

HW8: Due on Nov 20 in class.

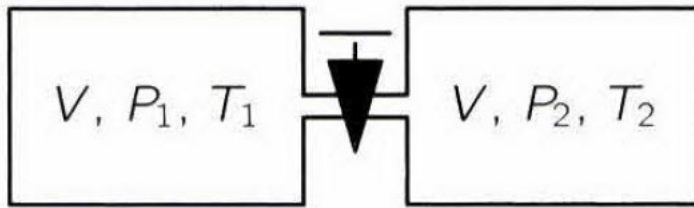
CODE NUMBER: _____ SCORE: _____

Problem #1: Adiabatic compression

We have derived that during an adiabatic process, the relation among the temperature T , pressure P , and volume V of an ideal gas.

- (a) Please derive the relation between P and T , and then between V and T during an adiabatic process.
- (b) Two samples of gas, A and B, of the same initial volume V_0 , and at the same initial absolute pressure P_0 , are suddenly compressed adiabatically, each to one-half its initial volume. How does the final pressure of each sample (P_A , P_B) compare with its initial pressure if γ_A is $5/3$ (monatomic) and γ_B is $7/5$ (diatomic)?
- (c) Find the ratio of work W_A/W_B required to perform the two compressions described.

Problem #2: Mixing and free expansion of gas



Two containers of volume $V_1 = V_2 = V$ are connected by a small tube with a valve, as shown in the figure.

- (a) Initially, the valve is closed and the two volumes contain monatomic gas at pressures P_1 and P_2 and temperatures T_1 and T_2 , respectively. After the valve is opened, what will be the final pressure P_f and temperature T_f inside the joint volume? (Neglect heat lost from the system.)
- (b) Now let's assume only one of the volumes is filled with a dilute gas; the other part is empty. We assume that there are no interactions between gas molecules. Remove the partition and wait until the final equilibrium condition is attained where the molecules of the gas are uniformly distributed throughout the entire container of volume $2V$ (Neglect the joint volume).
 - 1) Has the total energy of the gas been changed? Use this result to compare the average energy per molecule and the average speed of a molecule in the equilibrium situations before and after the removal of the partition.
 - 2) What is the ratio of the pressure exerted by the gas in the final situation to that of the pressure exerted by it in the initial situation?

Problem #3: Density distribution over a potential gradient

- (a) Imagine a tall vertical column of gaseous or liquid fluid whose density varies with height. Show that the pressure as a function of height follows the differential equation $dP/dh = -\rho(h)g$.
- (b) Solve this differential equation for the case of an ideal gaseous atmosphere of molecular weight μ , in which the temperature is constant as a function of h .

Problem #4: Diatomic molecular gas pressure

In class, we derived the pressure $PV = Nk_B T$. Let us consider an atomic gas and a diatomic molecular gas, and examine their similarity and difference. Consider two boxes with the same volume. They are filled with the same numbers of O_2 molecules and He atoms, respectively. The two systems are at the same temperature T .

- (a) Why do these two systems exhibit the same pressure in spite of the fact that the oxygen box actually has twice the number of atoms as the helium box? Please examine the collision processes with the box wall carefully.
- (b) Imagine that we could increase the temperature from room temperature to very very high, say, $10^5 K$, although no materials can sustain at such a high temperature. How do the pressures change in these two boxes? Are there any differences? Why?