#1: Angular momentum of a charge-monopole system

We learned that angular momentum conservation is a consequence of spatial isotropy. Below we examine a subtle case, an isotropic system but not a central-force one.

Consider a magnetic monopole fixed at the origin with a magnetic charge g. Similar to an electric charge, the magnetic field generated is $\vec{B} = g\vec{r}/r^3$. Below we study the motion of a particle with an electric charge q and mass m in this monopole magnetic field. You may use either the Gaussian unit or the SI unit for the Lorentz force.

1) Prove that the kinetic energy of this particle is conserved, or, equivalently, its speed v of motion is a constant.

2) The mechanical angular momentum is defined as usual $\vec{L}' = m\vec{r} \times \vec{v}$. Please show that it actually is NOT conserved, i.e., $\frac{d}{dt}\vec{L}' \neq 0$.

3) Since angular momentum conservation is a requirement of spatial isotropy, we should check why the above naive definition does not work. The answer is that there is an extra contribution $\vec{L}_{EM}$ not taken into account yet, which arises from the angular momentum of the E&M field. Then the total angular momentum is defined as $\vec{L} = \vec{L}' + \vec{L}_{EM}$.

Based on symmetry analysis, please prove that $\vec{L}_{EM}$ is along the radial direction (No actual calculation is needed for this statement.). Then we express $\vec{L}_{EM} = k\hat{r}$, where k is a constant to be determined and $\hat{r}$ is the unit vector along the radial direction.

We need to carefully choose the expression of k such that $\frac{d}{dt}\vec{L} = 0$. In order for this purpose, what is k?

4) In quantum mechanics, a non-zero angular momentum has a minimal quantized value. Apply this quantization to the charge-monopole system, what will you obtain from it?

#2: Why do we only see one side of the Moon?

We have heard of the Moon only shows one side towards the earth, which is due to the tidal locking. In this problem, let us examine why this could happen.

The Moon is moving around the earth with an orbit radius R and also spinning around its axis perpendicular to its orbiting plane. The revolution angular velocity is ω_r and its spin angular velocity is ω_s. Assume the Moon mass is M_m and its density is uniform. The Moon radius is r.

1) What are the orbital angular momentum L and the spin angular momentum S of the Moon, respectively? The moment of inertia of a uniform sphere solid is $\frac{2}{5}Mr^2$, where M is the mass and r is its radius.

2) Assume that the total angular momentum of Moon is conserved, under what condition its total kinetic energy is smallest? We assume the orbit radius is fixed here.