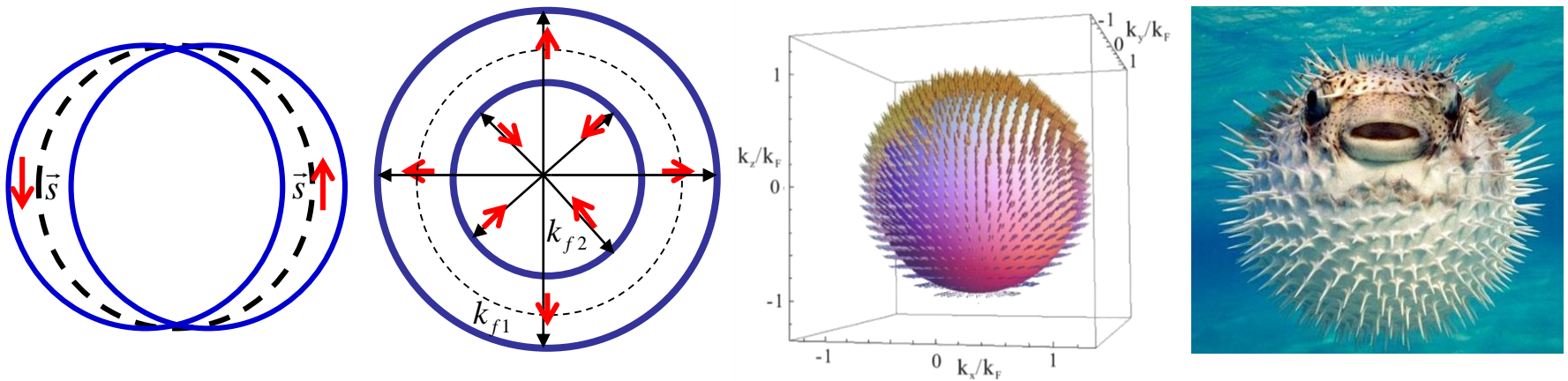


# Unconventional magnetism and spontaneous spin-orbit ordering

Congjun Wu (Univ. California, San Diego)



Ref. 1) C. Wu and S. C. Zhang, PRL 93, 36403 (2004);

2) C. Wu, K. Sun, E. Fradkin, and S. C. Zhang, PRB.75, 115103 (2007).

3) Y. Li, C. Wu, PRB 85, 205126 (2012).

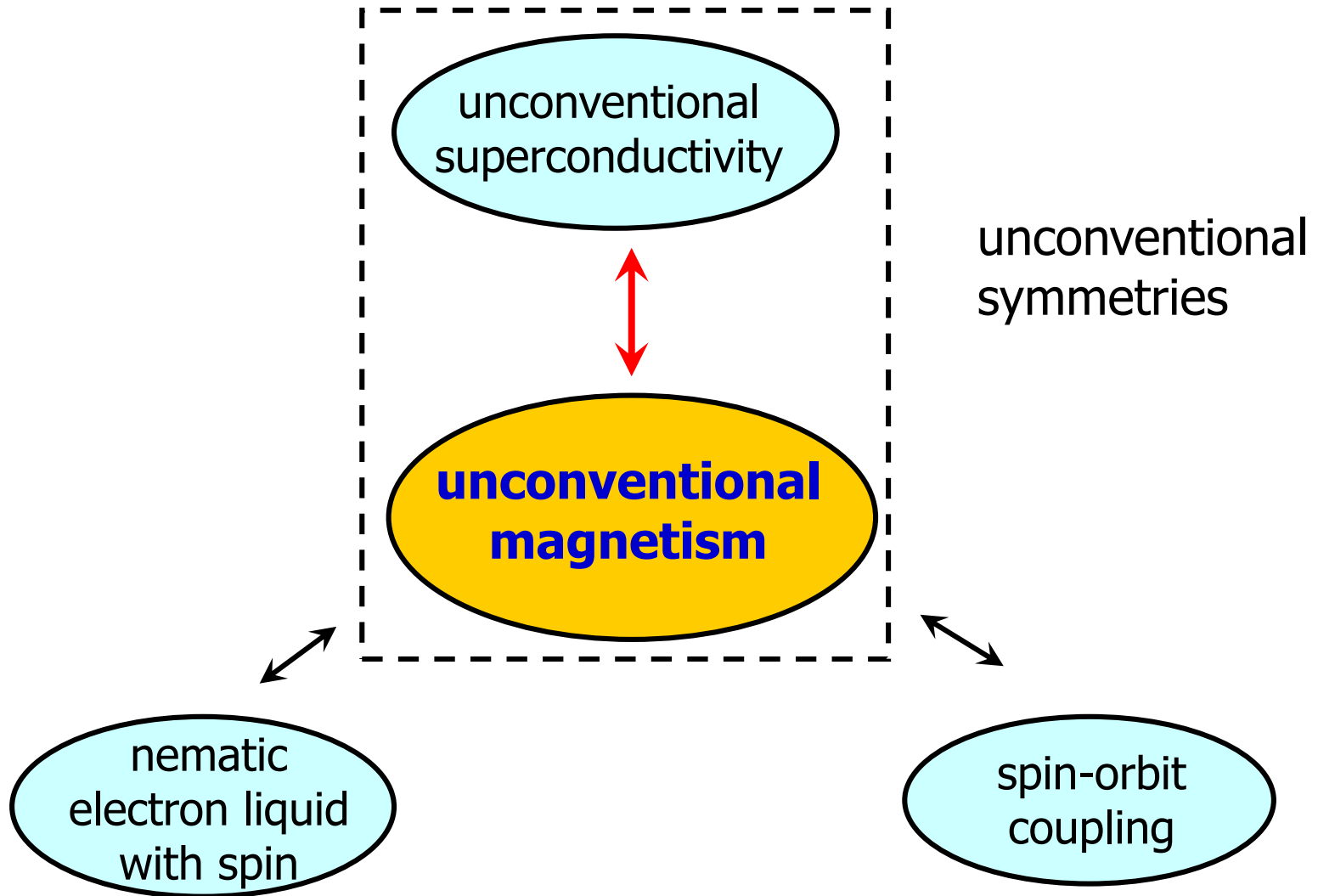
Jan 18, 2017, UCSD

# Collaborators

- S. C. Zhang, Stanford.
- E. Fradkin, UIUC.
- D. Arovas, UCSD
- K. Sun, UIUC (now at U. Michigan)
- W. C. Lee, UCSD (now at Binghamton, SUNY)
- Y. Li, UCSD (now at Johns Hopkins)
- C Xu and S. L. Xu, UCSD

Thanks to J. Hirsch, M. Beasley, A. L. Fetter, S. Kivelson, J. Zaanen, S. Das Sarma,, A. J. Leggett, L. Balents for stimulating discussions.

# Introduction

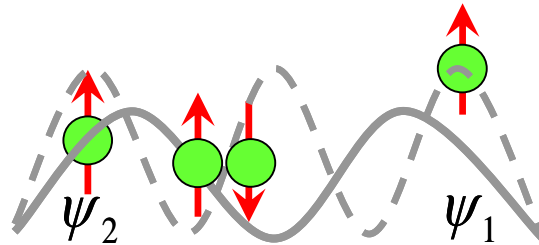


# Itinerant FM: Quantum origin!

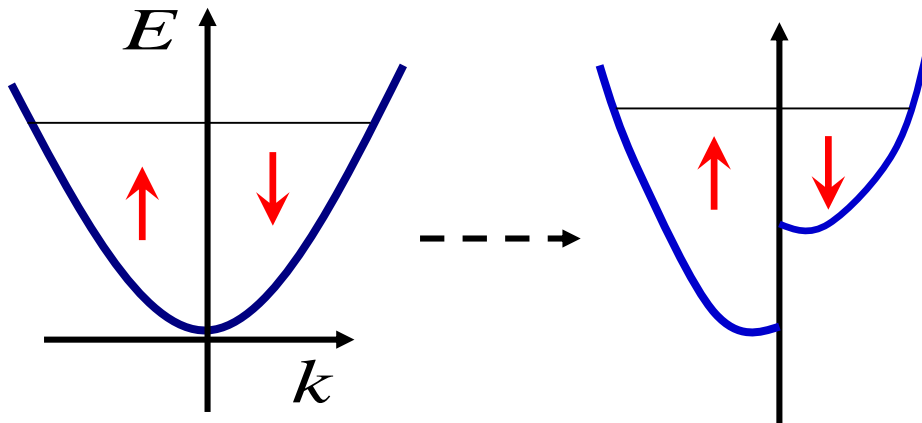


E. C. Stoner

- Q: How does spin-independent interaction induce spin polarization?
- Electrons with parallel spins avoid each other to reduce repulsion.



$$E_{\uparrow\uparrow} < E_{\uparrow\downarrow}$$



- **Stoner criterion:**

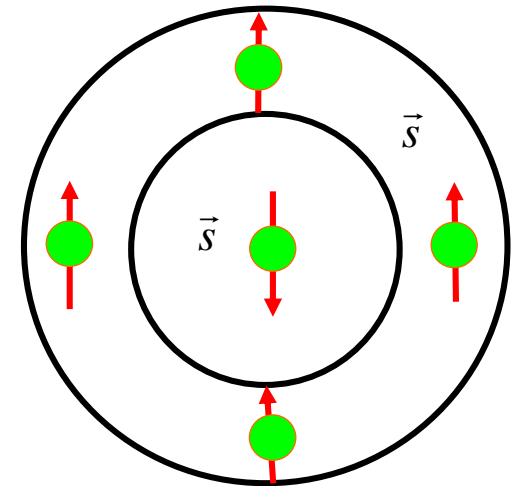
$$UN_0 > 1$$

U: interaction strength

# Itinerant ferromagnetism: *s*-wave

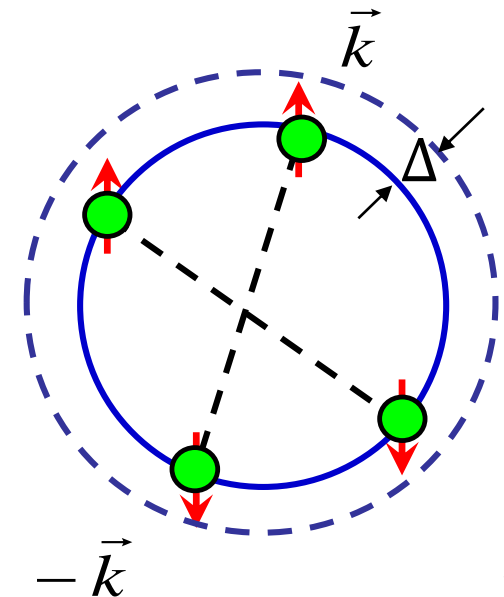
- Spin rotational symmetry is broken.

• Orbital rotational symmetry is **NOT** broken: spin polarizes along a **fixed direction**.



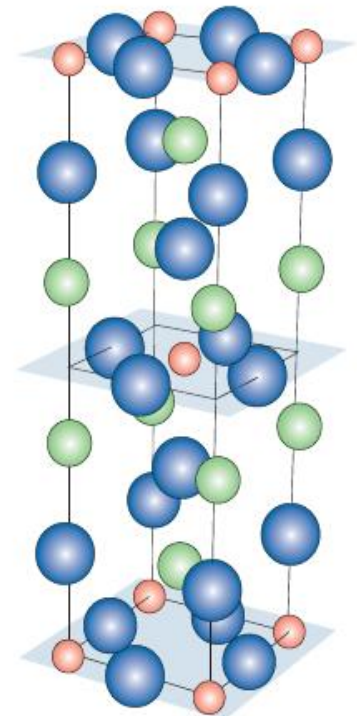
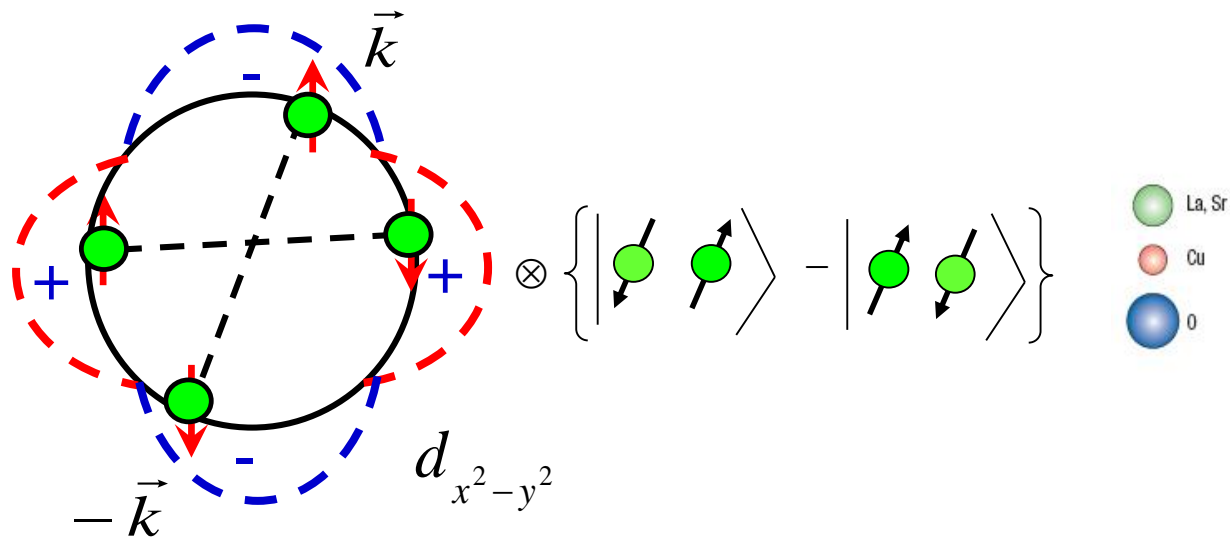
- *cf.* Conventional superconductivity.

*s*-wave: gap function invariant over the Fermi surface.



# cf. Unconventional superconductivity

- High partial wave pairing symmetries (e.g.  $p$ ,  $d$ -wave ...).
- $d$ -wave: high  $T_c$  cuprates.

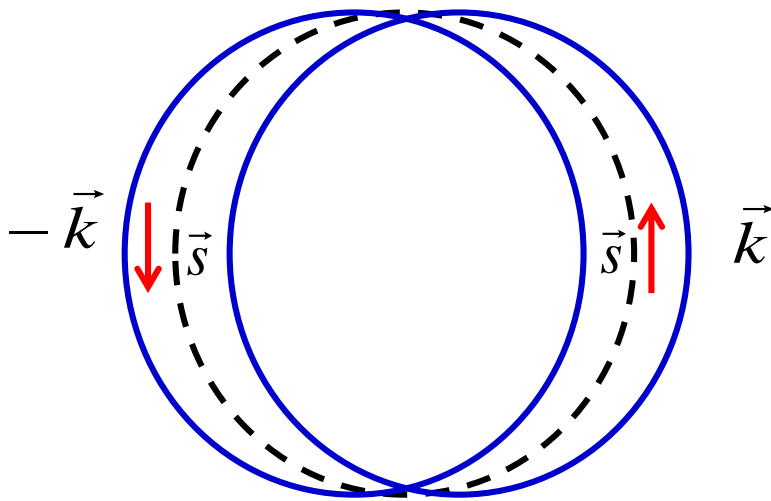


- $p$ -wave:  $\text{Sr}_2\text{RuO}_4$ ,  $^3\text{He}$ -A and B.

D. J. Van Harlingen, Rev. Mod. Phys. 67, 515 (1995); C. C. Tsuei et al., Rev. Mod. Phys. 72, 969 (2000).

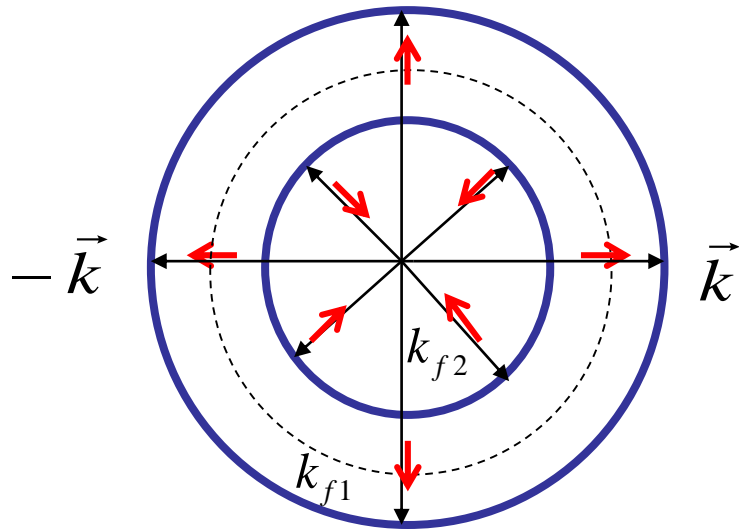
# New states of matter: unconventional magnetism!

- High partial wave channel magnetism (e.g.  $p$ ,  $d$ -wave...).
- Multi-polar spin distribution over the Fermi surface.



anisotropic  $p$ -wave state

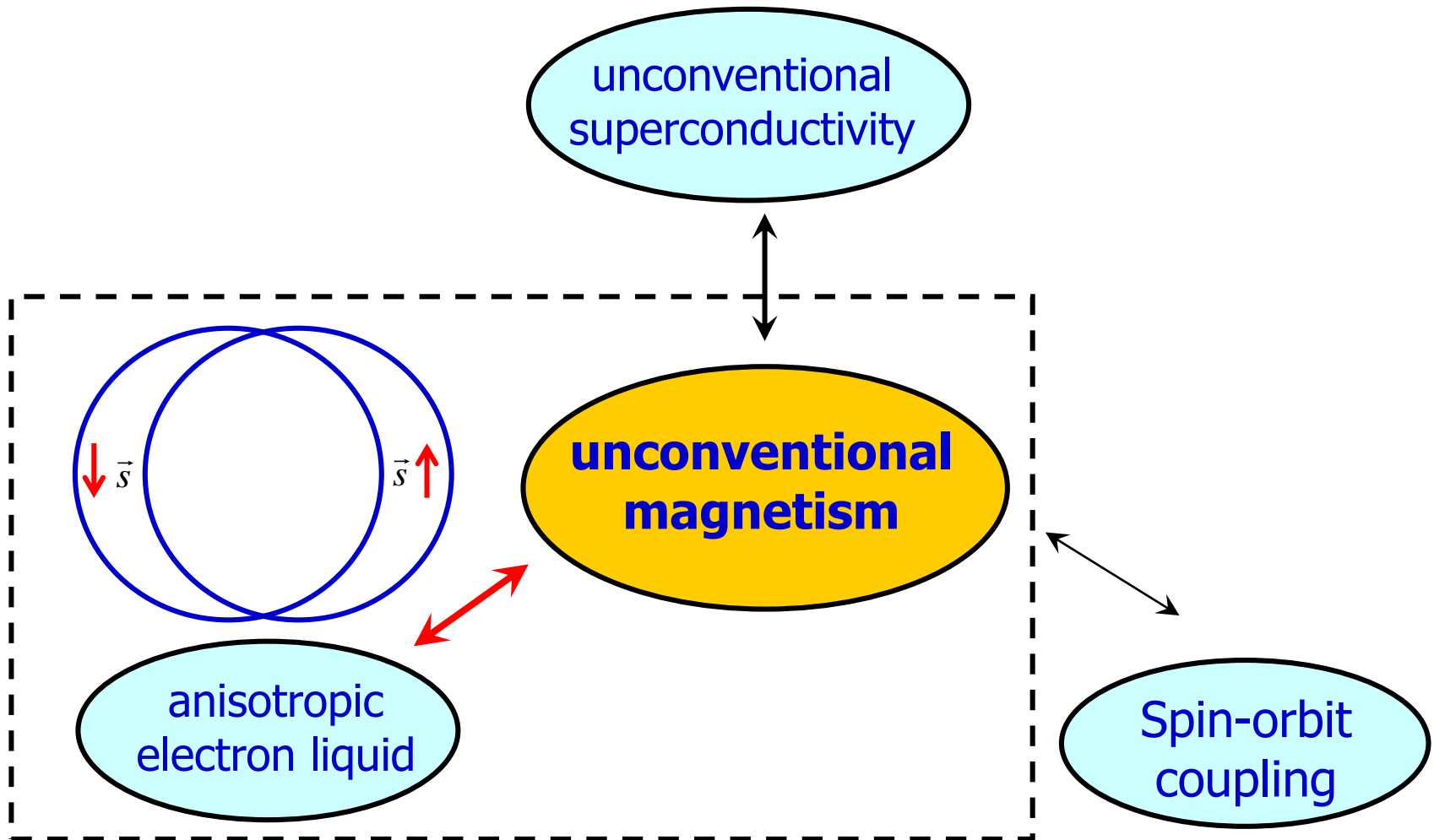
**spin-split state** by J. E. Hirsch, PRB 41, 6820 (1990); PRB 41, 6828 (1990).



isotropic  $p$ -wave state

spin flips the sign as  $\vec{k} \rightarrow -\vec{k}$

# Introduction: electron spin liquid-crystal

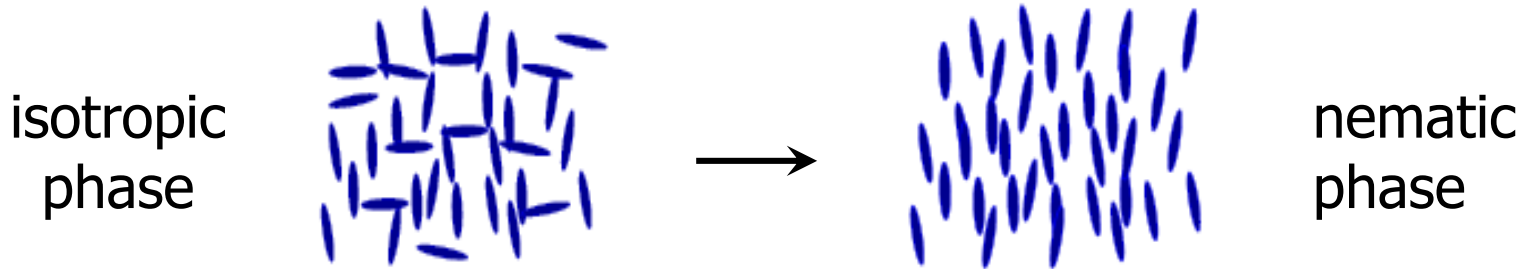




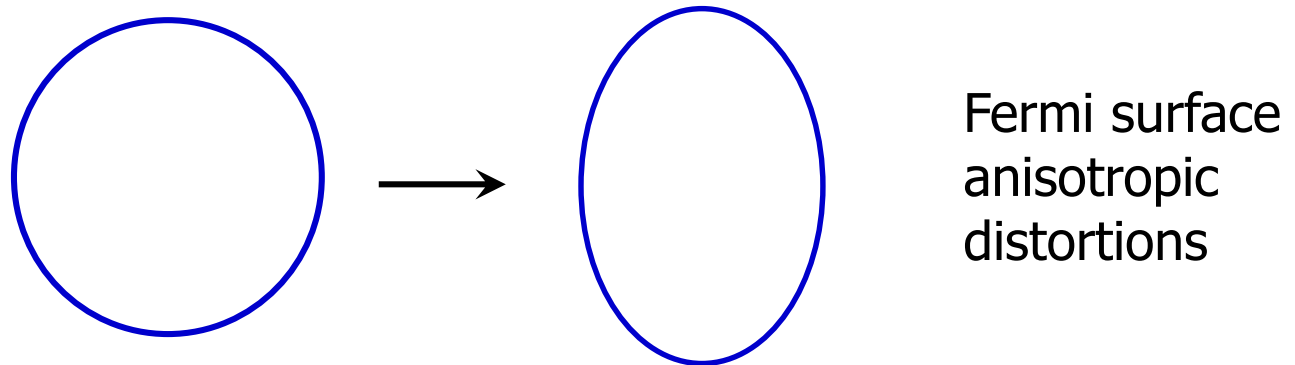
# Anisotropy: liquid crystalline order

- Classic liquid crystal.

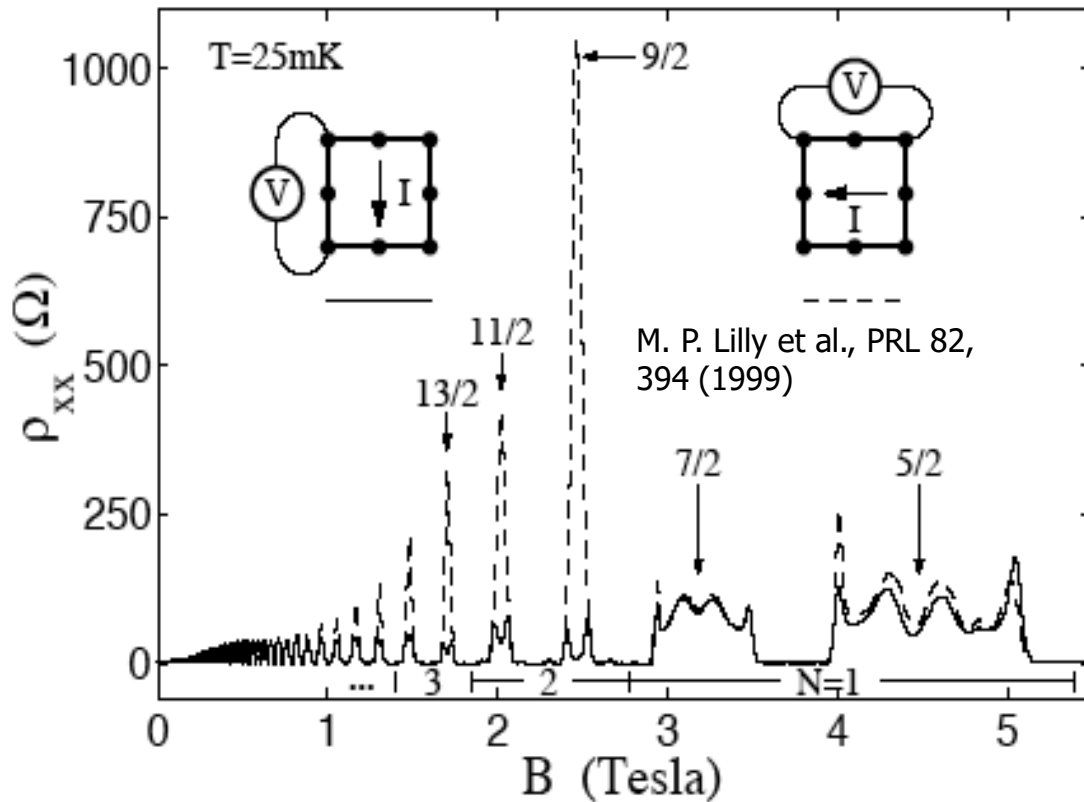
Nematic phase: rotational anisotropic but translational invariant.



- Quantum version of liquid crystal: **nematic electron liquid.**



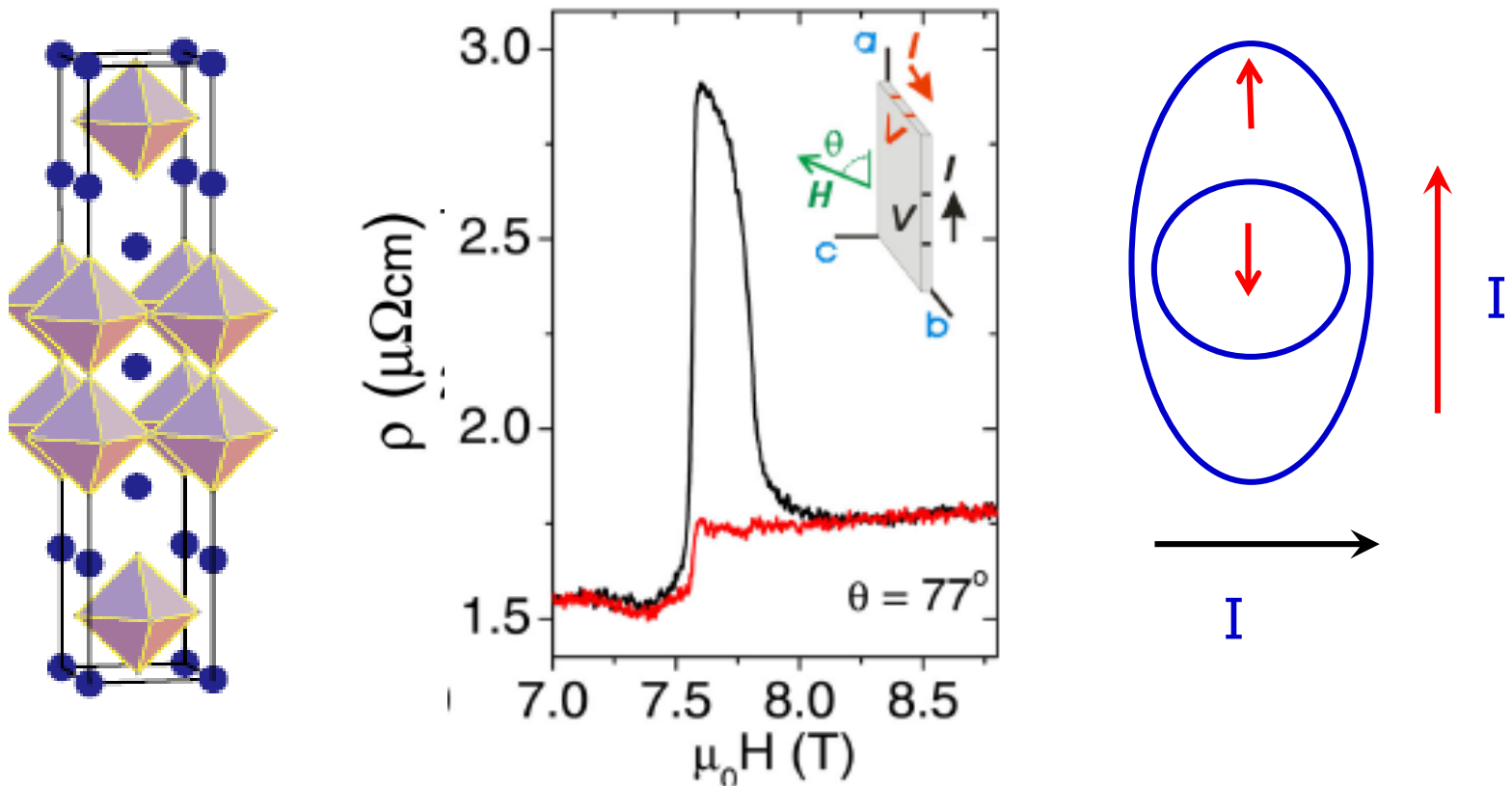
# Nematic electron liquid in 2D GaAs/AlGaAs at high B fields



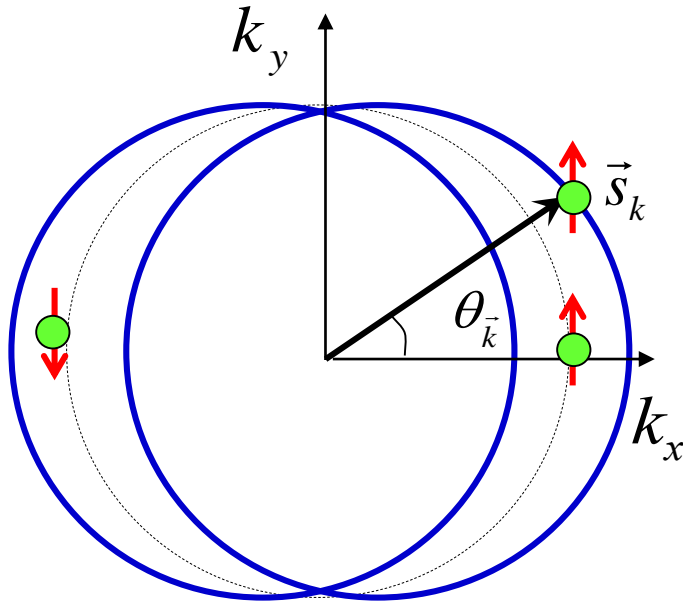
M. M. Fogler, et al, PRL 76 ,499 (1996), PRB 54, 1853 (1996); E. Fradkin et al, PRB 59, 8065 (1999), PRL 84, 1982 (2000).

# Nematic electron liquid in $\text{Sr}_3\text{Ru}_2\text{O}_7$ at high B fields

- Quasi-2D system; resistivity **anisotropy** at 7~8 Tesla.
- Fermi surface nematic distortions.



# Anisotropic unconventional magnetism: spin liquid-crystal phases!



**anisotropic  $p$ -wave  
magnetic phase**

- $p$ -wave distortion of the Fermi surface.
- Spin dipole moment in momentum space (not in coordinate space).

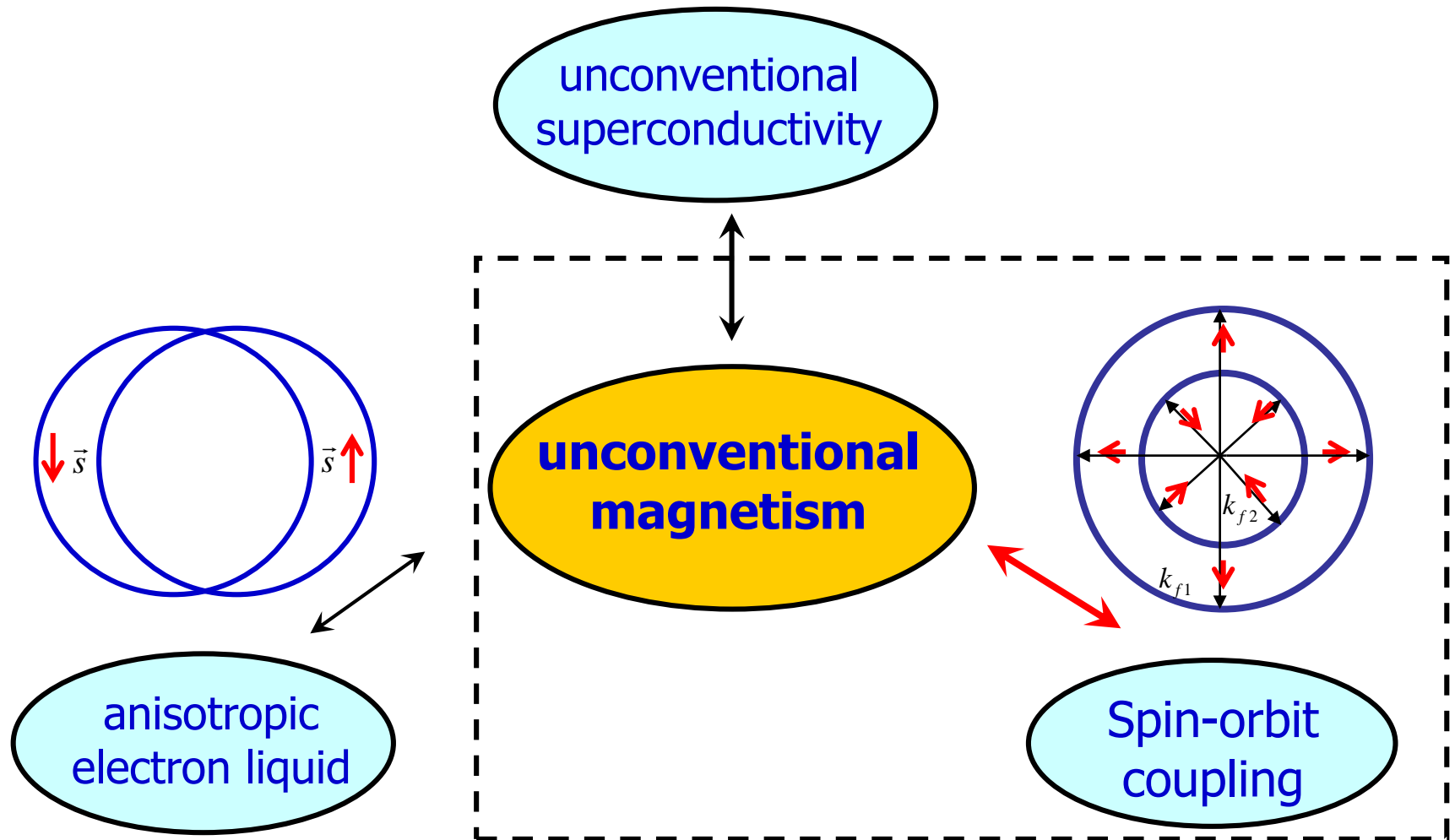
$$\vec{n}_1 = \sum_{\vec{k}} \vec{s}_k \cos \theta_k \neq 0$$

- Both orbital and spin rotational symmetries are broken.

**spin-split state** by J. E. Hirsch, PRB 41, 6820 (1990); PRB 41, 6828 (1990).

V. Oganesyan, et al., PRB 64,195109 (2001).  
C. Wu et al., PRL 93, 36403 (2004); Varma et al., Phys. Rev. Lett. 96, 036405 (2006)

# Introduction: dynamic generation of spin-orbit coupling



# Unconventional magnetism: dynamic generation of spin-orbit (SO) coupling!

- Conventional wisdom:

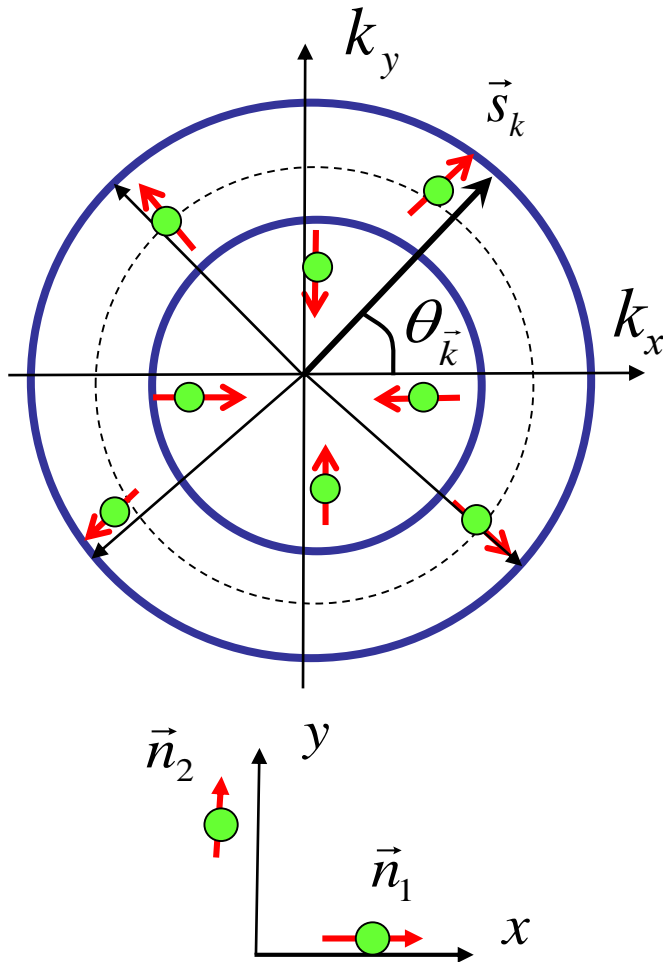
A **single-body** effect from the Dirac equation

- New mechanism (**many-body collective effect**):

Generate SO coupling through **unconventional magnetic phase transitions.**

- **Advantages:** tunable SO coupling by varying temperatures; new types of SO coupling.

# The isotropic $p$ -wave magnetic phase



- Helicity  $\vec{\sigma} \cdot \vec{k}$  is a good quantum number.
- No net spin-moment; spin dipole moment in momentum space.

$$\vec{n}_1 = \sum_{\vec{k}} \vec{s}_k \cos \theta_k, \quad \vec{n}_2 = \sum_{\vec{k}} \vec{s}_k \sin \theta_k$$

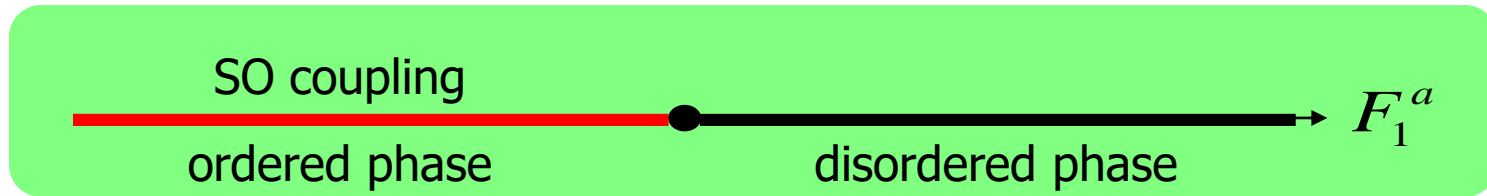
- **Isotropic phase with SO coupling.**

$$H_{MF} = H_0 + \bar{n} \sum_k \psi_\alpha^\dagger \vec{\sigma}_{\alpha\beta} \cdot \vec{k} \psi_\beta$$

$$\bar{n} = |\vec{n}_1| = |\vec{n}_2|$$

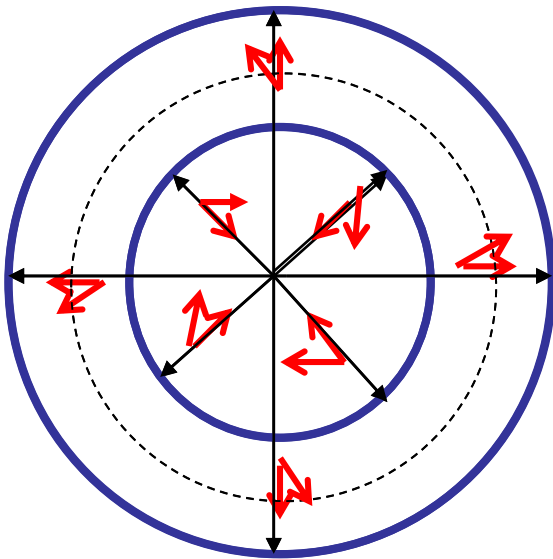
C. Wu et al., PRL 93, 36403 (2004);  
 C. Wu et al., PRB PRB.75, 115103  
 (2007). .

# The subtle symmetry breaking pattern



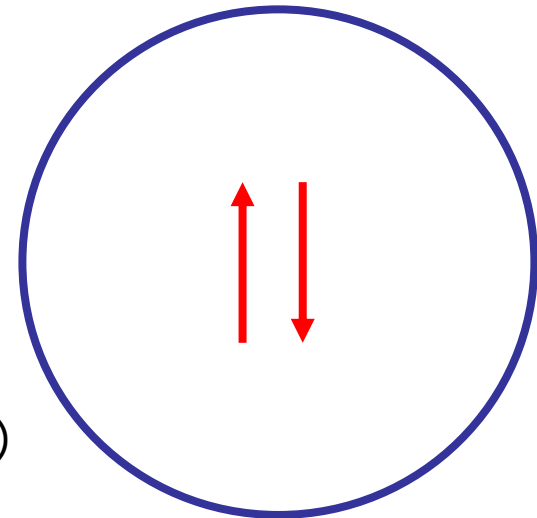
- $\vec{J}$  is conserved, but  $\vec{L}$ ,  $\vec{S}$  are not separately conserved.

- **Independent** orbital and spin rotational symmetries.



$$\vec{J} = \vec{L} + \vec{S}$$

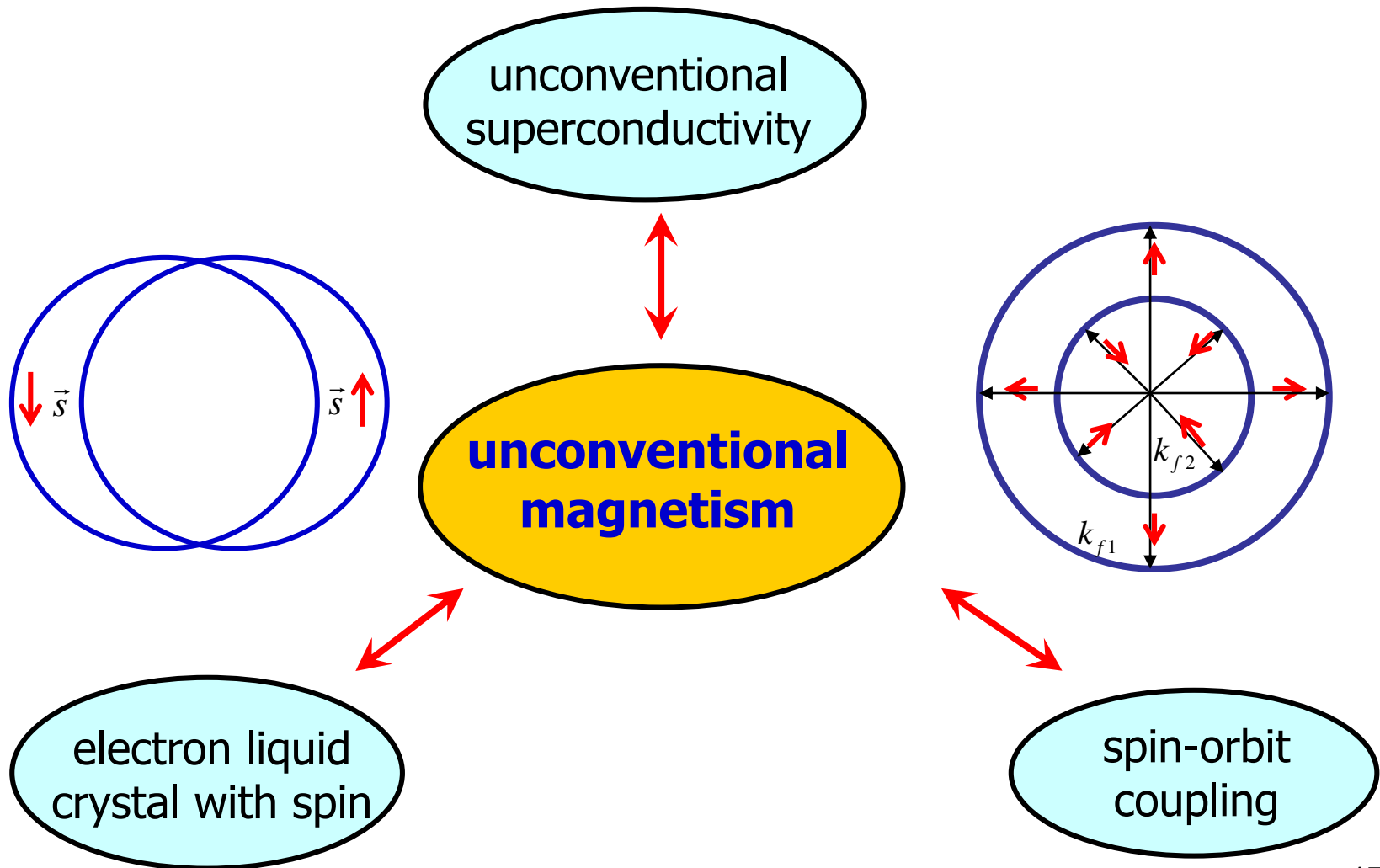
Leggett, Rev. Mod. Phys **47**, 331 (1975)



- **Relative spin-orbit** symmetry breaking.



# Summary of the introduction



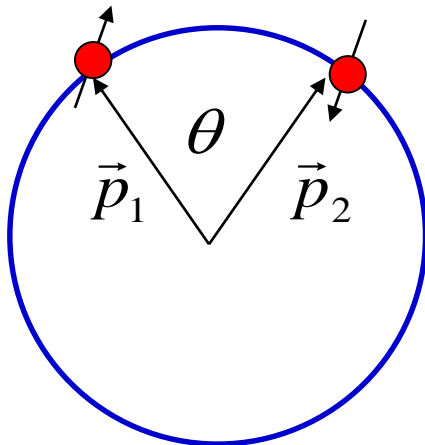
# Outline

- Introduction.
- **Mechanism for unconventional magnetic phase transitions.**
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
  - Collective modes and neutron spectroscopy.
- Spin-orbit coupled Fermi liquid theory – magnetic dipolar.
- Possible directions of experimental realization and detection methods.

# Landau Fermi liquid (FL) theory



L. Landau



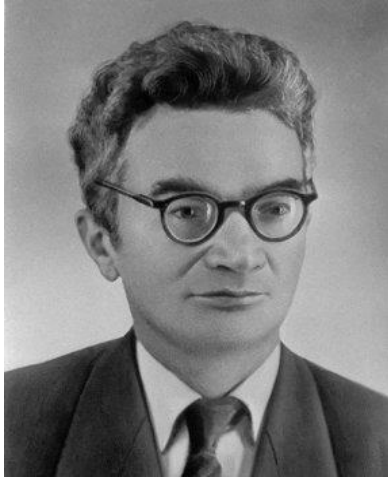
- The existence of Fermi surface.
- Electrons close to Fermi surface are important.
- Interaction functions:

$$f_{\alpha\beta,\gamma\delta}(\hat{p}_1, \hat{p}_2) = f^s(\hat{p}_1, \hat{p}_2) \quad \text{density} \\ + f^a(\hat{p}_1, \hat{p}_2) \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \quad \text{spin}$$

- Landau parameter in the  $l$ -th partial wave channel:

$$F_l^{s,a} = N_0 f_l^{s,a} \quad N_0 : \text{DOS}$$

# Pomeranchuk instability



I. Pomeranchuk

- Fermi surface: elastic membrane.

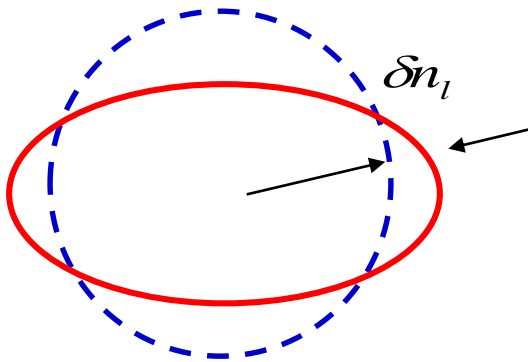
- Stability:

$$\Delta E_K \propto (\delta n_l^{s,a})^2$$

$$\Delta E_{\text{int}} \propto \frac{F_l^{s,a}}{2l+1} (\delta n_l^{s,a})^2$$

- Surface tension vanishes at:

$$F_l^{s,a} < -(2l+1)$$

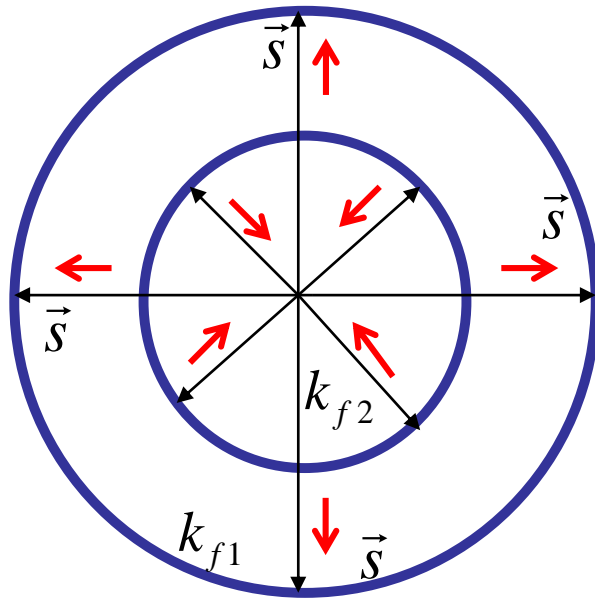


- Ferromagnetism: the  $F_0^a$  channel.

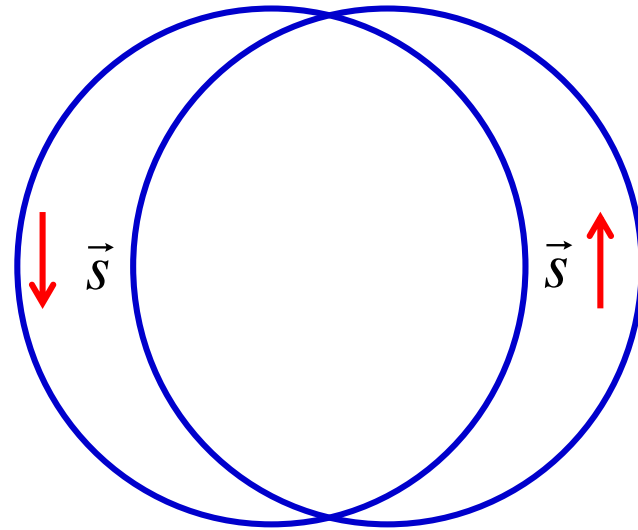
- Nematic electron liquid: the  $F_2^s$  channel.

# Unconventional magnetism: Pomeranchuk instability in the spin channel

$F_1^a$



$\beta$  - phase

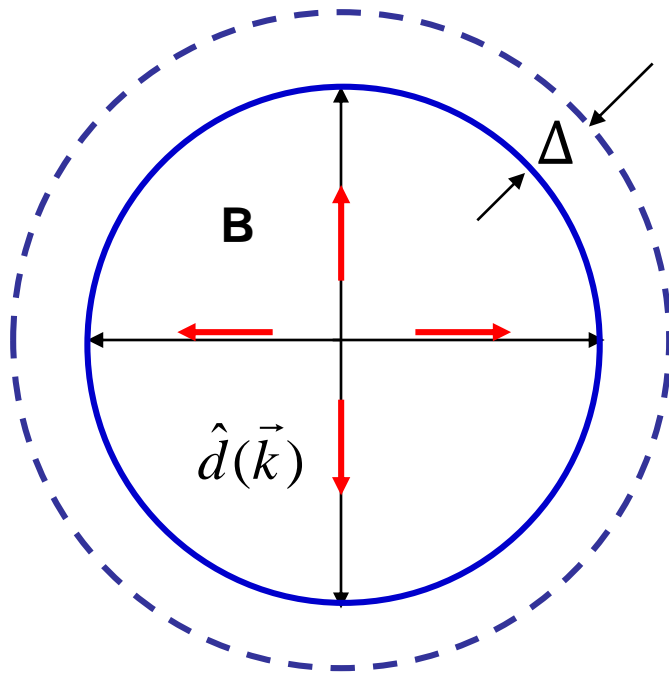


$\alpha$  - phase

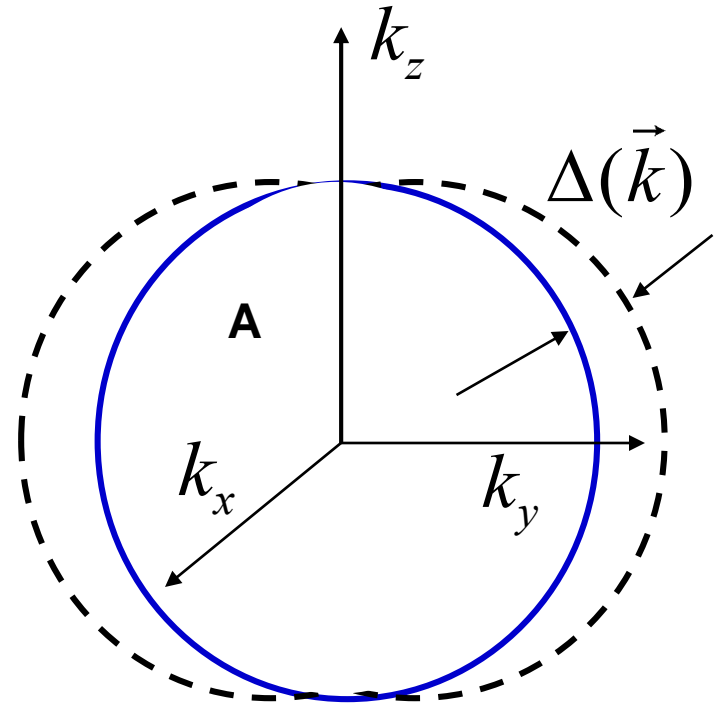
- An analogy to superfluid  $^3\text{He-B}$  (isotropic) and A (anisotropic) phases.

## cf. Superfluid $^3\text{He-B}$ , A phases

- $p$ -wave triplet Cooper pairing.



$$\vec{\Delta}(\vec{k}) = \Delta \hat{d}(\vec{k}) = \Delta \hat{k}$$



$$\vec{\Delta}(\vec{k}) = \Delta \hat{d}(\hat{k}_x + i\hat{k}_y)$$

- $^3\text{He-B}$  (isotropic) phase.

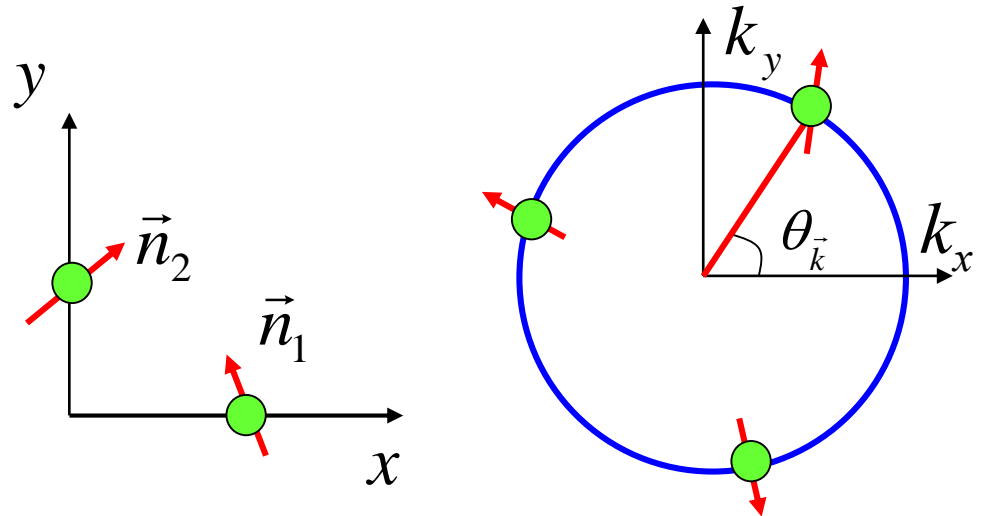
- $^3\text{He-A}$  (anisotropic) phase.

# The order parameters: the 2D $p$ -wave channel

- $F_1^a$  : Spin currents flowing along x and y-directions, or **spin-dipole moments in momentum space.**

$$\vec{n}_1 = \sum_{\vec{k}} \psi_k^+ \vec{\sigma} \psi_k \cos \theta_k$$

$$\vec{n}_2 = \sum_{\vec{k}} \psi_k^+ \vec{\sigma} \psi_k \sin \theta_k$$



- *cf.* Ferromagnetic order (s-wave):  $\vec{s} = \sum_{\vec{k}} \psi_k^+ \vec{\sigma} \psi_k$

- Arbitrary partial wave channels: spin-multipole moments.

$$F_l^a : \cos \theta_k \rightarrow \cos l \theta_k ; \sin \theta_k \rightarrow \sin l \theta_k$$

# Mean field theory and Ginzburg-Landau free energy

- The simplest non-*s*-wave exchange interaction:

$$F_1^a \quad H_{\text{int}} = \sum_q f_1^a(\vec{q}) \{ \vec{n}_1(\vec{q}) \cdot \vec{n}_1(\vec{q}) + \vec{n}_2(\vec{q}) \cdot \vec{n}_2(\vec{q}) \}$$

$$H_{MF} = \sum_k \psi^\dagger(k) [ \varepsilon(k) - \mu - (\vec{n}_1 \cos \theta_k + \vec{n}_2 \sin \theta_k) \cdot \vec{\sigma} ] \psi(k)$$

- Symmetry constraints: rotation (spin, orbital), parity, time-reversal.

$$F(\vec{n}_1, \vec{n}_2) - F(0) = r(|\vec{n}_1|^2 + |\vec{n}_2|^2) + v_1(|\vec{n}_1|^2 + |\vec{n}_2|^2)^2 + v_2 |\vec{n}_1 \times \vec{n}_2|^2$$

$$r = \frac{N_0}{2} \frac{1 + F_1^a / 2}{|F_1^a|}$$

$$F_1^a < -2$$

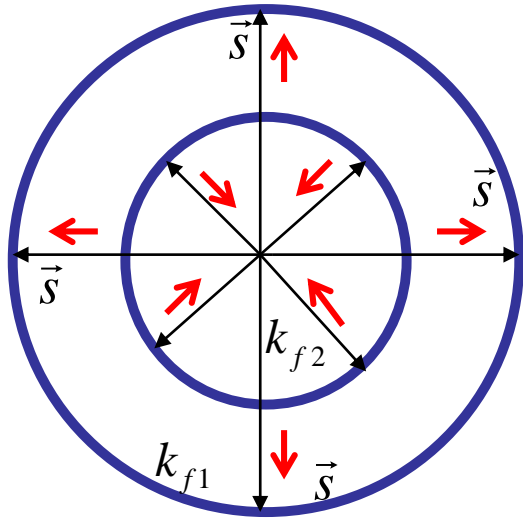


instability!



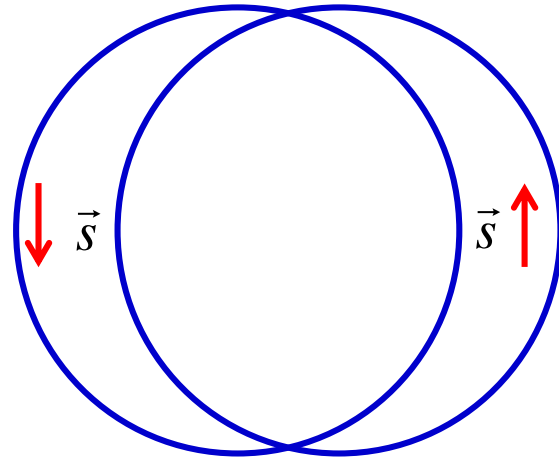
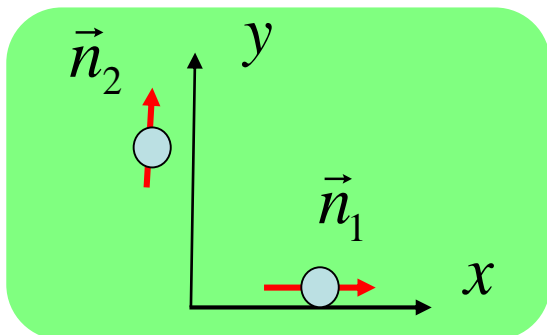
# $\beta$ and $\alpha$ -phases ( $p$ -wave)

$F_1^a$



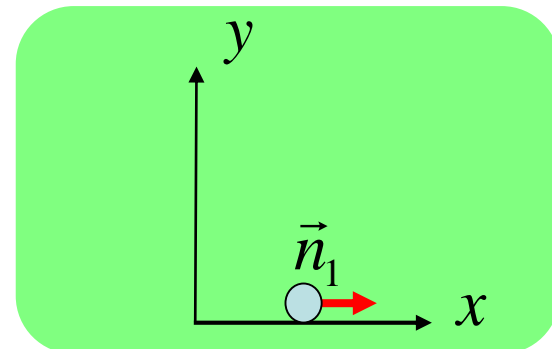
$v_2 < 0$ :  $\beta$ -phase

$\vec{n}_1 \perp \vec{n}_2$  and  $|\vec{n}_1| = |\vec{n}_2|$

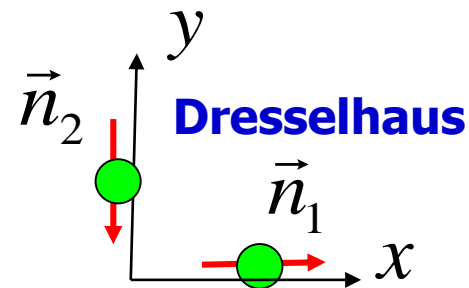
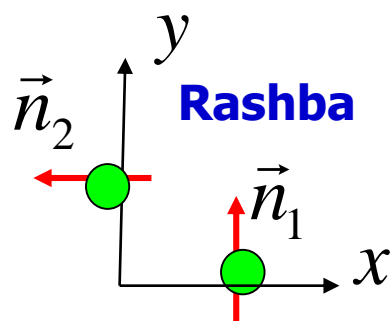
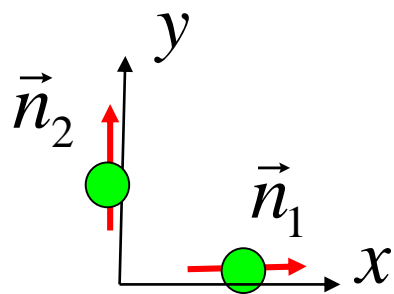
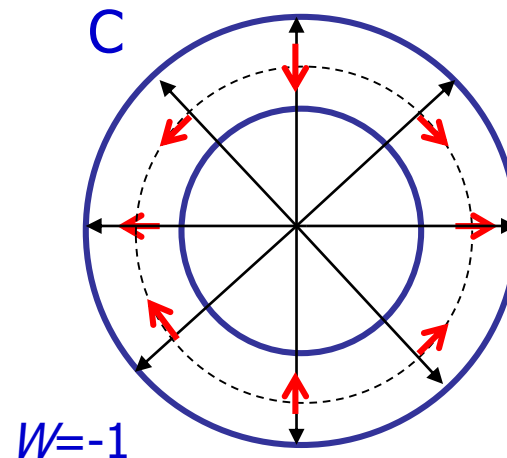
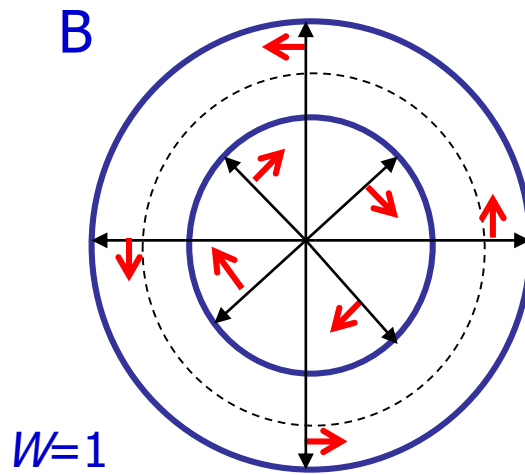
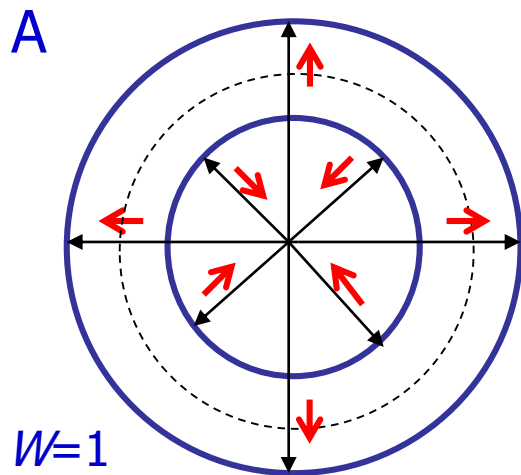


$v_2 > 0$ :  $\alpha$ -phase

$\vec{n}_1 \parallel \vec{n}_2$ ;  $|\vec{n}_2| \neq |\vec{n}_1|$  arbitrary



# The $\beta$ -phases: vortices in momentum space



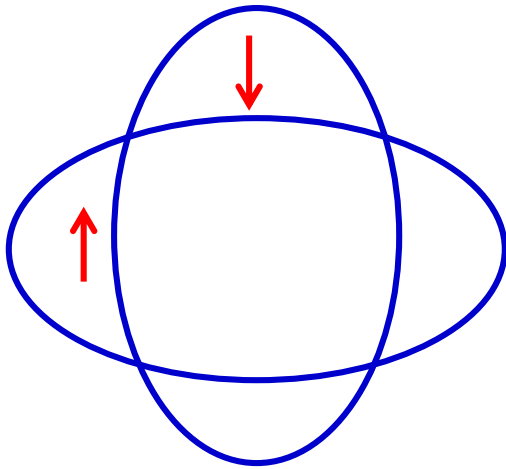
- Perform global spin rotations,  $A \rightarrow B \rightarrow C$ .

L. Fu's(PRL2015): gyro

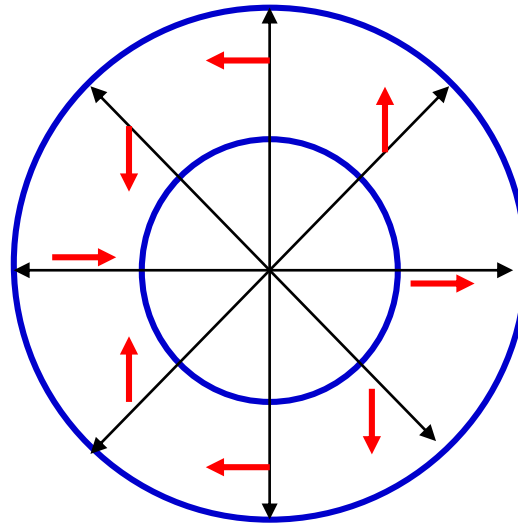
ferroelectric

muti-polar

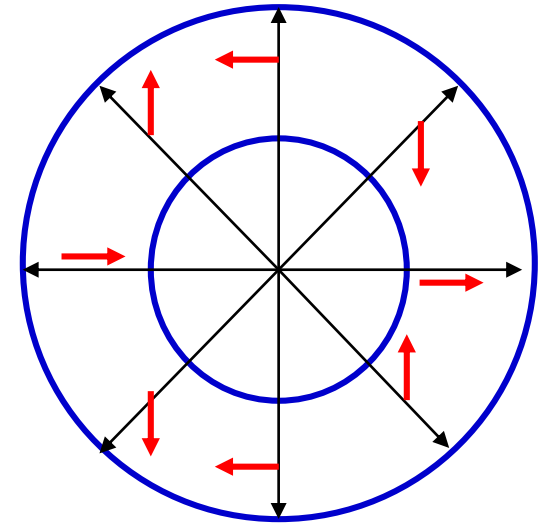
## 2D $d$ -wave $\alpha$ and $\beta$ -phases



$\alpha$ -phase



$\beta$ -phase:  $w=2$



$\beta$ -phase:  $w=-2$

# The $\alpha$ -phases: orbital & spin channel Goldstone (GS) modes

- Orbital channel GS mode: FS oscillations (intra-band transition).

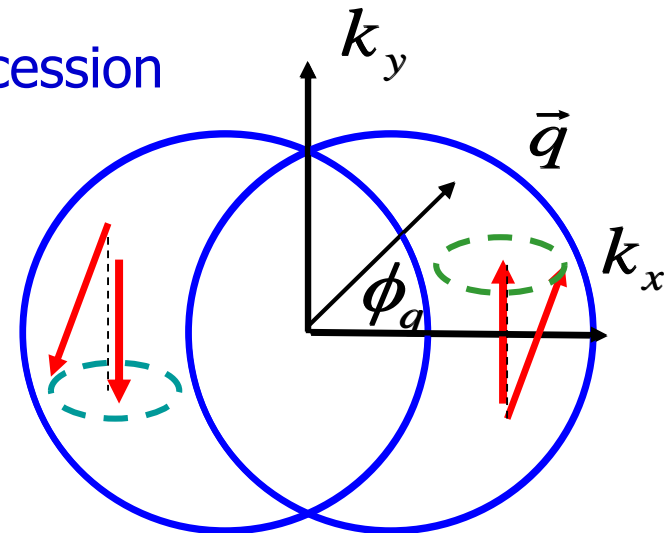
$$L_{FS}^{\alpha}(\vec{q}, \omega) = N_0 \left\{ \frac{(q\xi)^2}{|F_l^a|} - i \frac{\omega}{2v_f q} (1 + \cos 2\phi_q) \right\}$$

Anisotropic overdamping: The mode is maximally overdamped for  $q$  along the  $x$ -axis, and underdamped along the  $y$ -axis ( $l=1$ ).

- Spin channel GS mode: "spin dipole" precession (spin flip transition).

$$\omega_{x\pm iy}^2 = \frac{\bar{n}^2}{|F_l^a|} (q\xi)^2$$

Nearly isotropic, underdamped and linear dispersions at small  $q$ .

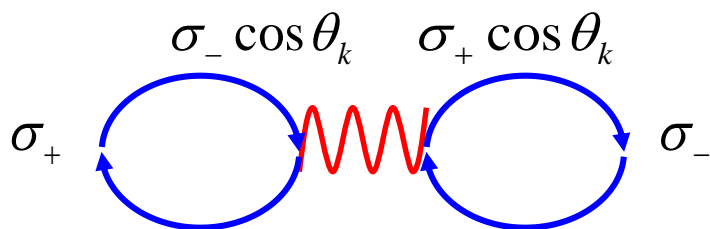


# The $\alpha$ -phases: neutron spectra

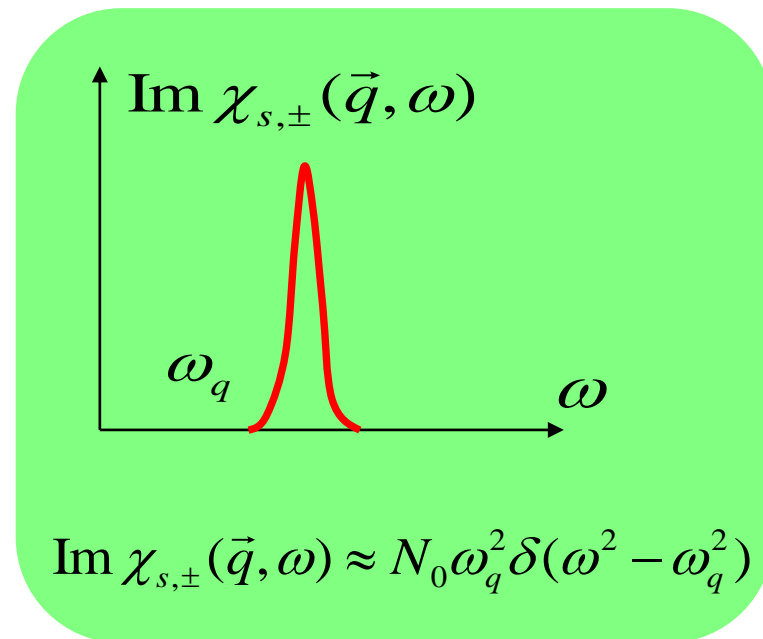
- No *elastic* Bragg peaks.
- $\vec{n}_{1,2}$  can couple with spin dynamically **at**  $T < T_c$  -- coupling between GS modes and spin-waves (spin-flip channel).

$$L = (\vec{n}_1 \times \partial_t \vec{n}_1 + \vec{n}_2 \times \partial_t \vec{n}_2) \cdot \vec{S}$$

$$\rightarrow \bar{n} (S_y \partial_t n_{1x} - S_x \partial_t n_{1y})$$



- *In-elastic*: **resonance peaks** develop **at**  $T < T_c$ .



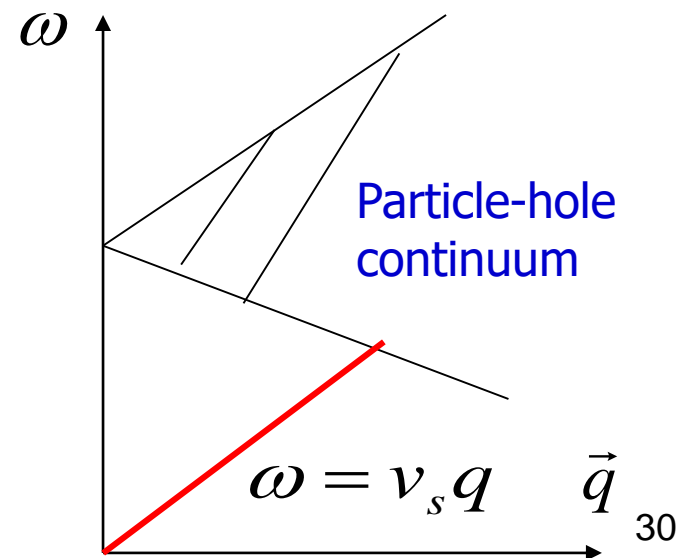
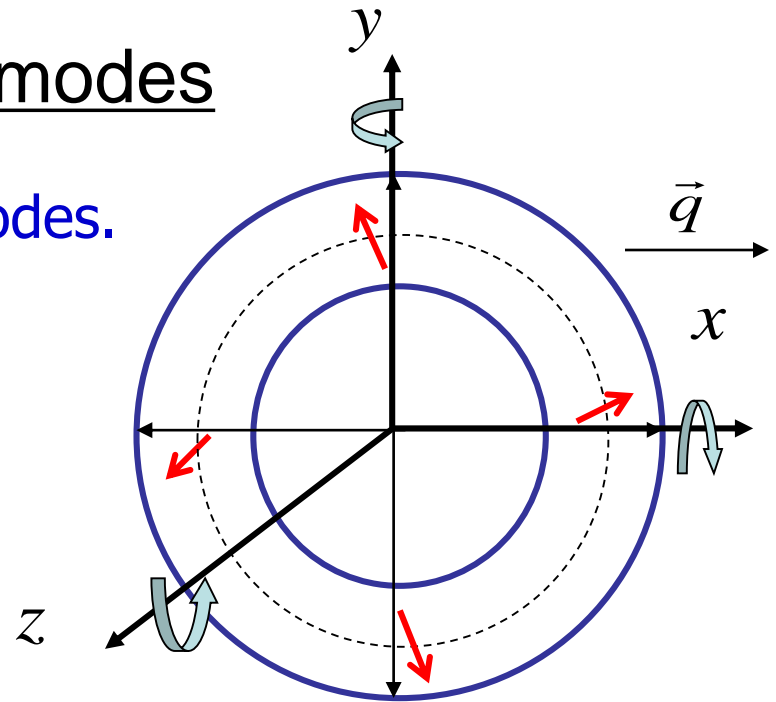
# The $\beta$ -phases: GS modes

- 3 branches of relative spin-orbit modes.

$$O_z = \frac{1}{\sqrt{2}}(n_2^x - n_1^y);$$

$$O_x = -n_2^z; \quad O_y = n_1^z;$$

- For  $l \geq 2$ , these modes are with linear dispersion relations, and underdamped at small  $q$ .
- Inelastic neutron spectra: GS modes also couple to spin-waves, and induce resonance peaks in both spin-flip and non-flip channels.

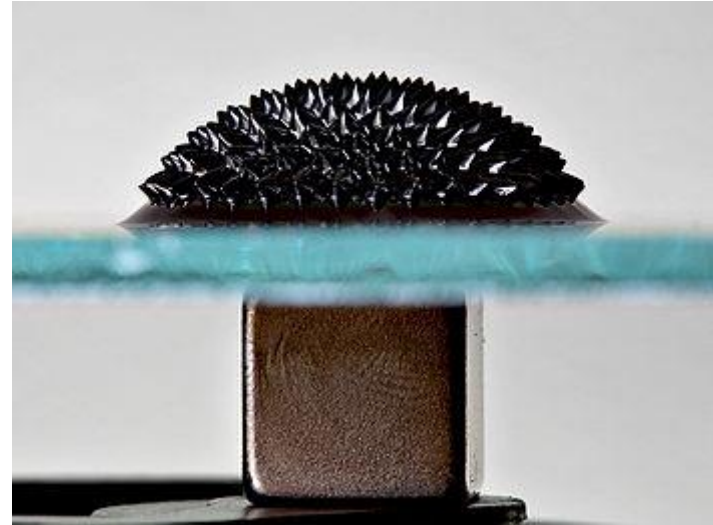


# Outline

- Introduction.
- Mechanism for unconventional magnetic phase transitions.
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
  - Collective modes and neutron spectroscopy.
- **Spin-orbit coupled Fermi liquid theory – magnetic dipolar interaction.**
- Possible directions of experimental realization and detection methods.

# Magnetic dipoles: from classic to quantum

- Ferro-fluid: iron powders in oil.



- In solids, magnetic dipolar interaction  $\ll$  Coulomb interaction.

$$r_s = \frac{d}{a_B} \quad E_m = \frac{\mu_B^2}{d^3} = \frac{\lambda_{cmp}^2 Ry}{a_B^2 r_s^3} = \frac{\alpha^2}{r_s^2} E_{el} \approx \frac{1.4}{r_s^3} meV$$



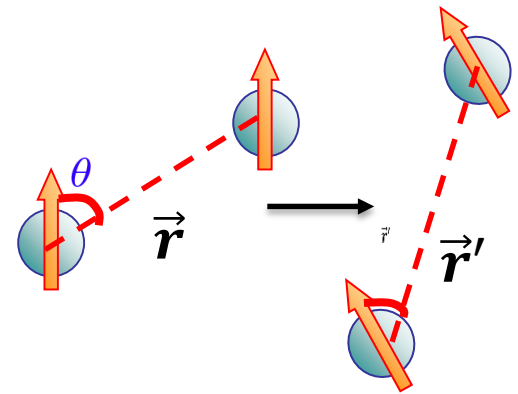
# Magnetic dipolar Fermi gases

- Itinerant magnetic dipolar system: ( $^{161}\text{Dy}$ ,  $^{163}\text{Dy}$ )  $10\mu_B$

$$n \approx 4 \times 10^{13} \text{ cm}^{-3}, T_F \approx 300 \text{ nK} \quad \lambda = \frac{E_d}{E_f} \approx 15\%$$

- SO coupling at the interaction level.

$$V(\vec{r}) = \frac{(g\mu)^2}{r^3} [\vec{F}_1 \cdot \vec{F}_2 - 3(\vec{F}_1 \cdot \hat{r})(\vec{F}_2 \cdot \hat{r})]$$



- SO coupled many-body physics (no Fermi surface splitting):

Weyl p-wave triplet pairing ( $L=S=J=1$ ) Y. Li, C. Wu, Sci. Rep., 2,392 (2012).

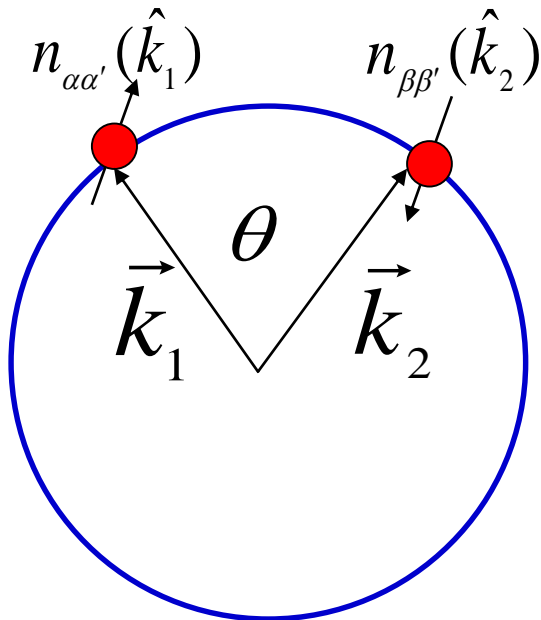
SO coupled Fermi liquid:

Y. Li, C. Wu, PRB 85, 205126 (2012).

# Spin-orbit (SO) coupled Fermi liquid theory

- Landau functions: SO harmonic partial-wave decomposition.

$$\frac{N_0}{4\pi} f_{\alpha\alpha',\beta\beta'}(\hat{k}_1, \hat{k}_2) = \sum_{JJ_zLL'} Y_{JJ_z;LS}(\hat{k}_1, \alpha\alpha') F_{JJ_zLS;JJ_zL'S} Y_{JJ_z;L'S}^+(\hat{k}_2, \beta\beta')$$



- Landau matrices: an eigenvalue  $< -1$   
 $\rightarrow$  Pomeranchuk instability
- $J = 1^-$  (odd parity),  $L = S = 1$ .
- Transfer SO coupling to the single particle level (Rashba like).

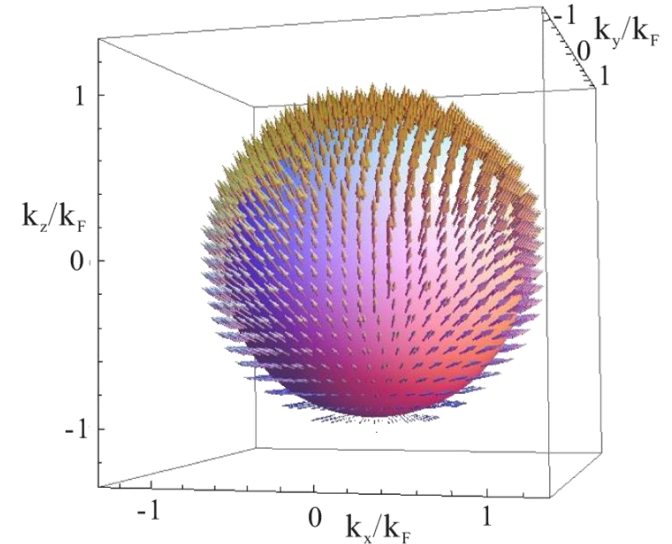
# Topological SO zero sound

- SO coupled Fermi surface oscillations.

$u_0$ : hedgehog distribution:  $J = 0^-$ ;

$u_1$ : longitudinal ferro:  $J = 1^+$ , and  $u_0 > u_1$

$$\vec{S}(\vec{r}, \vec{k}, t) = (u_0 \hat{k} + u_1 \hat{q}) e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$



- Underdamped mode

$s = \omega/(v_f q) > 1$  : sound velocity  $>$  Fermi velocity.

$$s_{\lambda \ll 1} \approx 1 + 2e^{-2(1+1/2F_+)} = 1 + 2e^{-2-12/7\pi\lambda}, \quad s_{\lambda \gg 1} \approx \frac{F_{\times}}{3} = \frac{\pi}{3\sqrt{3}}\lambda.$$

$$F_+ = F_{10;01} + F_{00;11} \quad F_{\times} = \sqrt{F_{10;01} F_{00;11}}$$

# Outline

- Introduction.
- Mechanism for unconventional magnetic phase transitions.
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
  - Collective modes and neutron spectroscopy.
- Spin-orbit coupled Fermi liquid theory – magnetic dipolar interaction.
- **Possible directions of experimental realization and detection methods.**

# A natural generalization of ferromagnetism

- The driving force is still exchange interactions, but in **non-s-wave** channels.

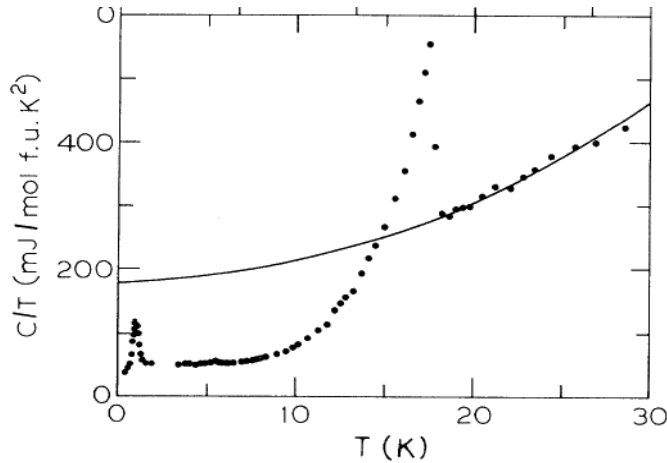
	<i>s</i> -wave	<i>p</i> -wave	<i>d</i> -wave
SC/SF	Hg, 1911	<sup>3</sup> He, 1972	high $T_c$ , 1986
magnetism	Fe, ancient time	?	?

- Optimistically, unconventional magnets are probably not rare.

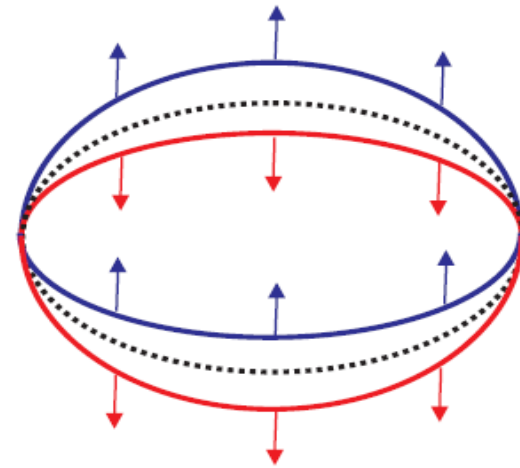
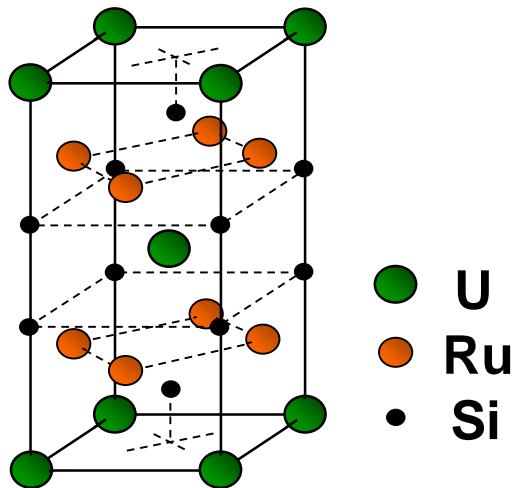
cf. Antiferromagnetic materials are actually very common in transition metal oxides. But they were not well-studied until neutron-scattering spectroscopy was available.

# Search for unconventional magnetism (I)

- $\text{URu}_2\text{Si}_2$ : hidden order behavior below 17.5 K.



T. T. M. Palstra et al., PRL 55, 2727 (1985);  
M. B. Maple et al, PRL 56, 185 (1986)

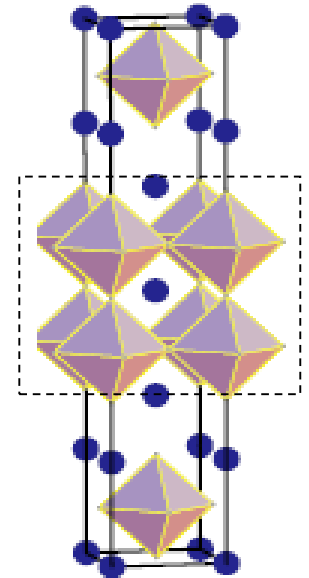
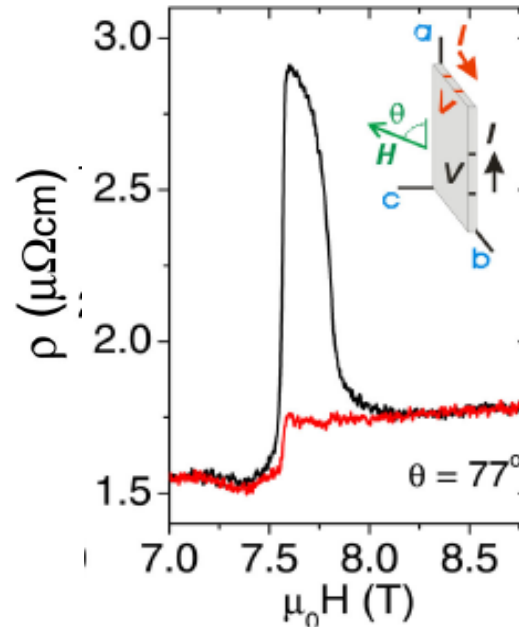
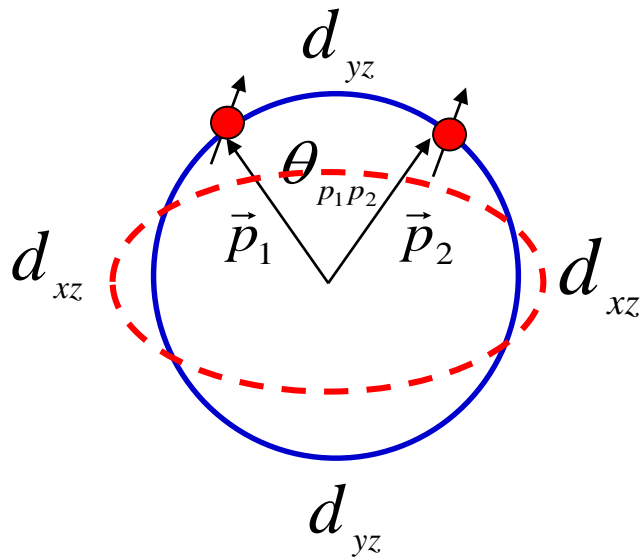


**helicity order (the  $p$ -wave  $\alpha$ -phase);**

Varma et al., Phys. Rev Lett. 96, 036405 (2006)

# Search for unconventional magnetism (II)

- $\text{Sr}_3\text{Ru}_2\text{O}_7$  in the external B field – Orbital-assisted unconventional meta-magnetic state.



$$f_{\uparrow\uparrow}(\vec{p}_1, \vec{p}_2) = V(q=0) - \frac{1}{2} [1 + \cos 2\theta_{p_1 p_2}] V(p_1 - p_2)$$

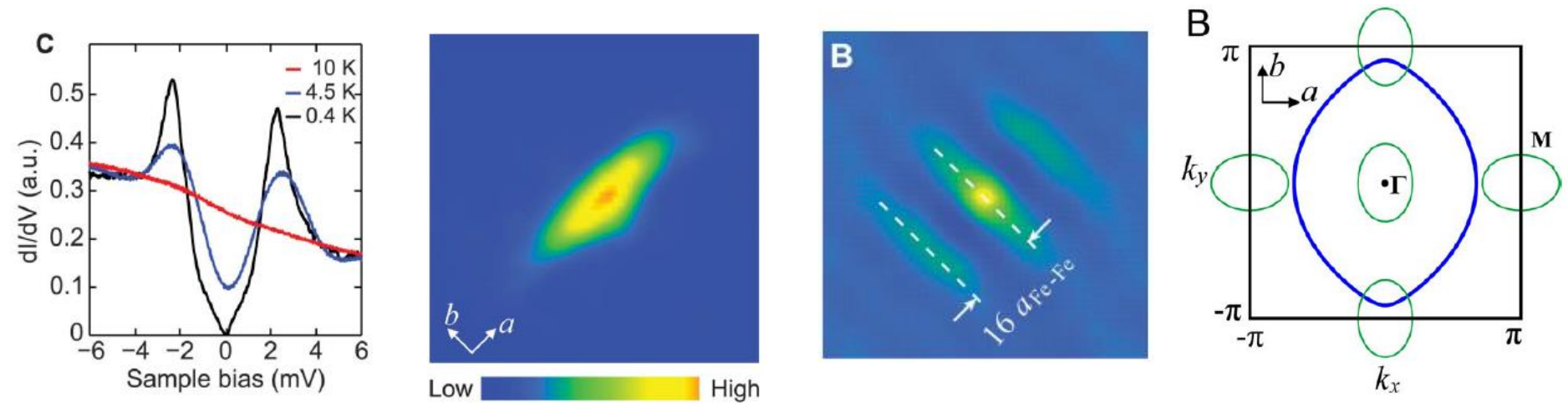
$$f_{\uparrow\downarrow}(\vec{p}_1, \vec{p}_2) = V(q=0)$$

W. C. Lee, C. Wu, PRB 80, 104438 (2009)

# Direct Observation of Nodes and Twofold Symmetry in FeSe Superconductor

Science 332, 1410 (2011)

Can-Li Song,<sup>1,2</sup> Yi-Lin Wang,<sup>2</sup> Peng Cheng,<sup>1</sup> Ye-Ping Jiang,<sup>1,2</sup> Wei Li,<sup>1</sup> Tong Zhang,<sup>1,2</sup> Zhi Li,<sup>2</sup> Ke He,<sup>2</sup> Lili Wang,<sup>2</sup> Jin-Feng Jia,<sup>1</sup> Hsiang-Hsuan Hung,<sup>3</sup> Congjun Wu,<sup>3</sup> Xucun Ma,<sup>2\*</sup> Xi Chen,<sup>1\*</sup> Qi-Kun Xue<sup>1,2</sup>



- Consistent with orbital ordering between  $dxz/dyz$  orbitals.

H. H. Hung, C. L. Song, Xi Chen, Xucun Ma, Q. K. Xue, C. Wu, Phys. Rev. B 85, 104510 (2012).

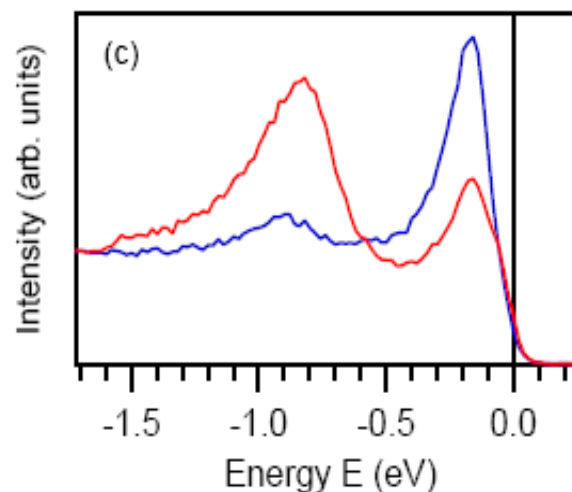


# Detection (I): ARPES

- Angular Resolved Photo Emission Spectroscopy (ARPES).

ARPES in spin-orbit coupling systems ( Bi/Ag surface), Ast et al., cond-mat/0509509.

**band-splitting for two spin configurations.**



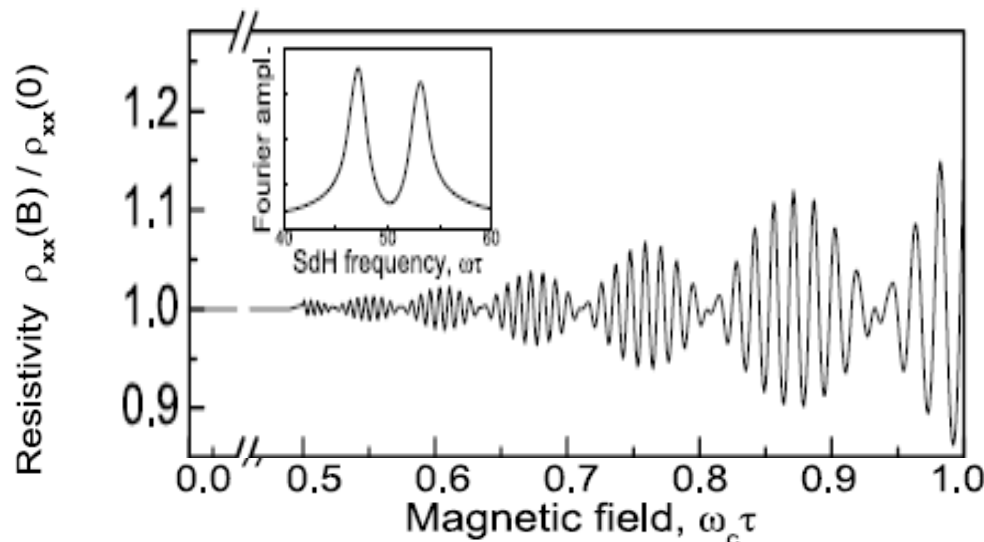
- $\alpha$  and  $\beta$ -phases (dynamically generated spin-orbit coupling):

The band-splitting is proportional to order parameter, thus is temperature and pressure dependent.

## Detection (II): neutron scattering and transport

- Elastic neutron scattering: no Bragg peaks;  
Inelastic neutron scattering: resonance peaks below  $T_c$ .
- Transport properties.

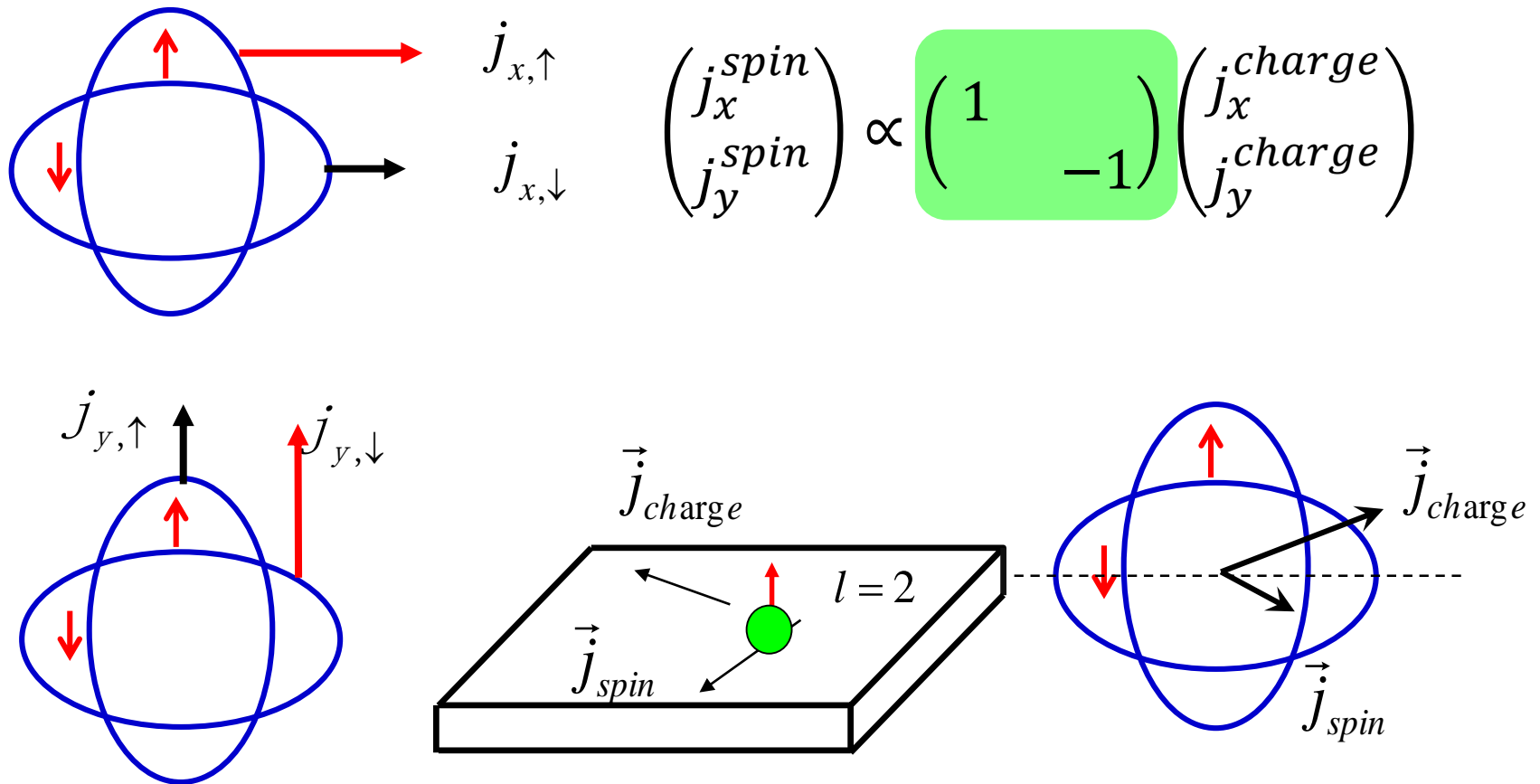
$\beta$ -phases: Temperature dependent beat pattern in the Shubnikov - de Hass magneto-oscillations of  $\rho(B)$ .



N. S. Averkiev et al.,  
Solid State Comm. 133,  
543 (2004).

# Detection (III): transport properties

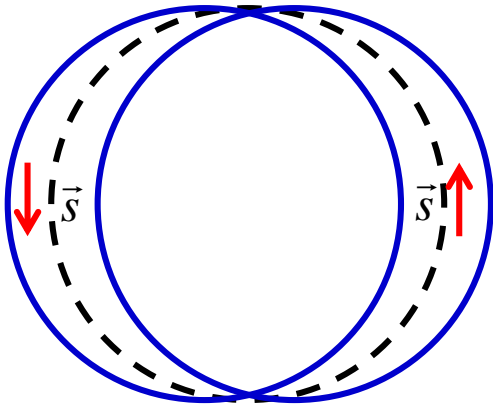
- Spin current induced from charge current (d-wave).  
Their directions are symmetric about the x-axis.



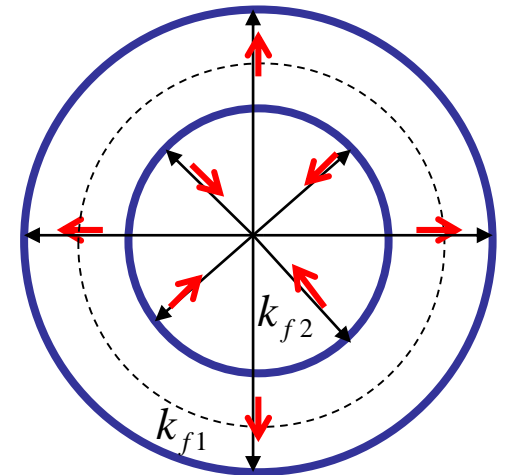
# Summary

unconventional  
superconductivity

$\alpha$ -phase



$\beta$ -phase



unconventional  
magnetism

electron liquid crystal  
with spin



spin-orbit  
coupling