Problem 1: Angular momentum conservation

An air puck of mass m moves on the surface of a horizontal table, guided by a string attached to the puck and passing downward through a small hole in the table top. Initially the length of string above the table is r_1, and the puck is set moving at speed v_1 in a circular path of this radius. The string is then pulled downward through the hole until an amount r_2 remains above the table. Find:

1. The final speed v_2 of the puck.
2. The work W required to pull the string through the hole from r_1 to r_2.
3. The magnitude of the force F needed to hold the radius at a constant r.

Problem 2 – Parallel axis theorem and more

If the rotation axis does not pass the center of mass, denote the distance between the mass center and the rotation axis as r_c. Prove that

$$I_z = I_{cz} + Mr_c^2, \tag{1}$$

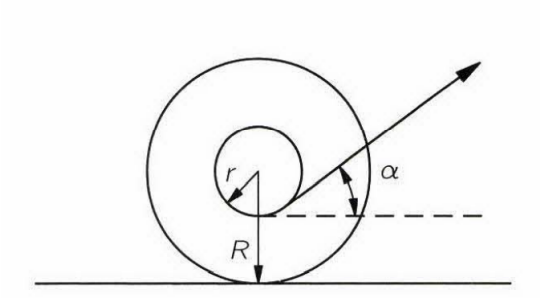
where I_{cz} is the moment of inertia if the rotation axis is parallelly shifted to pass the center of mass.

2) A rigid body is doing a composed motion of translation and rotation. In its center of mass frame, it is rotating around the z-axis with the angular velocity ω, and its center of mass is moving at the velocity v_c. Prove that its kinetic energy can be expressed as

$$E_k = \tfrac{1}{2} I_{cz}\omega^2 + \tfrac{1}{2} M v_c^2. \tag{2}$$

Problem 3 A toy of yo-yo

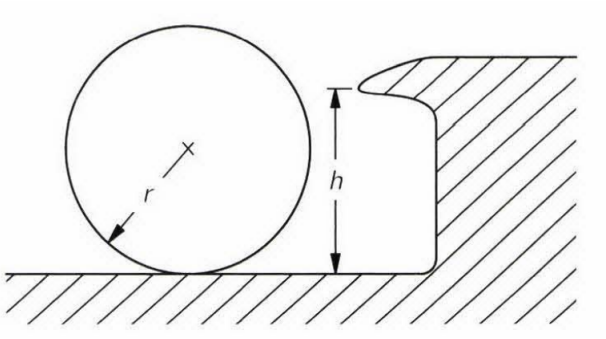
A yo-yo of mass M, outer radius R, and moment of inertia I, on a horizontal table is free to roll without slipping. A force F is applied at the inner radius r at an angle α with the horizontal, as shown in figure.



1. Find the acceleration a of the yo-yo, if the yo-yo does not rise from the table top.
2. How strong a force F at the angle α is needed in order to lift the yo-yo off the table?

Problem 4 Bounce back

Find the ratio h/r of the height of the cushion of a billiard table to the radius of the balls, as shown in figure, such that a ball that approaches the cushion with a pure rolling motion will rebound with a pure rolling motion even if the coefficient of friction between the ball and the table is negligible. Assume that the force exerted on the ball by the cushion during the impact is in the horizontal direction.



Problem 5 Leaning rod falling down

This is quite difficult problem. Try your best.

A rod of length 2L leans against a wall. It starts to slip downward without friction.

1) Check the center of mass of the rod. What does the trajectory look like?

2) Show that the top of the rod will finally lose contact with the wall. When does it happen?

Problem 6 Larmor precession

In the class, you have learnt the Eular equation can help you solve the rigid body motion problem. Here we consider a disk, shown in the figure, of mass m and radius a. And we wish to constrain the disk to rotate about a fixed axis at an angle θ from the normal axis with a constant angular velocity ω. Please figure out the torque that we need to put on that disk.

We choose principal axes as indicated, with the z axis normal and the x axis in the plane determined by ω and $\hat{\mathbf{z}}$.