HW12: Due on Dec 18 in TA's office. CODE NUMBER: ———— SCORE: ————

**Problem #1: Statistical Mechanics**

(a) In the canonical ensemble, show that the fluctuations in energy $\Delta E^2 = \langle E^2 \rangle - \langle E \rangle^2$ are proportional to the heat capacity.

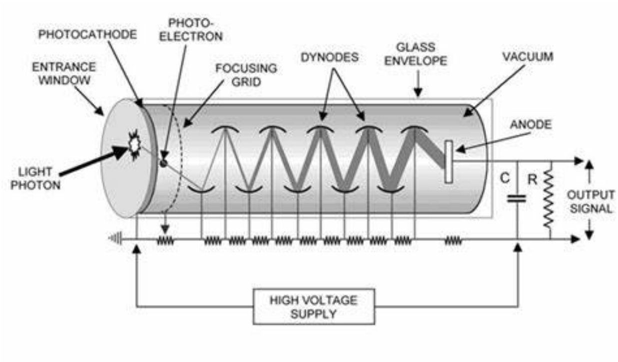
(b) Show that in the canonical ensemble the Gibbs entropy can be written as

$$S = k_B \frac{\partial}{\partial T}(T \log Z)$$

**Problem #2: Drift in a mixed gas**

Two gases, A and B, are at density $\rho_A$ and $\rho_B$ at a certain temperature $T_0$. A particular ion is observed to have a mobility $\mu_a$ in gas A and $\mu_b$ in gas B. What mobility $\mu$ would you expect the ion to have in a mixture of these gases, at density $\rho_A+\rho_B$ and temperature $T_0$?

**Problem #3: Photomultiplier tube**




Photomultiplier tubes (PMTs) are common light detectors in experiments which operates based on the photoelectric effect. The PMT is used to detect low levels of light by converting photons into an electrical signal. During the experiment, you shine light of varying frequencies onto the photocathode of the PMT and measure the output current. After the initial photoelectrons are emitted, the PMT amplifies this signal through a series of dynodes. Each photoelectron strikes a dynode, causing the emission of several secondary electrons. This process is repeated across multiple dynodes, resulting in a substantial amplification of the initial signal.

(a) Calculate the threshold frequency for the photocathode material if the work function is known to be 2.5 eV. Assume the Planck constant ($h$) is $6.626\times10^{-34}$ J·s and the speed of light (c) is $3.00\times10^{8}$ m/s.

(b) Consider the implications of using a photocathode material with a different work function on the performance and sensitivity of the PMT.

(c) Discuss the impact of temperature on the dark count rate of the PMT, which is a measure of the PMT's noise level in the absence of light. Compare the rates between room temperature (300K) and liquid xenon temperature (165K), and explain why using what you have learnt so far.

**Problem #4: Thermal conductivity**

In class we discussed about the diffusion and drift, while same principle can be used to compute the *thermal conductivity* of a gas. The transfer of heat from the hotter gas on top to the colder gas at bottom is by the diffusion of the "hot" molecules downward and the diffusion of the "cold" molecules upward (so we can neglect the convection). To compute the flow of thermal energy we can ask about the energy carried downward across an element of area by the downward-moving molecules, and about the energy carried upward across the surface by the upward-moving molecules. The difference will give us the net downward flow of energy. Show that, in the absence of convection, the thermal conductivity $\kappa$ of a gas is

$$\kappa = \frac{knlv}{\gamma - 1},$$

or in terms of the cross section $\sigma_c$,

$$\kappa = \frac{1}{\gamma - 1}\frac{kv}{\sigma_c},$$

where $n$ is the number of gas molecules per unit volume, $v$ is their average velocity, $l$ is their mean free path, and $kT/(\gamma - 1)$ is the average energy of a molecule at the temperature T.