Problem 1: Euler equations

A rigid body is rotating freely, subject to zero torque.
1) Use the Euler equation to prove that the magnitude of the angular momentum **L** is a constant.

2) Prove that the kinetic energy of rotation $T = \frac{1}{2}(\lambda_1\omega_1^2 + \lambda_2\omega_2^2 + \lambda_3\omega_3^2)$ is a constant.

Problem 2: Space station

An axially symmetric space station (principle axis along $\mathbf{e}_3$ and $\lambda_1 = \lambda_2$) is floating in free space. It has rockets mounted symmetrically on either side that are firing and exert a constant torque Γ about the symmetry axis. Solve Euler's equations for $\vec{\omega}$ (relative to the body axis) and describe the motion. At $t = 0$, take $\vec{\omega} = (\omega_{10}, 0, \omega_{30})$.

Problem 3: A discrete wave

In class, we directly derive the wave equation based on the continuum. Now we construct a wave from the discrete side by examining the normal mode of a set of blocks connected by springs. This can be viewed as an elementary model of a particular type of lattice vibrations of solids, i.e., the longitudinal phonon.

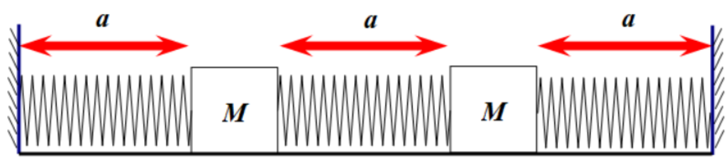

Figure 1: A two-block vibration problem.

Consider two blocks of mass M slide on a frictionless surface, which are connected to rigid walls and coupled to each other by springs as shown in Fig. 1. The blocks are small such that their widths are neglected. All springs are identical with zero mass, spring constant K and relaxed length a_0. At equilibrium position, each spring is stretched to length a (Not necessarily equal to a_0).

We use $u(j,t)$ $(j = 1, 2)$ to denote the displacement of the j-th block relative to its equilibrium position.

1) Show that there are two different types of solutions. One with $u(1,t) = u(2,t)$ and the other $u(1,t) = -u(2,t)$. Each of them exhibits a single frequency, which is called a normal mode. Find the frequencies of these two normal modes, respectively.

2) Please show that these normal modes can be labeled by the wave number $k_n = n\pi/L$ with $n = 1, 2$ as

$$u_{k_n}(j,t) = A\sin(k_n ja)e^{i(\omega_{k_n}t+\phi)}, \tag{1}$$

where $L = 3a$. Can you find a single expression of ω_{k_n} for both modes?

3) Now consider three blocks connected with one another by springs, and other settings are the same as before. Write down the equations of motion for the three blocks. Again solutions to these equations can be written in

the form of Eq. 1 with $L = 4a$, $j = 1, 2, 3$, and $n = 1, 2, 3$. Figure out the frequencies ω_{k_n}.

4) Generalize your results to the case of N-blocks. Write down the equation of motion for every block. How about the general solution in this case?