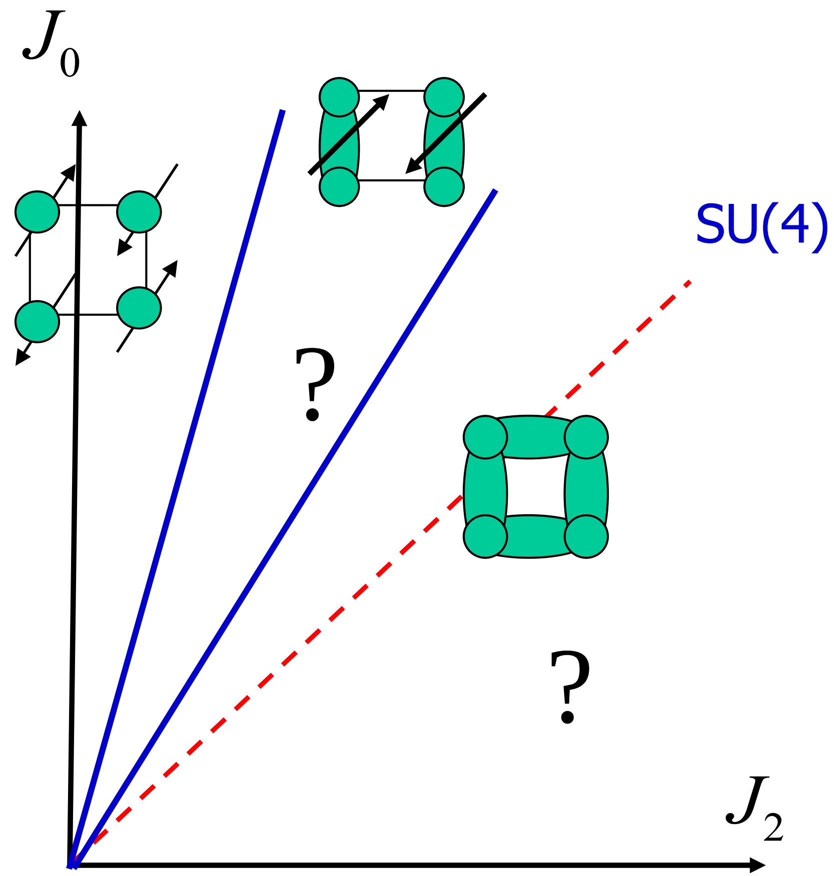
- J_2>0, no conclusive results! Difficult both analytically and numerically.

2D Plaquette ordering at the SU(4) point?

Exact diagonalization on a 4*4 lattice

Bossche et. al., Eur. Phys. J. B 17, 367 (2000).

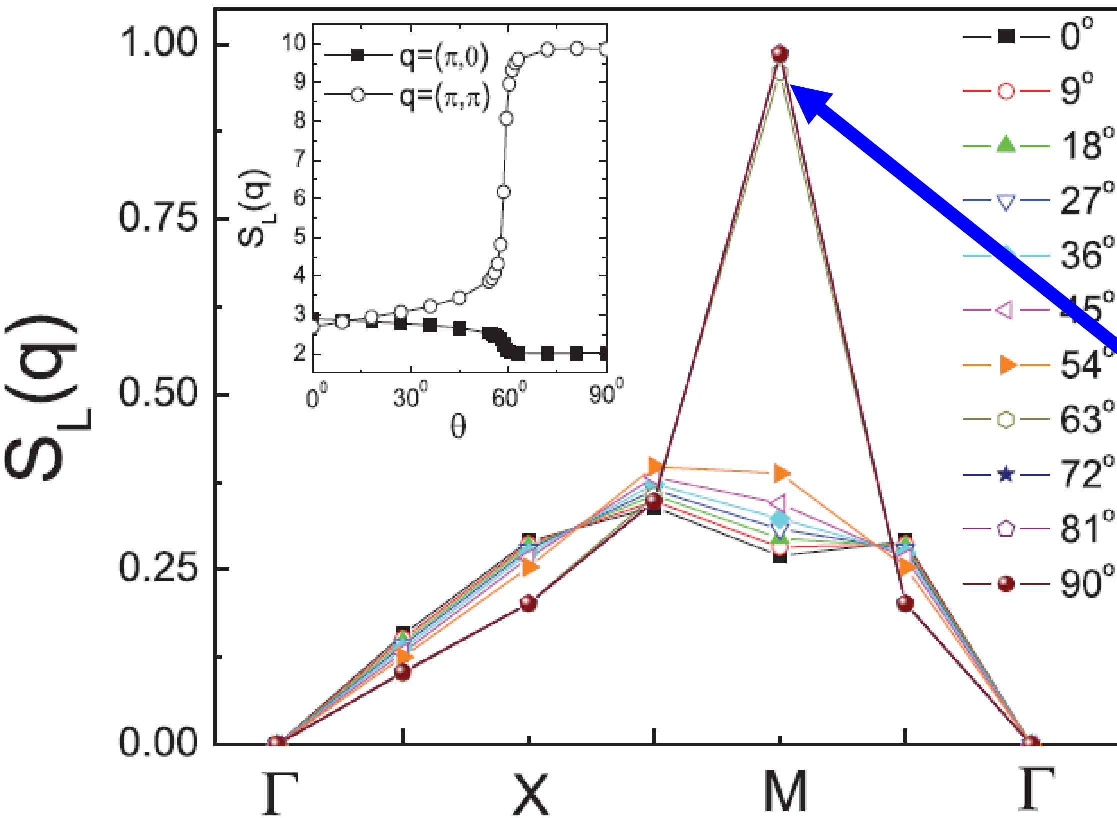
- Phase transitions as J_0/J_2? Dimer phases? Singlet or magnetic dimers?

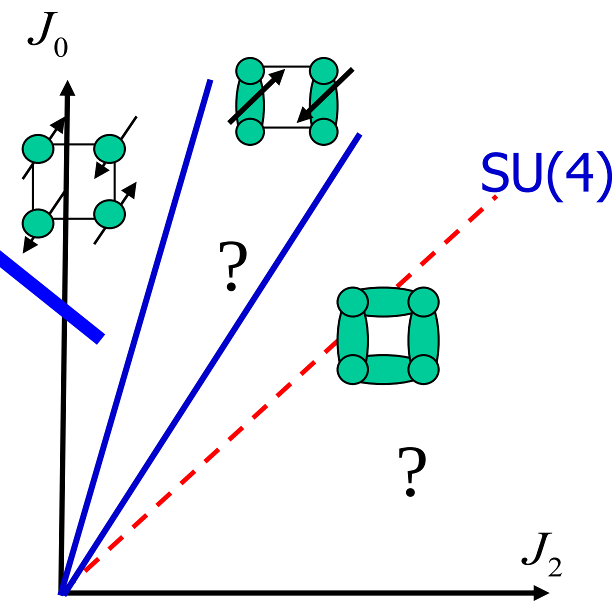


4x4 Exact diag. (I): Neel correlation

- Spin structure form factor peaks at (π,π) at $\theta > 60^\circ$, indicating strong Neel correlation.

$$S_L(q) \sim \sum_{1\le a<b\le 5} \sum_{i,j} \langle L_{ab}(i) L_{ab}(j) \rangle e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$





H. H. Hung, Y. P. Wang, C. Wu, Phys. Rev. B 84, 054406, (2011).

4x4 Exact Diag. (II): Dimer correlation

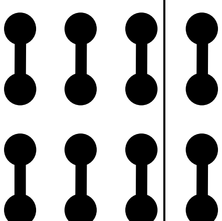
- Susceptibility: $H(\delta) = H_{exc} + \delta * H_{perp}$ $\quad E(\delta) = E(0) - \frac{1}{2}\chi\delta^2,$

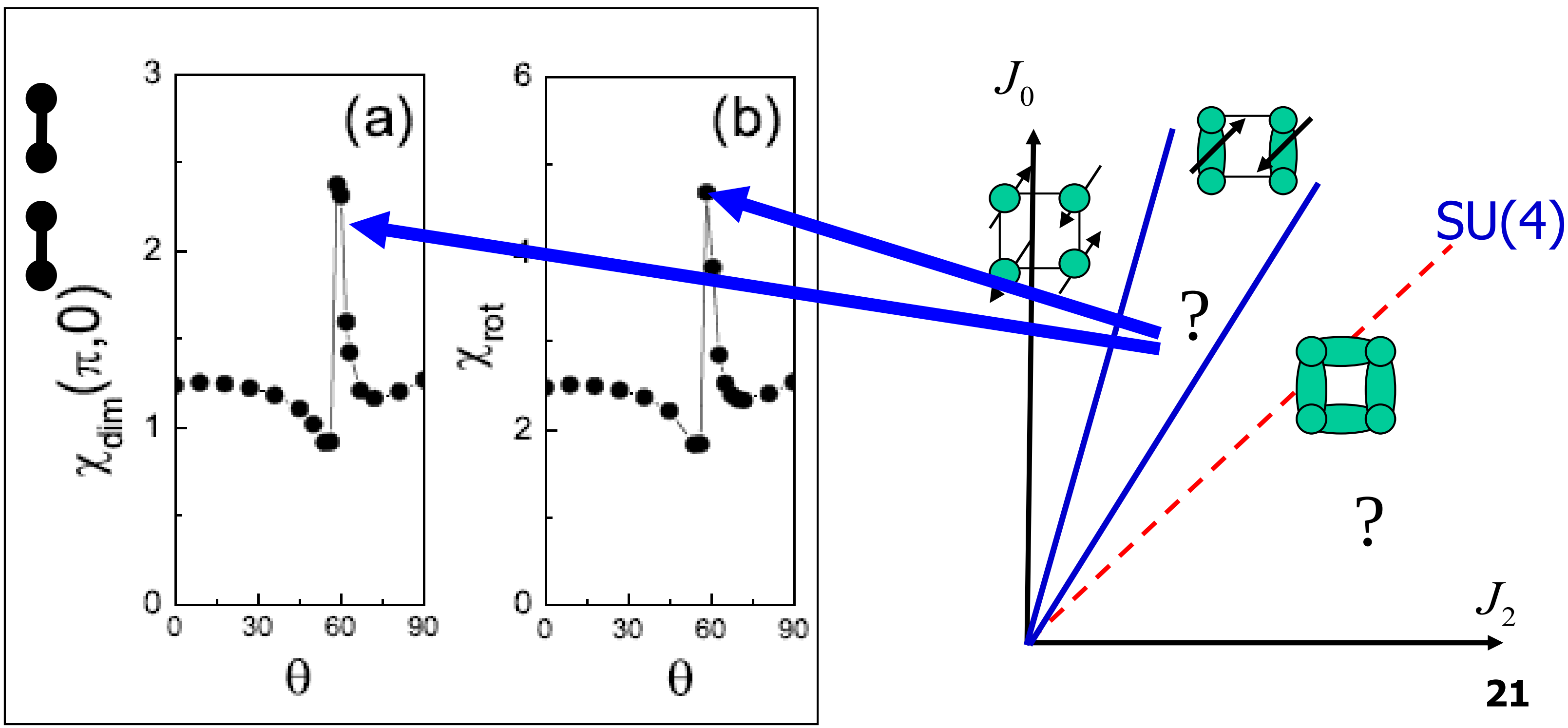
- a) Break translational symm:

$$H_{pert} = \sum_i \cos(\vec{Q}\cdot\vec{r}_i)\ H_{ex}(i,i+x),$$

- b) Break rotational symm:

$$H_{pert} = \sum_i H_{ex}(i,i+x) - H_{ex}(i,i+y)$$



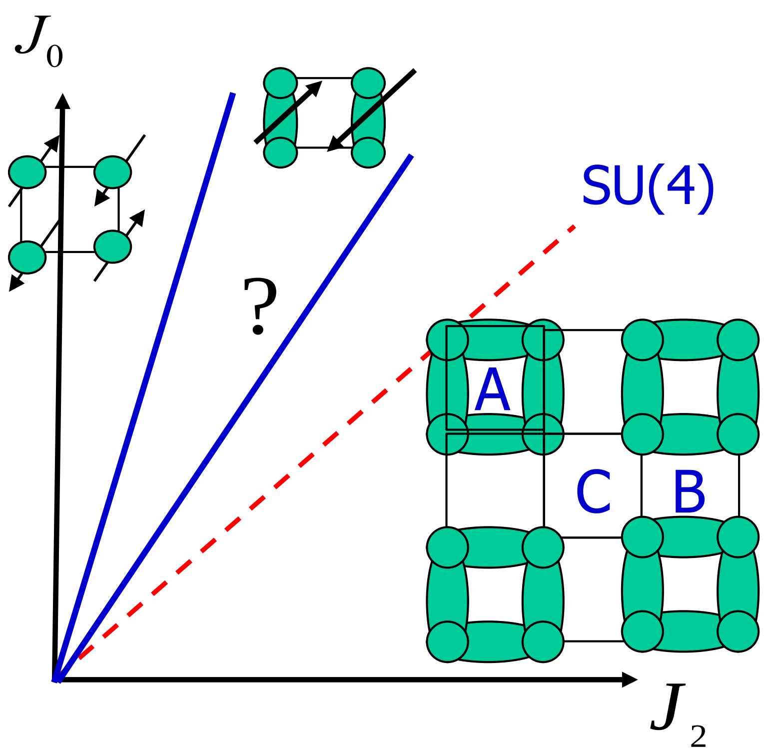


4x4 Exact Diag. (III): Plaquette formation?

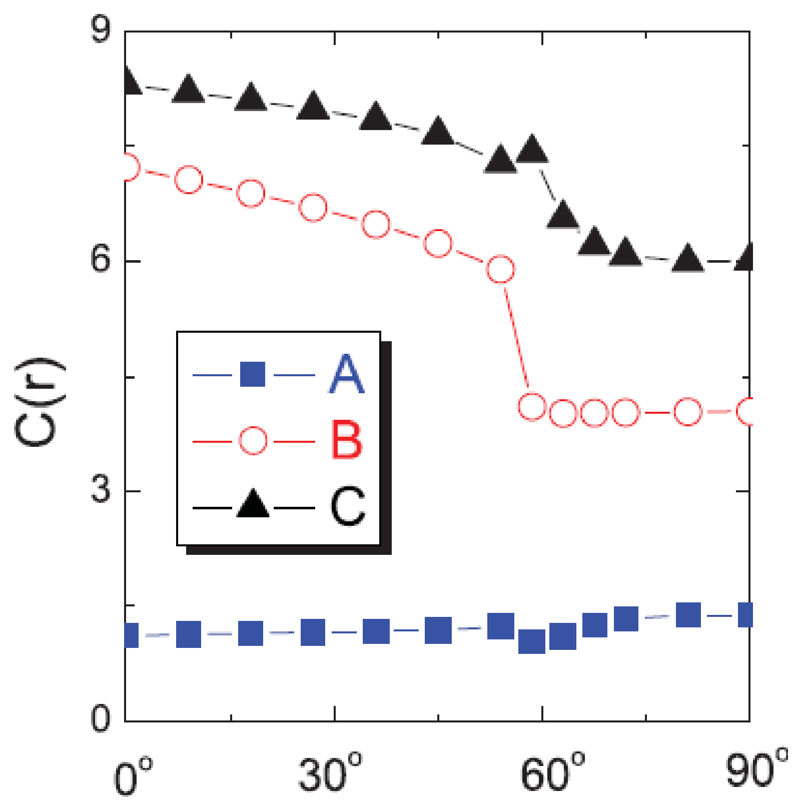
- Local Casimir; analogy to total spin of SU(2).

$$C(r) \sim \left\langle \sum_{1\le a<b\le 5} \{ \sum_{i\in plaquette\ r} L_{ab}(i) \}^2 \right\rangle$$

C(r) →0: singlet



Open boundary condition



More technical details

Brief Review

Modern Physics Letters B, Vol. 20, No. 27 (2006) 1707–1738

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HIDDEN SYMMETRY AND QUANTUM PHASES IN SPIN-3/2 COLD ATOMIC SYSTEMS

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Conclusion

- **Large-spin cold fermions are quantum-like NOT classical**.

- The simplest case of spin-3/2 fermions are characterized by a generic Sp(4) (SO(5)) symmetry without fining tuning.

- Spin-3/2 Hubbard model unifies AF, SC and CDW phases with exact symmetries.

- Exotic “color magnetism” exhibits dominant multi-particle correlations.

Our other work:

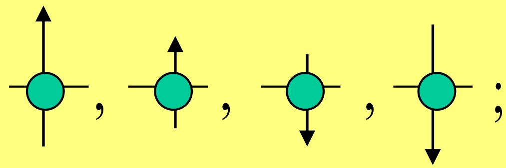
- Quintet pairing superfluid and SO(4) Cheshire charge.

- 4-fermion baryon-like superfluidity

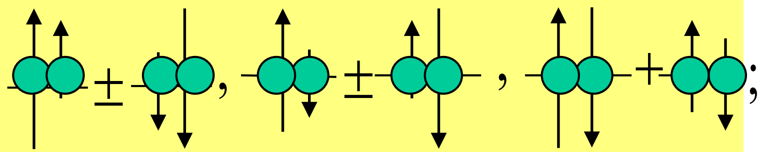
Sp(4) (SO(5)) symmetry: the single site problem

$E_0 = 0$

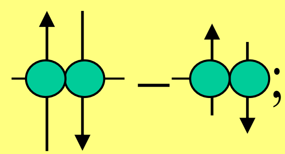
$E_1 = -\mu$



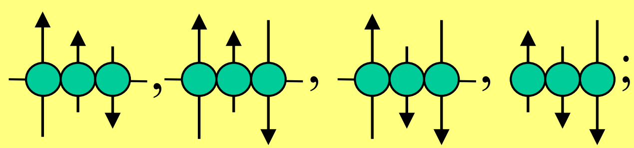
$E_2 = U_2 - 2\mu$



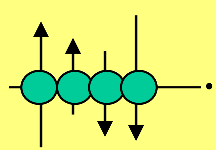
$E_3 = U_0 - 2\mu$



$E_4 = \frac{1}{2}U_0 + \frac{5}{2}U_2 - 3\mu$



$E_5 = U_0 + 5U_2 - 4\mu$



2^4=16 states.

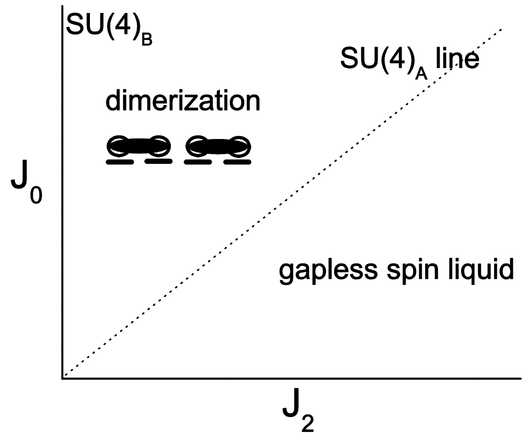
	SU(2)	SO(5)	degeneracy
$E_{0,3,5}$	singlet	scalar	1
$E_{1,4}$	quartet	spinor	4
E_2	quintet	vector	5

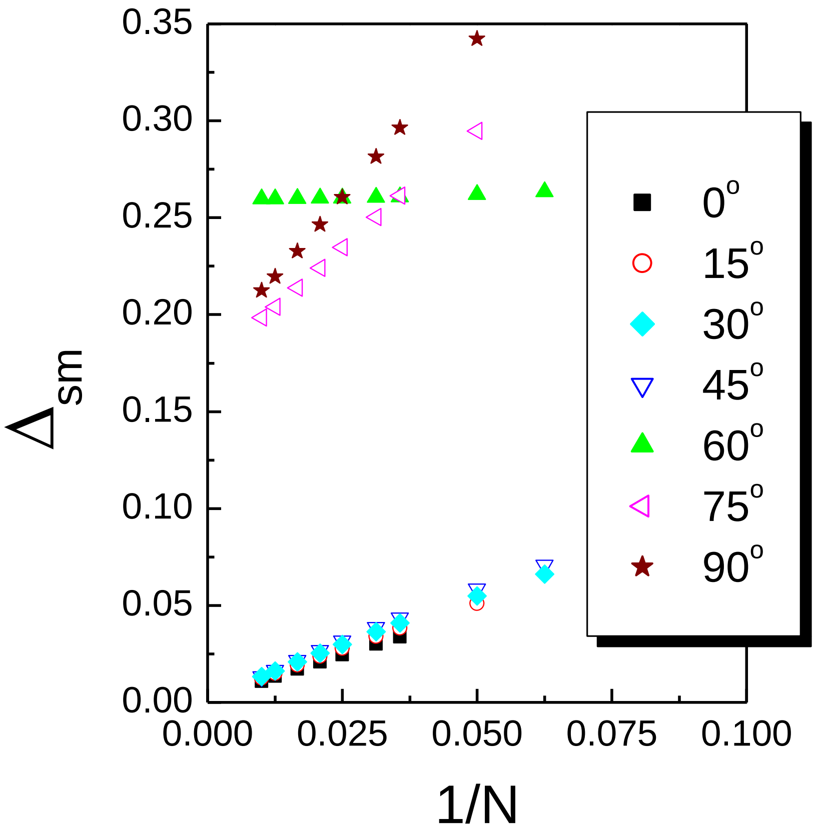
- $U_0=U_2=U$, SU(4) symmetry.

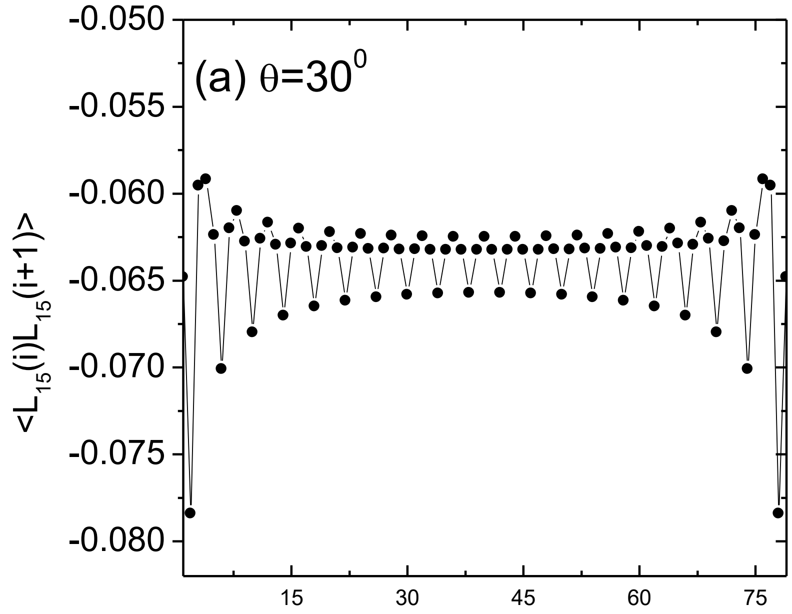
$$H_{\text{int}} = \frac{U}{2} n(n-1)$$

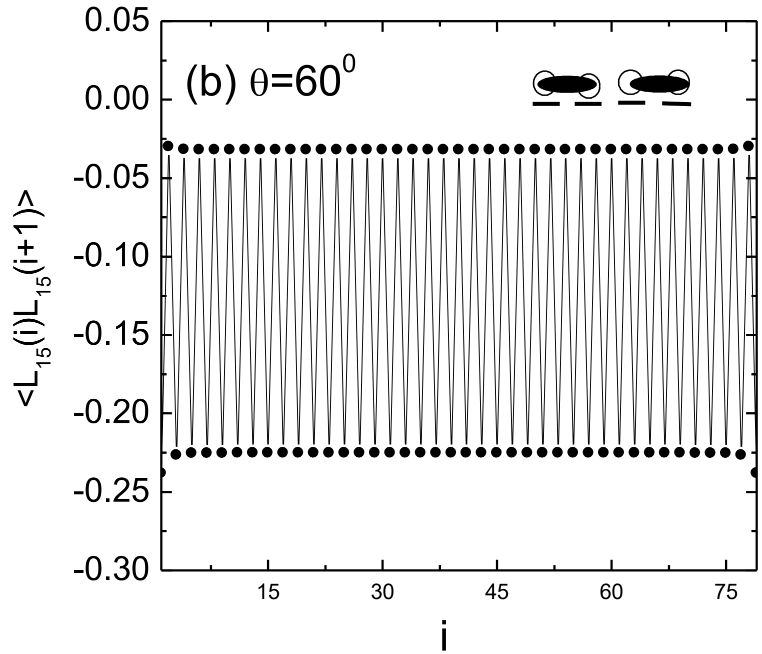
- Except 3-spin, what are the 7 **hidden** conserved quantities?

DMRG results in 1D

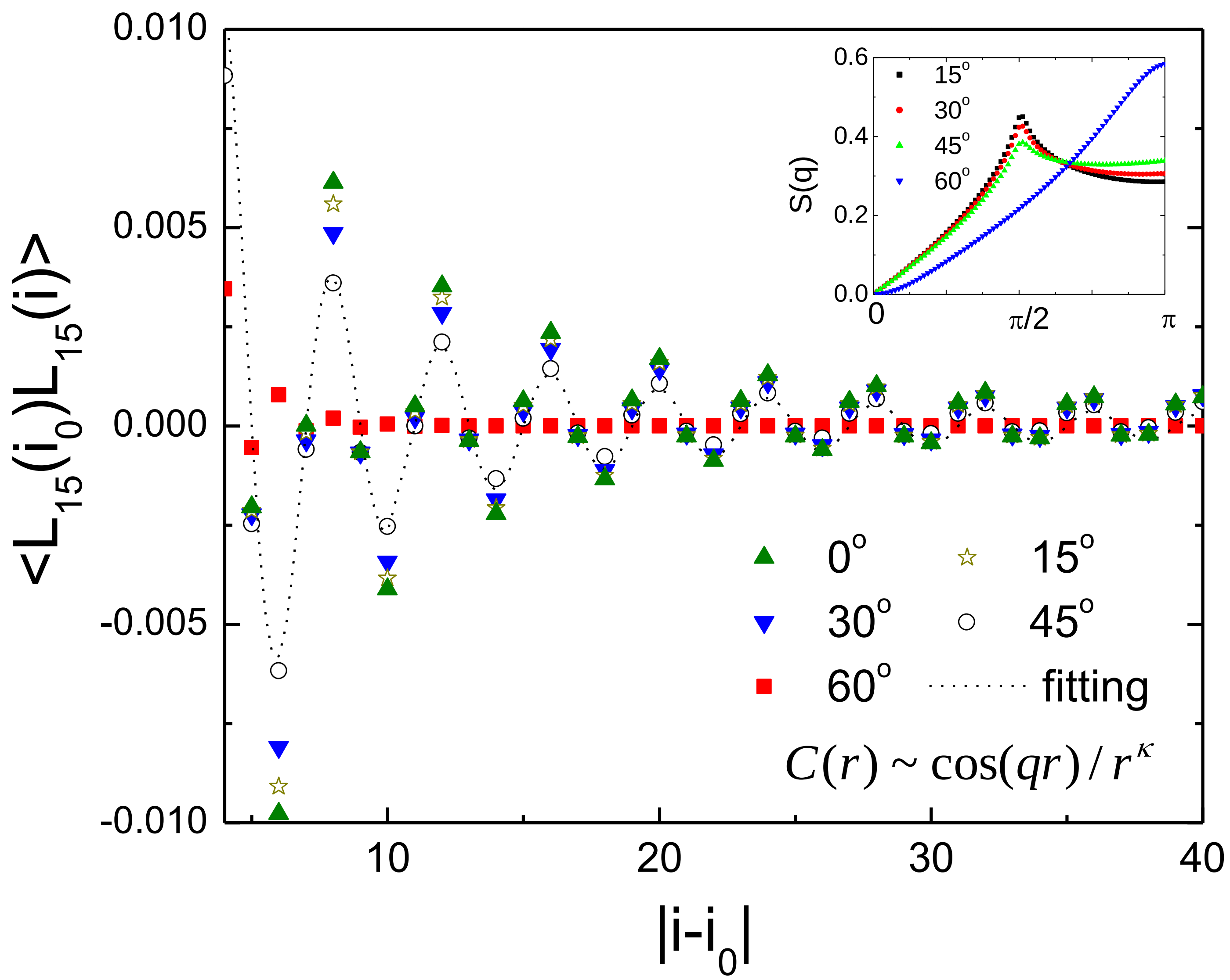








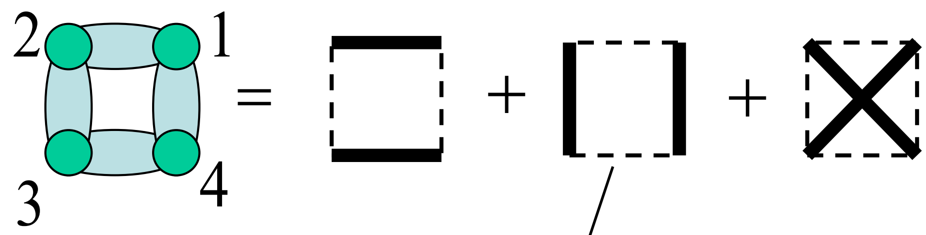
Two-point correlations show four-site periodicity



Sp(4) magnetism: a four-site problem

- Bond spin singlet:

- Plaquette SU(4) singlet:



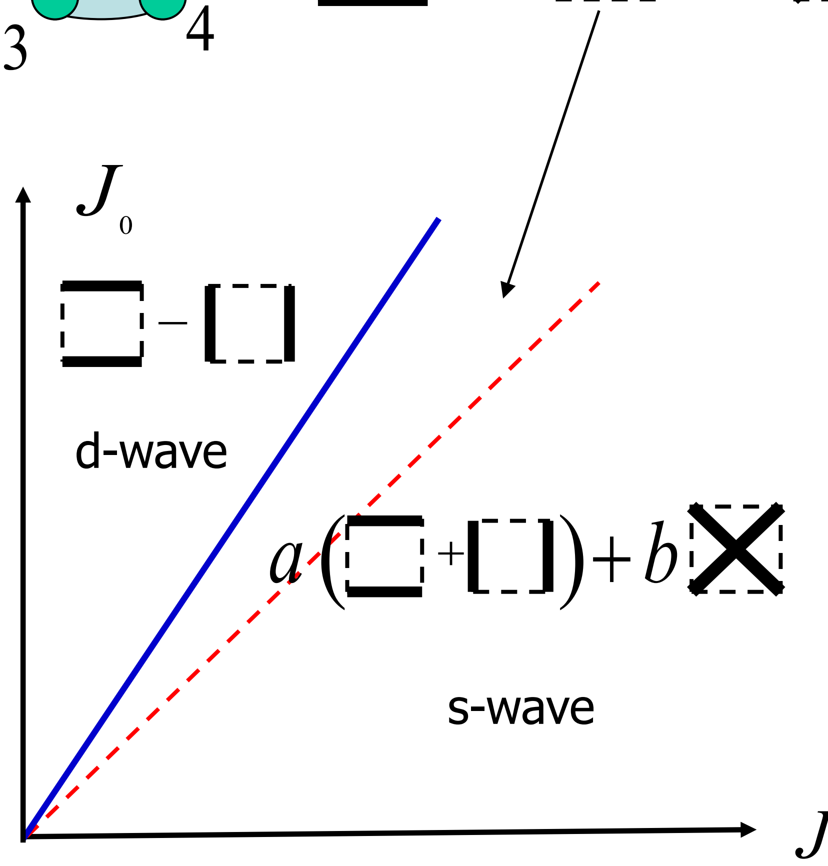
$$\frac{\varepsilon_{\alpha\beta\gamma\delta}}{4!}\psi^+_\alpha\psi^+_\beta\psi^+_\gamma\psi^+_\delta\left|0\right\rangle$$

4-body EPR state; no bond orders

- Level crossing:

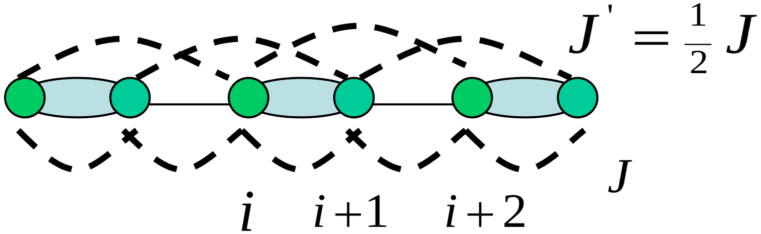
d-wave to s-wave

- Hint to 2D?



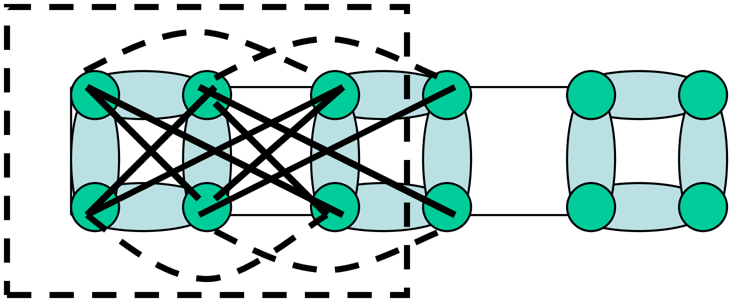
Exact result: SU(4) Majumdar-Ghosh ladder

- Exact dimer ground state in spin 1/2 M-G model.

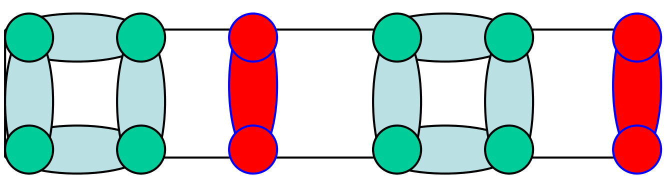


$$H = \sum_i H_{i,i+1,i+2}, \quad H_{i,i+1,i+2} = \frac{J}{2}(\vec{S}_i + \vec{S}_{i+1} + \vec{S}_{i+2})^2$$

- SU(4) M-G: plaquette state.



$$H = \sum_{\text{every six-site cluster}} H_i$$



$$H_i = (\sum_{\text{six sites}} L_{ab})^2 + (\sum_{\text{six sites}} n_a)^2$$

SU(4) Casimir of the six-site cluster

- Excitations as fractionalized domain walls.

S. Chen, C. Wu, S. C. Zhang and Y. P. Wang,
Phys. Rev. B 72, 214428 (2005).