What does the earth get from the Sun?

General Physics I

Lect.20 Entropy

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2023.11

Thermodynamic temperature

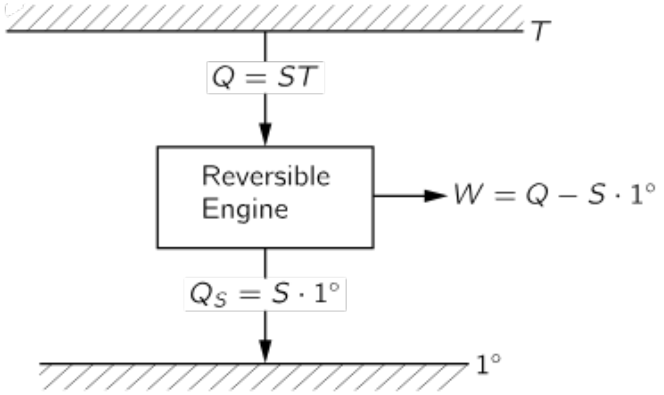
- For a moment, let's forget in L16 we have defined that **kT** is a measure of mean kinetic energy (ideal gas temperature)

- From last lecture we learnt that for an ideal/reversible engine

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_s}{1^\circ}$$

- Since the reversible engine is a universal property, we can define a "new temperature" independent of any particular substance, the obvious case is that f(T)=T/ 1° → (we now call 1° as Kelvin)

$$Q=Q_s f(T)=ST, \text{ where } Q_s=S\cdot 1^\circ$$



"Temperature" = heat absorbed at **that** temperature / heat released at the unit temperature

Entropy in a reversible process

Now again imagine a reversible engine, let’s look at this equation in a new way:

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

- We find that heat Q_1 at temperature T_1 is “equivalent” to Q_2 at T_2; there is no gain or loss of Q/T. This Q/T is called ***entropy*** **（熵）**, and we say “there is no net change in entropy in a reversible cycle.”

- We have found another quantity which is a function of the **condition**, i.e., the entropy of the substance. Mean that it is not dependent of its path . We shall prove it later.

Entropy change in a reversible transformation

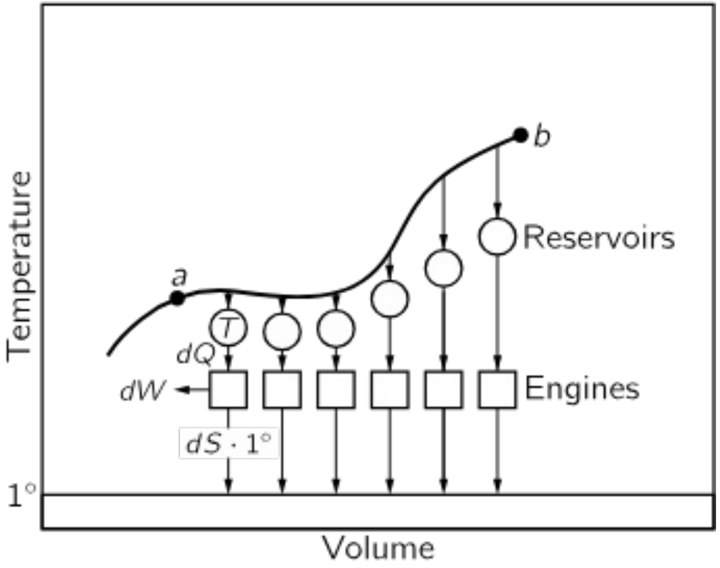
In a reversible transition, from a to b, we take heat dQ from the substance at each little step at the temperature T corresponding to the point on the path.

Connect all these reservoirs, by reversible heat engines, to a single reservoir at the *unit temperature*. At each step, the change of entropy dS delivered at the unit temperature is

$$dS = dQ/T.$$

The entropy difference from a to b, evaluated at unit temperature

$$S_b - S_a = \int_a^b \frac{dQ}{T}$$

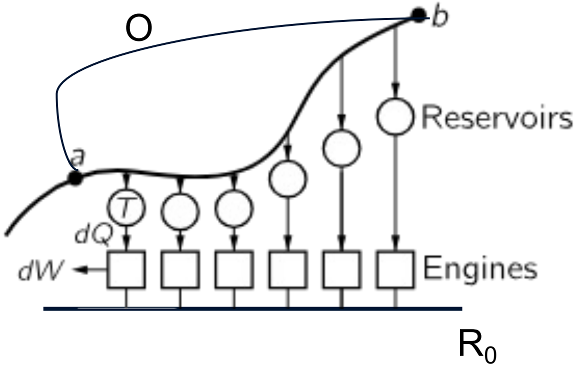


Transition from a to b for a substance.

Q: does the entropy difference depend upon the path taken?

Clausius' theorem

- We bring Carnot engine working between the R_0 and each heat reservoir R_i. Let the Carnot engine input Q_i back to R_i, for each Q_i taken out during the cycle O. At such, each reservoir has zero net heat transfer. Meanwhile Q_{0i} is transferred from the standard reservoir to the Carnot engine. The total amount of heat extracted from the standard reservoir:



$$Q_{tot} = \sum_i Q_i^0 = T_0 \sum_{i=1}^{N} \frac{Q_i}{T_i}.$$

Hence, the net effect is to extract Q_{tot} from the standard reservoir R_0 and convert it to work. This is impossible if $Q_{tot} > 0$, which would violate the 2nd law of thermodynamics. So,

$$\sum_{i=1}^{N} \frac{Q_i}{T_i} \leqslant 0 \xrightarrow{\text{continuous}} \oint \frac{dQ}{T} \leqslant 0.$$

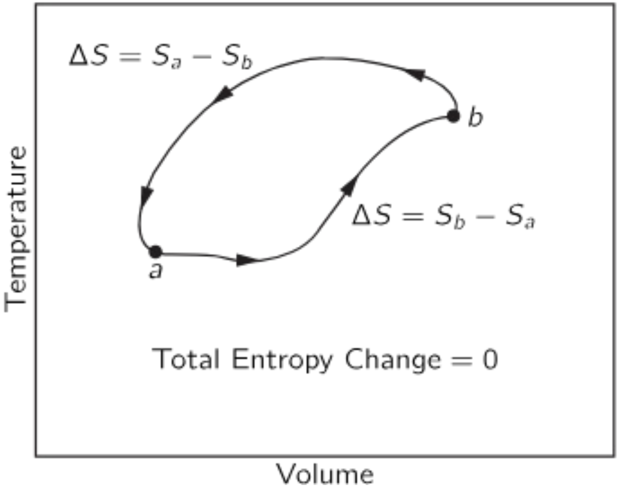
Entropy as a function of state

- If the cycle O is reversible, we can reverse all the steps around the path, s.t.

$$\oint -\frac{dQ}{T} \leqslant 0.$$

- Together with the previous result, we have for a reversible process O

$$\oint \frac{dQ}{T} = 0,$$



- The path integral of dS=dQ/T from a to b, should be exactly the opposite to that from b to a, independent of the path chosen. The entropy difference between state a and b can be written as a function of state.

$$S(b) - S(a) = \int_a^b dS \underset{\uparrow}{=} \int_a^b \frac{dQ}{T}$$

For reversible process only

- Also note that "<" applies when the process is irreversible, due to the inefficiency of an irreversible process, where

$$\frac{dQ_{irev}}{T} < \frac{dQ_{rev}}{T} = dS$$

The law of increase of entropy

Unlike reversible cycle, we shall see the entropy **increases** for irreversible cycles (directionality of a closed system):

1. We do irreversible work on an object by friction, generating a heat Q on some object at temperature T. The entropy is increased by Q/T, where the heat Q = W.
2. If we put together two objects that are at different temperatures, and assume $T_1>T_2$. A certain amount of heat will flow from one to the other by itself. The entropy of the hotter object will loose $\Delta Q/T_1 (\Delta Q>0)$, and the cooler one will increase by $\Delta Q/T_2$, so the total entropy increases

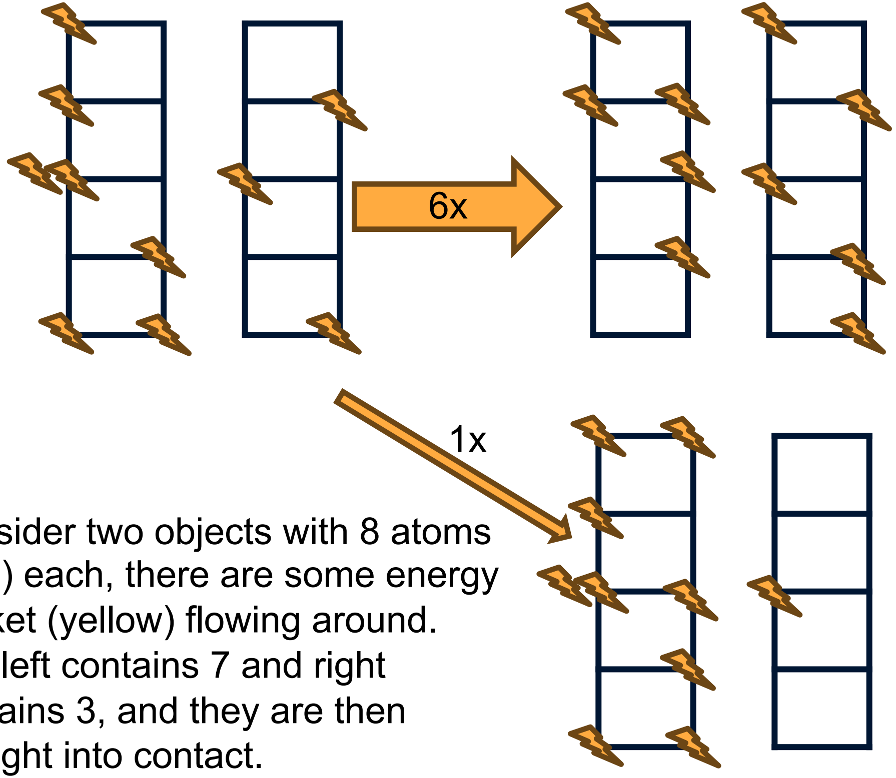
$$\Delta S = \frac{\Delta Q}{T_2} - \frac{\Delta Q}{T_1}$$

$$\boxed{{}^1/_T = \left(\frac{\partial S}{\partial Q}\right)_{rev}}$$

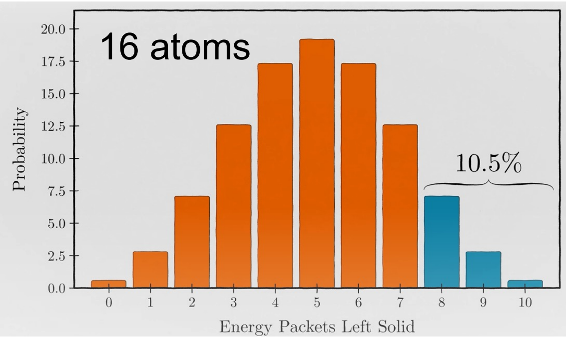
3. **Joule expansion:** Expansion of gas into vacuum: $\Delta U = \Delta Q = \Delta W = 0$

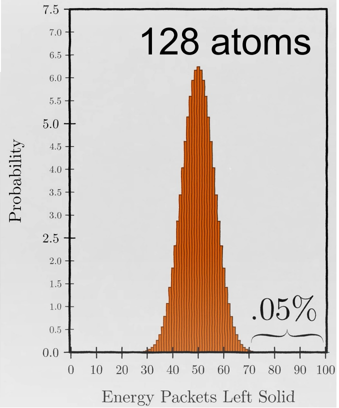
$\Delta U = 0$ $\Delta T = 0$ for an ideal gas T = T'. We know the temperature is unchanged, so is the U. but the entropy need to increase as a result of or an irreversible process.

Another view: energy tend to spread out



Consider two objects with 8 atoms (grid) each, there are some energy packet (yellow) flowing around. The left contains 7 and right contains 3, and they are then brought into contact.

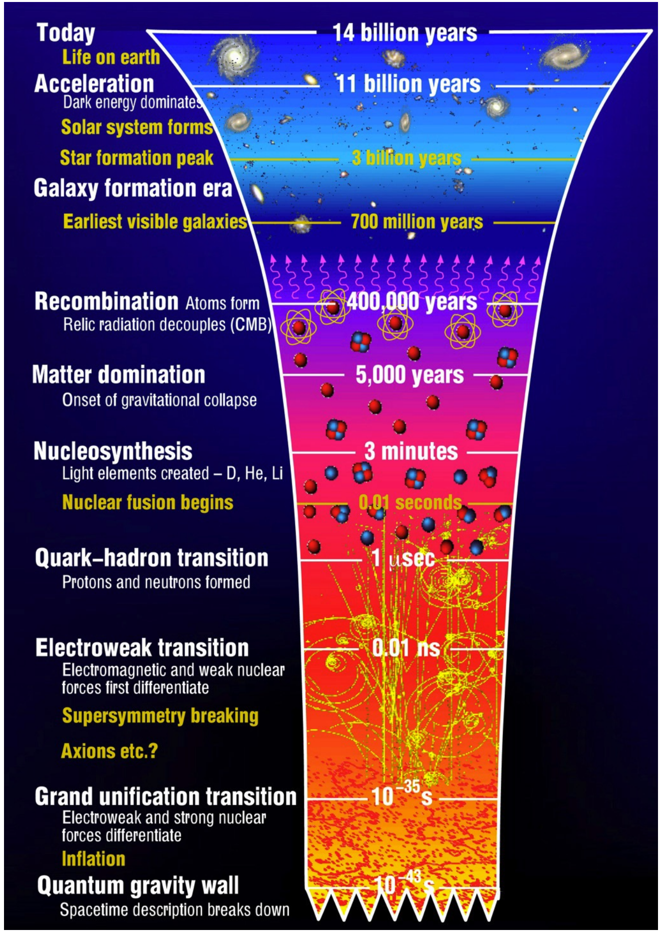




- The probability of the left object containing more energy than before is not **impossible** (10.5%), but less **probable** (e.g. compared to 5 and 5).
- When we increase the number of atoms (128) and number of energy packet (100), the probability that the energy flowing from the colder to hotter becomes more unlikely (0.05%).
- In real life, with number of atoms O(N_A), the spontaneous process with a net effect of heat flowing from cold to hot *almost* never happens.
- This probability point of view for entropy is essential in information theory.

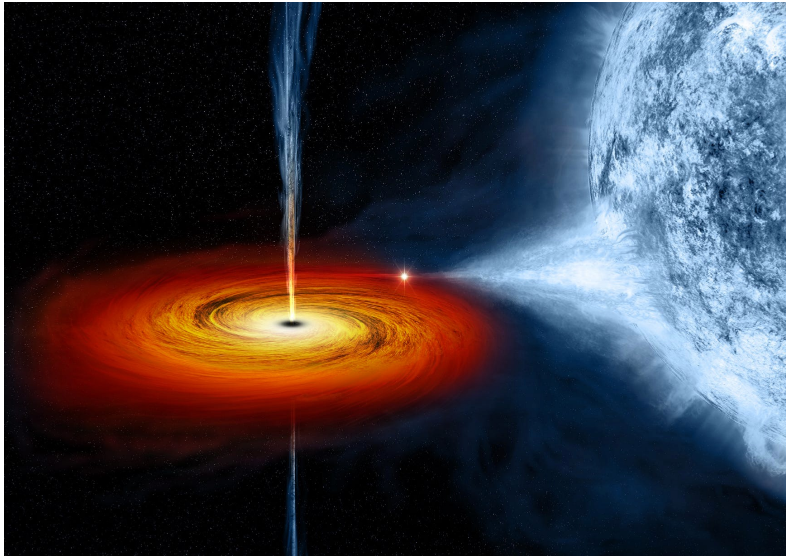
Reading: Entropy of the Universe

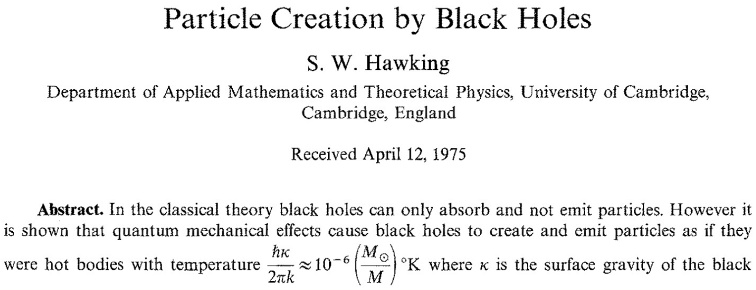
- Our universe is certainly an isolated system so its entropy cannot decrease.
- The big bang model suggests that the universe began in thermal equilibrium at high temperatures (low entropy), expanding and cooling. New particle, stars, planets got formed and accumulate under gravity. Some energy from the early universe turned into the kinetic energy of matter, and then converted to a less “useful” energy of heat due to collision and friction.
- Eventually, with increased entropy, the universe will come to thermal equilibrium—the “heat death”. The 2nd law of thermodynamic gives a **direction** that our universe evolve. And it also seems to suggest that there was a time the entropy of our universe equal to zero.



Reading: Entropy of a blackhole

- Assuming blackholes carry zero entropy, meaning its swallowing another object tend to decrease the total entropy of the universe, thus violate the 2nd law of thermodynamics.
- Bekenstein conjected that blackholes have nonzero entropy, $S = \frac{k_B A}{4 l_p^2}$
- Recall from thermodynamics, 1/T=dS/dQ. This means that blackholes also has a temperature, and thus emit *radiation---the* Hawking radiation.
- In fact, in the late stage of the universe, BHs will constitute most of the entropy of our universe.





Particle Creation by Black Holes

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Received April 12, 1975

Abstract. In the classical theory black holes can only absorb and not emit particles. However it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{\hbar\kappa}{2\pi k} \approx 10^{-6}\left(\frac{M_\odot}{M}\right)$ °K where κ is the surface gravity of the black

Laws of Thermodynamics (reinterpreted)

First law:

Heat put into a system and the work done on a system add up to the increase in internal energy of the system.

The energy of the universe is constant.

$$\boxed{\begin{array}{c} dU=dW+dQ=pdV+dQ \\ \mathbf{dU=pdV+TdS\ (iff\ reversible)} \end{array}}$$

Second law:

A process whose only net result is to take heat from a reservoir and convert it to work is impossible.

The entropy of the universe tend to a maximum.