GR - HOMEWORK 2

1. In the co-moving frame of the electron, the equation of motion for its spin is given by

$$\frac{d\vec{S}}{dt} = \vec{S} \times \left(\frac{ge}{2mc} \vec{B}' - \vec{\omega}_T \right).$$

Show that this equation of motion can be derived from the following coupling energy:

$$U = -\vec{S} \cdot \left(\frac{ge}{2mc} \vec{B}' - \vec{\omega}_T \right).$$

2. A particle moves in a 2D plane under the action of external forces. Its velocity is given by $\vec{v}(t)$, and the external forces exert no torque on the particle. After some time, the particle returns to its initial position, forming a closed trajectory. According to Thomas precession, determine the net rotation angle of the particle's instantaneous rest frame over this motion.

Hint: You may work out the problem in the velocity space. Assume that $|\vec{v}(t)| \ll c$.

3. Starting from the action

$$S = \int d\tau (-mc + \frac{q}{c} A_\mu \frac{dx^\mu}{d\tau}),$$

derive the equation of motion for a charged particle in the electromagnetic field.