Problems 1, 2, 3 on AC circuit

Work out the following problems in Berkeley Physics Course Volume II, Problems
8.4
8.7
8.16
of Chapter 8.

Problem 4. Angular momentum of an electric charge and a monopole

In a previous lecture, we worked out the angular momentum of a dyon system – an electric charge and a magnetic monopole. Now we will show that the extra contribution of angular momentum actually comes from that of the E&M field.

Consider a monople charge g located at $\mathbf{R}_m$ and an electric charge e located at $\mathbf{R}_e$. Then the magnetic field $\mathbf{B}$ and the electric field $\mathbf{E}$ read

$$\begin{aligned} \mathbf{B} &= g\frac{\mathbf{r}-\mathbf{R}_m}{|\mathbf{r}-\mathbf{R}_m|^3} \\ \mathbf{E} &= q\frac{\mathbf{r}-\mathbf{R}_e}{|\mathbf{r}-\mathbf{R}_e|^3}. \end{aligned} \tag{1}$$

Then the field angular momentum is defined as

$$L_{em} = \int d^3\mathbf{r}\ \mathbf{r}\times\mathbf{p}, \tag{2}$$

where $\mathbf{p} = \frac{1}{4\pi c}\mathbf{E}\times\mathbf{B}$ is the momentum density of the E&M field.

1) Prove that the definition of $\mathbf{L}_{em}$ is independent of the choice of origin.

Hint: you need to prove that $\int d^3\mathbf{r}\ \mathbf{p} = 0$.

Then without of loss of generality, we put q and g along the z-axis with g located at $z = a$, and e located at $z = -a$.

2) Prove that only L_z is nonzero.

3) Calculate the value of $\mathbf{L}_{em} = \frac{qg}{c}\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is the unit vector pointing from q to g.

Hint: you may need to use the integral

$$\int_{-\infty}^{+\infty} dt \int_0^{+\infty} \frac{s^3 ds}{[(s^2+t^2+1)^2-4t^2]^{\frac{3}{2}}} = 1. \tag{3}$$