Problems 1, 2, 3 on dielectric materials

Work out the following problems in Berkeley Physics Course Volume II, Problems
10.15
10.16
10.20
of Chapter 10.

Problem 4. The interaction energy between two electric dipoles

Consider two dipoles with the moments $\mathbf{p_1}$ and $\mathbf{p_2}$. Put $\mathbf{p_1}$ at the origin and $\mathbf{p_2}$ at the position $\mathbf{r}$.

1) Calculate the interaction energy $V(\mathbf{p_1}, \mathbf{p_2}, \mathbf{r})$ between them.

Hint: In class, we derived that the electric potential generated by the dipole $\mathbf{p_1}$ is $U = \frac{\mathbf{p_1} \cdot \mathbf{r}}{r^2}$. View $\mathbf{p_2}$ as a pair of charges $\pm q$ separated at $\mathbf{r} \pm \mathbf{d}/2$ with $p_2 = qd$ taking the limit $d \to 0$ while p_2 is kept constant.

2) Consider the configurations in the figure of Problem 10.17 of Berkeley Vol II. Which ones exhibit attractive interactions and which ones exhibit repulsive interactions?

3) Consider that we apply rotations to $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{r}$. Under what kinds of rotations, the interaction $V(\mathbf{p_1}, \mathbf{p_2}, \mathbf{r})$ is invariant?

Problem 5. Dielectric ball in a uniform electric field

In class, we have derived the electric field generated by a uniformly polarized dielectric sphere. Here we consider a dielectric sphere with the dielectric constant ϵ in an external electric field $\mathbf{E_0}$. Actually the sphere will be uniformly polarized with the polarization density $\mathbf{p}$ (you do not need to prove this).

1) What is the direction of $\mathbf{p}$?

2) Please work out the polarization density $\mathbf{p}$ inside the sphere.

3) Please work out the total electric field $\mathbf{E}(\mathbf{r})$ both inside and outside the sphere.