GR - HOMEWORK 4

1. Using the following expression for the Christoffel symbols,

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\lambda} \left(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu} \right), \tag{1}$$

derive their transformation law under a general coordinate transformation.

2. Using the definition of the covariant derivative

$$D_\sigma A_\mu = \partial_\sigma A_\mu - \Gamma^{\lambda}_{\sigma\mu} A_\lambda, \tag{2}$$

answer the following questions:

(a) Derive the transformation law of $D_\sigma A_\mu$ under a general coordinate transformation.

(b) Obtain the explicit expression for the covariant derivative of a contravariant vector A^μ.