Problem 1: Electric potential of a charged sphere

Try to find the electric potential function $V(\mathbf{r})$ of a sphere with a uniform charge distribution. The radius of sphere is R and the total charge is Q.

1. Explain in terms of symmetry why the function $V(\mathbf{r})$ is only dependent on $|\mathbf{r}|$.

2. Perform the following integral

$$V(\mathbf{r}) = \frac{Q}{4\pi} \int d\Omega' \frac{1}{|\mathbf{r}-\mathbf{r}'|} \tag{1}$$

   to find $V(\mathbf{r})$ for both $r > R$ and $r < R$. $\mathbf{r}'$ is located on the sphere, and $d\Omega'$ is the solid angle element.

3. Use the Gauss's law to check the above result.

Problem 2: Uniqueness theorem with conductors

The uniqueness theorem states that there is a unique solution of the electric field distribution once the boundary condition are fixed. In class, we have proved the case of the 1st type boundary condition, i.e., the electric potential distributions on boundaries are fixed. Now we consider a different type of boundary condition.

Suppose that the system contains conductors $C_i, i = 1, 2, ..., N$. Each of them is charged with Q_i, respectively. Please prove that the electric field distribution is unique. The outer boundary is either a conductor with total charge Q, or, is just set up to infinite.

(Hint: Although the charge distribution of the conductor is not given, the electric potential is a constant for each conductor.)

Problem 3: Method of images

Suppose there exists a grounded conducting ball with the radius R. Place an electric charge q at a distance of $a > R$ from the center of the sphere.

1. Use the method of images and find the position and the value of the image charge. (Hint: You need to design suitably the image charge such that the electric potential on the sphere satisfies the boundary condition.)
2. Find the electric potential outside the ball.
3. Please calculate the force acting on the conducting ball and compare it with the force on the point charge q.

Problem 4: Relativity

Please go through my paper, and derive the Lorentz transformation law of E and B-fields.