

GR - HOMEWORK 3

1. Derive explicit expressions for the gradient of a scalar field f , as well as the divergence and curl of a vector field \vec{F} , in cylindrical and spherical coordinate systems.

2. We have previously derived the expression for the Christoffel symbol in flat space. We now extend this result to a curved space, for example, a sphere. We define the covariant derivative of a vector field in curved space as

$$D_k v^i = \partial_k v^i + \Gamma_{kj}^i v^j = \frac{v^i(x + dx) - v^{i,*}(x + dx)}{dx^k}. \quad (1)$$

where $v^{i,*}(x + dx)$ denotes the vector obtained by parallel transporting $v^i(x)$ to $x + dx$, and Γ_{kj}^i is the Christoffel symbol. Based on this definition, derive the explicit expression for Γ_{kj}^i on a sphere.