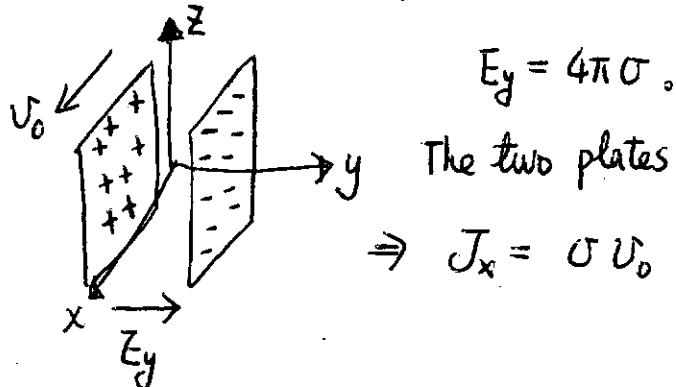


Lect 4 : Transformation of E-M fields

Let us consider two plates in the xz -plane. The surface charge density $\pm \sigma$ in Frame F



$$E_y = 4\pi\sigma.$$

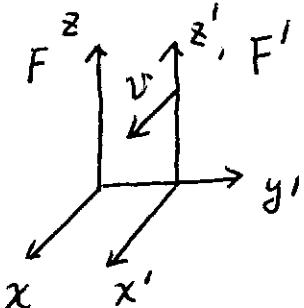
The two plates move along x -direction at the speed v_0

$$\Rightarrow J_x = \sigma v_0, \text{ thus } B_z = \frac{4\pi J_x}{c} = \frac{4\pi\sigma v_0}{c}.$$

we consider another frame F' , which moves on the speed v along x -axis respect with F , what the fields observed in F' ?

In F' , the velocity of the two plates

$$v'_0 = \frac{v_0 - v}{1 - \frac{v_0 v}{c^2}} = c \frac{\beta_0 - \beta}{1 - \beta_0 \beta} \quad \leftarrow \begin{matrix} \beta_0 = \frac{v_0}{c} \\ \beta = \frac{v}{c} \end{matrix}$$



in fram F' , the charge desity $\sigma' = \frac{\sigma}{\gamma_0}$, where $\gamma_0 = \frac{1}{\sqrt{1 - \beta_0^2}}$

$$\gamma_0' = \frac{1}{\sqrt{1 - \frac{(\beta_0 - \beta)^2}{(1 - \beta_0 \beta)^2}}} = \frac{1 - \beta_0 \beta}{\sqrt{(1 - \beta_0^2)(1 - \beta^2)}} \Rightarrow \sigma' = \sigma \frac{1 - \beta_0 \beta}{\sqrt{1 - \beta^2}} = \sigma (1 - \beta_0 \beta)$$

$$\text{thus } J'_x = \sigma' v'_0 = \sigma (1 - \beta_0 \beta) v'_0 = \sigma (1 - \beta_0 \beta) c$$

$$\Rightarrow E'_y = 4\pi\sigma' = 4\pi\sigma \gamma (1 - \beta_0 \beta) = \gamma [4\pi\sigma - \frac{4\pi\sigma v_0}{c} \left(\frac{v}{c} \right)]$$

$$B'_z = \frac{4\pi}{c} J'_x = \gamma \left[\frac{4\pi\sigma v_0}{c} - 4\pi\sigma \left(\frac{v}{c} \right) \right]$$

or $E'_y = \gamma(E_y - \beta B_z)$

$B'_z = \gamma(\dots - \beta E_y + B_z) .$

We can derive the rules for other components: F' is moving at speed of v along the x -direction, respect to F , then.

$$\begin{aligned} E'_x &= E_x, \quad E'_y = \gamma(E_y - \beta B_z), \quad E'_z = \gamma(E_z + \beta B_y) \\ B'_x &= B_x, \quad B'_y = \gamma(B_y + \beta E_z) \quad B'_z = \gamma(B_z - \beta E_y) \end{aligned}$$

first order

$$\vec{E}' = \vec{E} + \frac{\vec{v}}{c} \times \vec{B}$$

$$\vec{B}' = \vec{B} - \frac{\vec{v}}{c} \times \vec{E}$$

Suppose in the Frame F , that $B=0$, \Rightarrow

$$E'_x = E_x, \quad E'_y = \gamma E_y \quad E'_z = \gamma E_z$$

$$B'_x = 0, \quad B'_y = \dots \quad B'_z = \dots - \gamma \beta E_y$$

then

$$\vec{B}' = -\left(\frac{\vec{v}}{c}\right) \times \vec{E}'$$

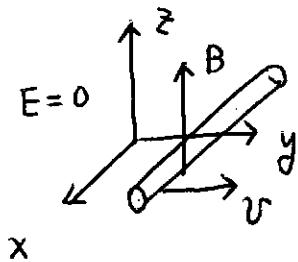
\vec{v} is the velocity of F' respect to F .

Similarly, if in the frame F in which $E=0$, then in the frame F'

we have

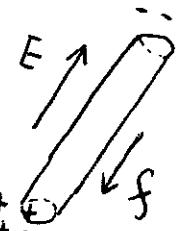
$$\vec{E}' = +\left(\frac{\vec{v}}{c} \times \vec{B}'\right)$$

§ A conducting rod moving in B -field



in the F-frame, $E=0$, $B=B\hat{z}$, the rod is moving along \hat{y} .

$$\text{Lorentz force } \vec{f} = \frac{q}{c} \vec{v} \times \vec{B}$$

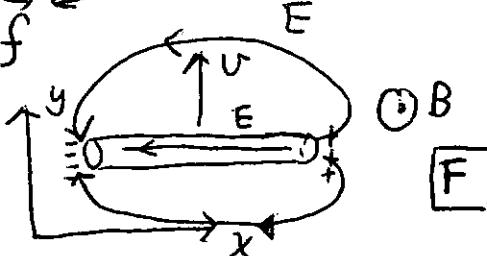


f drives charge accumulate at ends

that there's internal electric field

$$\Rightarrow q\vec{E} = -\vec{f}$$

in frame F



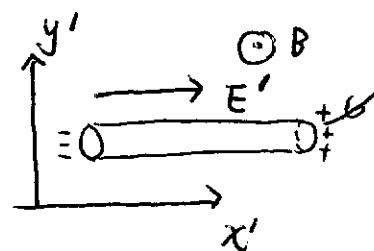
$$\vec{E} = -\frac{1}{c} \vec{v} \times \vec{B}$$

Now let us sit the co-moving frame F' with F. For the moment, we neglect the rod, the we will see in the frame F' , there exist B' and E' .

$$\vec{B}' \approx \vec{B} - \frac{\vec{v} \times \vec{E}}{c} \approx \vec{B} \text{ up to } \beta^2.$$

$$\vec{E}' = \frac{\vec{v}}{c} \times \vec{B}'.$$

in F' , the rod is at rest. \vec{E}' field



induces charge distributions on the rod. There's no electric field inside the rod! Thus no motion of electric charge!

