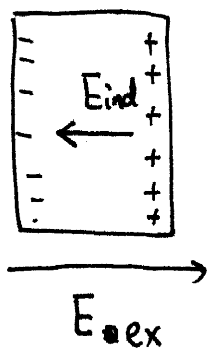
Lect 11. Conductors — metal

§ Electral static properties of conductors

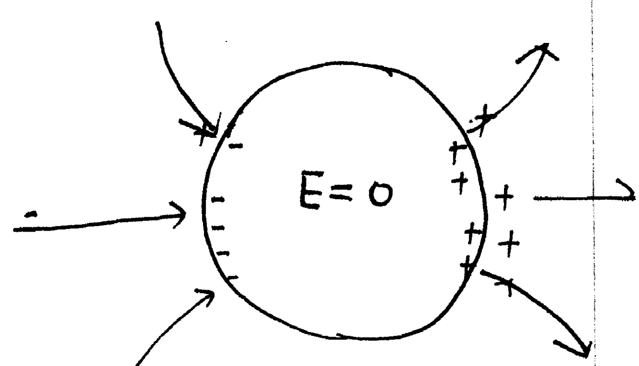
① $E = 0$ inside a conductor. — electro-static screening



mobil charge. External fields cause charge redistribution, to screen the external field which generate compensate fields

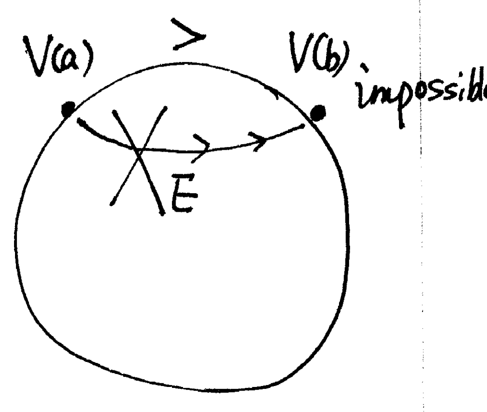
if $\vec{E}_{tot} = \vec{E}_{ex} + \vec{E}_{ind} \neq 0$, then charge will still be driven to move. Equilibrium is reached at $\vec{E}_{tot} = 0$.

Even inside ~~an~~ metallic shell, if there's no charge inside, you also have $E = 0$.



② the surface of a conductor has equal potential.

otherwise, electric charge will move follow the potential difference.

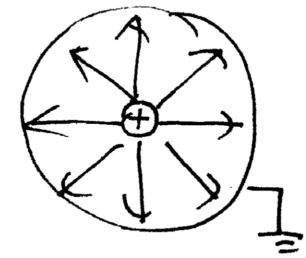


Inside a metallic shell, ~~others~~ if there's no charge. There should be no electric field lines. Otherwise, the field line has to

$\boxed{\text{Electric line can't form loop in electro-statics.}}$

~~start~~ from the shell and end at shell. Then the shell is n't equal-potentialed! Thus $\vec{E}=0$ inside the metallic shell.

But if there does exist charge inside the shell, $\vec{E}$ can exist.



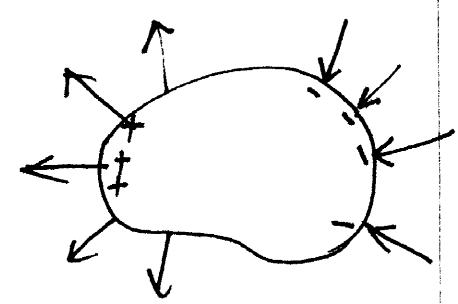
3 no free charge inside the bulk of metal.

Any net charge is on the surface.

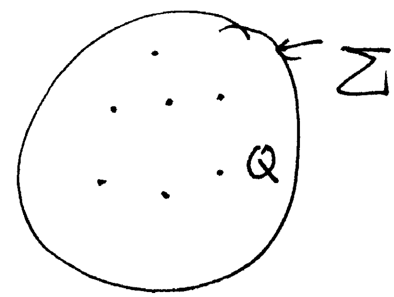
4 $\vec{E}$ is perpendicular to surface of conductor.

$\vec{E} = -\nabla V(r)$. the surface of conductor is equal-potential

$\Rightarrow \vec{E} \perp$ surface.



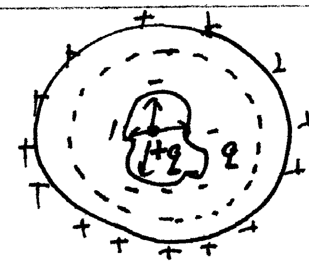
Question: Suppose we confine the charge within some region with the boundary of Σ.



$\longrightarrow$ the minimum electron-static energy configuration requires that all charges move to the surface!

This the same proplem of the equilibrium charge distribution of a metal with net charge.

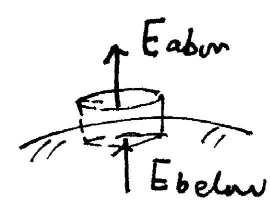
examp: electric field of a spherical sphere with a cavity.



$+q$ inside the cavity attracts $-q$ on the inner surface. Then $+q$ charge goes uniformly to the outer surface of the sphere. Because the electric field in the bulk is zero, the only part ~~important~~ is the outer surface.

$$\Rightarrow \quad \vec{E} = \frac{q}{r^2}\,\hat{e}_r .$$

§ surface charge



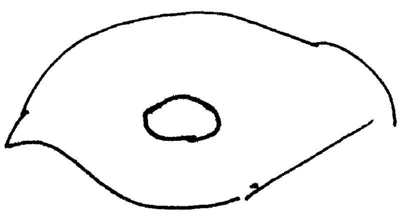
according to the boundary condition

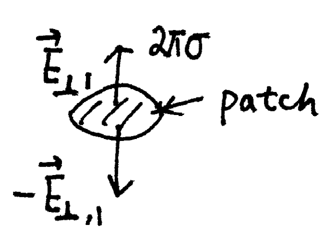
$$E_\perp^{above} - E_{\perp\, below} = 4\pi\sigma$$

and $E_{\perp,\, below} = 0 \;\Rightarrow\; E_{\perp\, above} = 4\pi\sigma$, or $\sigma = \dfrac{-1}{4\pi}\dfrac{\partial V}{\partial n}$.

What's the force exerted on the surface?

You might think the force density: $\vec{f} = \sigma \cdot E_\perp \hat{n}$, but actually this is not true. The force ~~the~~ exerted on one patch is by the fields generated by other parts of the surface.





Let us conside a small patch with charge density σ, it generates $\vec{E}$ along the norm in the oppositeed direction of $\pm 2\pi\sigma\hat{n}$.

The field generated by other parts of the surface should be continous at this patch $\vec{E}_{other}$.

we know $\vec{E}_{other} - \vec{E}_{\perp} = 0$ (inside)

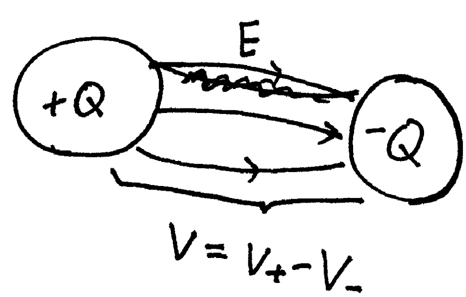
$$\vec{E}_{other} + \vec{E}_{\perp} = 4\pi\sigma\hat{n}$$

$$\Rightarrow \vec{E}_{other} = 2\pi\sigma\hat{n} \qquad \Rightarrow \boxed{\vec{f} = \vec{E}_{other}\,\sigma = 2\pi\sigma^2\hat{n}.}$$

outward electro-static pressure on the surface.

§ Capacitors — the ability to store charge

$$V = -\int_{-}^{+} \vec{E}\cdot d\vec{l}$$



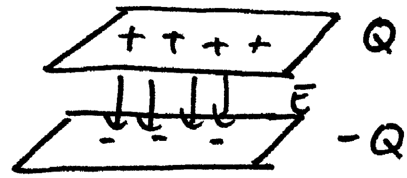
$$C = \frac{Q}{V}.$$

Q is proportional to V. $\Rightarrow$ C is a const which depends on the shape of conductors.

If we double the charge distribution on each surface, ~~by a factor~~ this is a linear system, the $\vec{E}$ field doubles $\Rightarrow$ V doubles.

C is unchanged!

example: ① planar ~~pl~~ plate capacitor

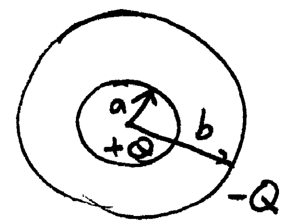


$$E = 4\pi\sigma = \frac{Q}{4\pi A}, \quad V = E\cdot d$$

$$\Rightarrow C = \frac{Q}{V} = \frac{Q\,4\pi A}{Q\,d} = 4\pi A/d$$ — in Gauss unit

C has the unit of length

② $$E = \frac{Q}{r^2}$$



$$V = +\int_a^b \vec{E}\cdot d\vec{r} = Q\left[\frac{1}{a} - \frac{1}{b}\right] = Q\,\frac{b-a}{ab}$$

$$\Rightarrow C = \frac{Q}{V} = \frac{ab}{a-b}$$

Energy stored in the capacitor: Considering a charging process

$$dw = V\,dq = \frac{q}{c}\,dq$$

$$\Rightarrow W = \int_0^Q \frac{q\,dq}{C} = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Check for the ~~plate~~ capacitor

$$\cancel{W = Vol\cdot\frac{E^2}{8\pi} = A\cdot d\cdot\frac{(4\pi\sigma)^2}{8\pi} = 2\pi\sigma^2 Ad = \frac{(A\sigma)^2\,4\pi d}{2\,A}}$$

$$\cancel{= \frac{Q^2}{2}}$$