Problem 1:

Some one-dimensional kinematics problems can always be reduced to doing an integral. Here is a class of problem: Show that if the net force on a one-dimensional particle depends only on position, $F = F(x)$, then Newton's second law can be solved to find v as a function of x given by

$$v^2 = v_0^2 + \tfrac{2}{m}\int_{x_0}^{x} F(x')dx'$$

[Hint: Use the chain rule to prove the following handy relation, which we call the "vdv/dx rule": If you regard v as a function of x, then

$$\dot{v} = v\tfrac{dv}{dx} = \tfrac{1}{2}\tfrac{d(v^2)}{dx}$$

Use this to rewrite Newton's second law in the separated form $md(v^2) = 2F(x)dx$ and then integrate from x_0 to x.] Comment on your result for the case that $F(x)$ is actually a constant. (You may recognize your solution as a statement about kinetic energy and work, both of which we shall be discussing in later courses.)

Problem 2:

A charged particle of mass m and positive charge q moves in uniform electric and magnetic fields, $\mathbf{E}$ pointing in the y direction and $\mathbf{B}$ in the z direction (an arrangement called 'crossed E and B fields'). Suppose the particle is initially at the origin and is given a kick at time $t = 0$ along the x axis with $v_x = v_{x0}$ (positive or negative).

1. Write down the equation of motion for the particle and resolve it into its three components. Show that the motion remains in $z = 0$ plane.
2. Prove that there is a unique value of v_{x0} called the drift speed v_{dr}, for which the particle moves undeflected through the field. (This is the basis of velocity selectors, which select particles traveling at one chosen speed from a beam with different speeds.)
3. Solve the equations of motion to give the particle's velocity as a function of t, for arbitrary values of v_{x0}. [Hint: Try to relate the equations with harmonic oscillator.]
4. Integrate the velocity to find the position as a function of t and sketch the trajectory for various values of v_{x0}.

Problem 3:

A gun can fire shells in any direction with the same speed v_0(ignoring air resistance). Let's take cylindrical polar coordinate with the gun at the origin and z measured vertically up. Show that the gun can hit any object inside the surface

$$z = \frac{v_0^2}{2g} - \frac{g}{2v_0^2}\rho^2$$

Problem 4: Fall with resistance

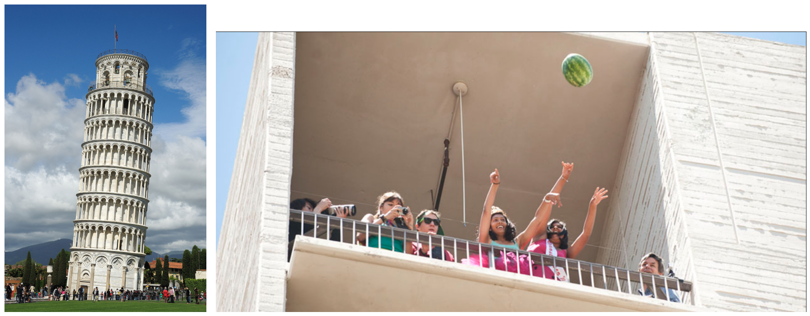

Figure 1: a) The leaning tower of Pisa. Legend says that Galileo did the free-fall experiments to falsify Aristotle's theory. b)500 years later, undergraduate students at University of California, San Diego developed a tradition to commemorate Galileo. The watermelon-drop tradition from the Urey Hall.

1. Legend has it that Galileo did the free-fall experiments on the leaning tower of Pisa. He dropped two iron balls with different weights and they arrived on the ground simultaneously. However, due to the air resistances, rigorously speaking, they should fall differently.

   Suppose that the radius of the bigger iron ball is 2cm, and that of the smaller one is 1cm. The leaning tower of Pisa is 55m. Using the knowledge and parameter values provided in the lecture note, calculate the difference between the falling times of these two iron balls. Would Galileo be able to tell the difference?

2. I spent 14 years working at University of California, San Diego. There is a long tradition of the Watermelon Drop, which is a great even at the last instruction day of each school year to commemorate Galileo's free-fall experiment. Undergraduate students vote to elect a girl as the Miss Watermelon. Miss Watermelon carries a watermelon to the top floor of the Urey Hall, and drops it. Meanwhile, the student band plays joyful music as the last relaxation before the final exam.

   Urey Hall has 8 floors, which is roughly estimated as 30 meters high. The radius of watermelon is about 20cm. Now try to evaluate the speed of watermelon when it hits the ground by taking into account the air resistance.