

GR - HOMEWORK 5

1. Using polar coordinates, derive the geodesic equations for a two-dimensional plane, and solve them to obtain the explicit form of the geodesics.

2. Obtain the explicit form of the geodesics on a sphere.

3. Study parallel transport on a cone, whose metric is given by

$$ds^2 = dr^2 + r^2 \sin^2 \theta_0 d\varphi^2, \quad (1)$$

where θ_0 is a constant.

(a) Derive the explicit expressions for the Christoffel symbols.

(b) Compute the components of the Riemann curvature tensor.

(c) Consider parallel transport of an arbitrary vector along a circle of constant r (*i.e.*, a latitude circle). Using the results from part (a), determine the rotation angle of the vector after it returns to its starting point.

(d) Explain the consistency between the results obtained in parts (b) and (c).