GR - HOMEWORK 1

1. Verify that the continuity equation is invariant under Galilean transformation.

2. In clock synchronization, the frame F_2 moves at velocity v relative to F_1, so that a group of clocks in F_1 can be synchronized. To synchronize a group of clocks in F_2, we need to know the velocity of F_1 relative to F_2. Try to prove that this velocity is $-v$.

3. Assume that Ampère's law and Gauss' law are valid in R frame. Based on this, we can derive the velocity addition law between frames F and R, where frame F moves at velocity v relative to R. Now assume that there are two inertial frames F_1 and F_2 moving at velocity v_1 and v_2 relative to R, respectively. Try to build the velocity addition law between F_1 and F_2.

4. Based on the Newton's second law $\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$, try to derive that $\boldsymbol{p} = m_0\gamma_u\boldsymbol{u}$.

5. Try to prove that the coefficients λ_e, λ_b and λ' in the Lorentz transformation of electromagnetic field are all equal to 1.

6. Again, assume that Ampère's law and Gauss' law are valid in R frame. Based on this, try to use Lorentz transformation to derive the Maxwell equations. If we further assume that all the solutions of the derived Maxwell equations are physical, then we have established the electrodynamics.