* Why do we need to learn classic mechanics ?

Three representations

- Newtonian : $F = ma$. Convenient for simple systems.

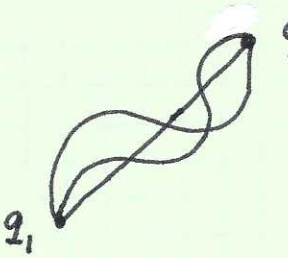
Not good for quamtum Mechanics (QM). Because of the uncertainty principle, acceleration is not well defined. Force is not as fundamental as potential (scalar and vecter potentials).

- Lagrangian : least action principle

$$\mathcal{L}(q,\dot{q}) \rightarrow S = \int_{t_1}^{t_2} dt\, \mathcal{L}(q,\dot{q}) \longrightarrow \delta S = 0 \Rightarrow \frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)$$

$\longrightarrow$ path integral formalisim for QM,

Quantum field theory



- Hamiltenian: $H(p, q) \leftarrow p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$

$$H = p\dot{q} - \mathcal{L} \quad \Rightarrow \begin{cases} \dot{p} = -\frac{\partial H}{\partial q} \\ \dot{q} = \frac{\partial H}{\partial p} \end{cases}$$ Canonical Equation.

$\longrightarrow$ Canonical quantization for QM

$$\{q, p\}_p = 1 \longrightarrow [q, p] = i\hbar$$

In Phys 110B, we will learn

- Newtonian (advanced)
  - ① non-inertial frame $\left\{ \begin{array}{l} \text{acceleration} \\ \text{rotation} \end{array} \right.$
  - ② rigid body (tops)
- Hamiltonian (elementary)
  phase space,
- Many-degree-of-freedom system
  elasticity, waves, collective motions.
- Relativity – E & M

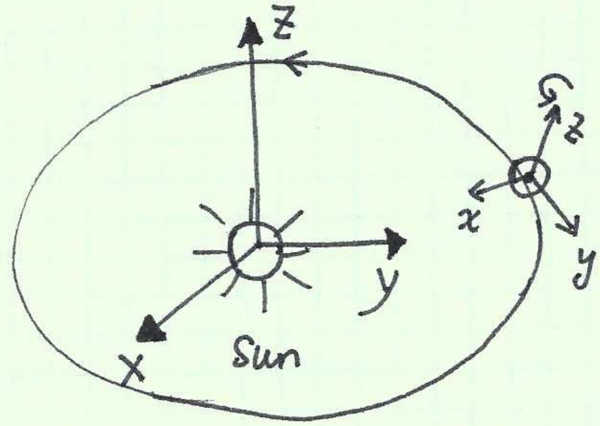
Lecture 1 - Non-inertial frame without rotation

1. motivation: why?

- Newton's law are only valid in inertial frames, i.e. no acceleration and rotation. Nevertheless, in many situations, non-inertial frames are more convenient:

  an accelerating car, train, plane, space-shuttle, ...

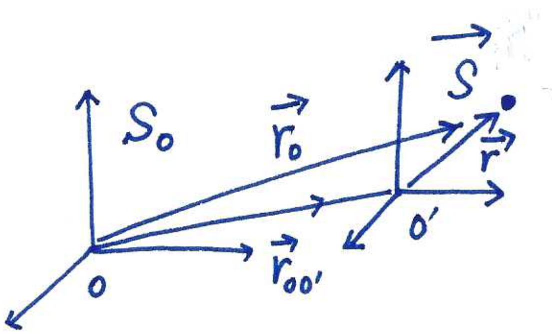
  earth is not an inertial frame (but very close) — spin, orbital motion



xyz is the frame fixed on the earth; it's not an inertial frame but convenient to use.

- Novel phenomena
  1. tides (12 hours period not 24 hours)
  2. Centrifugal force
  3. Coriolis force ← (Lorentz force)
     free falling object easternly deflection;
     typhone, hurricane ...
  4. Foucault pendulum ← see the spin of the earth on ground
- General relativity: non-inertial frame ⟶ gravity.

§2. acceleration without rotation



S_0 : inertial frame

S : non-inertial frame

$\vec{r}_0$: coordinate in S_0

$\vec{r}$: coordinate in S

$\vec{r}_{OO'}$: relative vecter between $S_0 . S$

newton's law in S_0 : $m\ddot{\vec{r}}_0 = F$

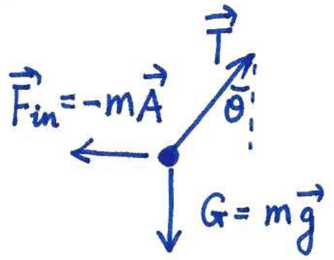
relation between S & S_0 : $\vec{r}_0 = \vec{r}_{OO'} + \vec{r} \Rightarrow \ddot{\vec{r}} = \ddot{\vec{r}}_{OO'} + \ddot{\vec{r}}$

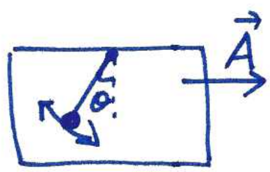
$$= \vec{A} + \ddot{\vec{r}}$$

$\Rightarrow$ law in frame S : $m\ddot{\vec{r}} = \vec{F} - m\vec{A} = \vec{F} + \vec{F}_{inertial}$.

where $\boxed{\vec{F}_{inertial} = -m\vec{A}}$

Simple example: pendulum in an accelerating car



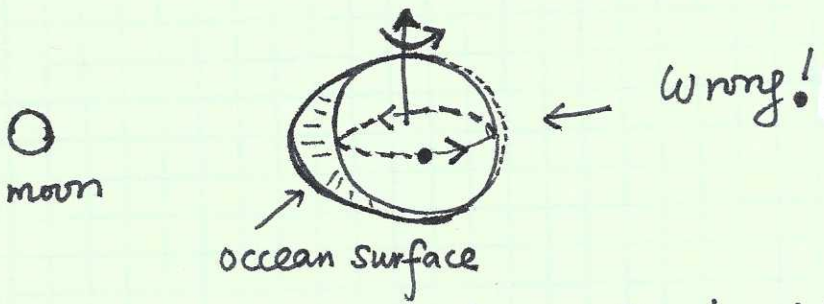


$$\Rightarrow \quad m\ddot{\vec{r}} = \vec{T} + \vec{G} + \vec{F}_{in} = \vec{T} + m(\vec{g} - \vec{A})$$

$$\vec{g}_{eff} = \vec{g} - \vec{A} \Rightarrow |g_{eff}| = \sqrt{g^2 + A^2}, \quad tg\theta = \frac{A}{g}$$

frequency $\omega = \sqrt{\frac{g_{eff}}{l}} = \sqrt{\frac{g}{l}}\sqrt{1 + (\frac{A}{g})^2} = \omega_0\sqrt{1 + (\frac{A}{g})^2}$

Application: tides — gravity of moon



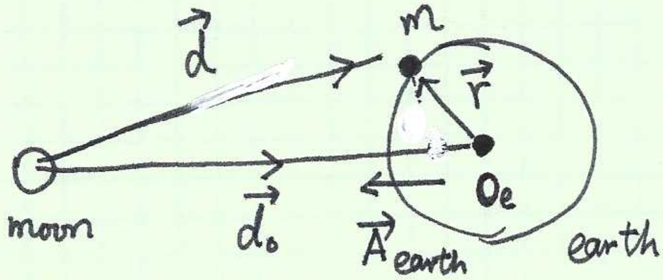
If the earth was an inertial frame, the height of the ocean surface would exhibit dipolan response to moon's gravity.

$\longrightarrow$ a single high tide per day, but this is not true!

$\boxed{\text{why? the earth is a non-inertial frame!}}$

The moon-earth system is orbiting around their center of mass.

The acceleration of the earth is $\vec{A}_{earth} = -\dfrac{G\,m_{moon}}{d_0^2}\hat{d}_0$



The earth is falling to the moon at $\vec{A}_{earth}$.

Oe is the center of earth;

In the earth frame, the mass point

$$\begin{cases} m\ddot{\vec{r}} = \vec{F} - m\vec{A}_{earth} \\ \vec{F} = m\vec{g} + \vec{F}_{other} - \dfrac{G\,M_{moon}\,m\,\hat{d}}{d^2} \end{cases}$$

$$m\ddot{\vec r} = m\vec g + \vec F_{other} + (-GM_{moon}m)\left[\frac{\hat d}{d^2} - \frac{\hat d_0}{d_0^2}\right]$$

$$\vec F_{tide} = -\frac{GM_{moon}m}{}\left(\frac{\hat d}{d^2} - \frac{\hat d_0}{d_0^2}\right)$$

$\vec d = \vec d_0 + \vec r$ compare to d_0, $\left|\frac{r}{d_0}\right| \sim \frac{6.4\times10^3\,km}{3.8\times10^5\,km} \sim 0.02 \ll 1$

$$\Rightarrow \frac{\hat d}{d^2} = \frac{\vec d}{d^3} = \frac{\vec d_0 + \vec r}{d_0^3\left(1 + \left(\frac{r}{d_0}\right)^2 + \frac{2\vec r\cdot\vec d_0}{d_0^2}\right)^{3/2}} \simeq \frac{\vec d_0 + \vec r}{d_0^3\left(1 + \frac{3\vec d_0\cdot\vec r}{d_0^2}\right)}$$

$$\simeq \frac{\vec d_0}{d_0^3} + \left[\frac{\vec r}{d_0^3} - \frac{3\vec d_0}{d_0^3}\frac{\vec d_0\cdot\vec r}{d_0^2}\right]$$

$$\Rightarrow \vec F_{tide} = \frac{GM_{moon}m}{d_0^2}\left[-\frac{\vec r}{d_0} + \frac{3\hat d_0(\hat d_0\cdot\vec r)}{d_0}\right]$$

The tide force is the gradient of the gravity force from the moon, i.e.

the gravity difference between the $\vec r$ and

the gravity at the earth center Oe.

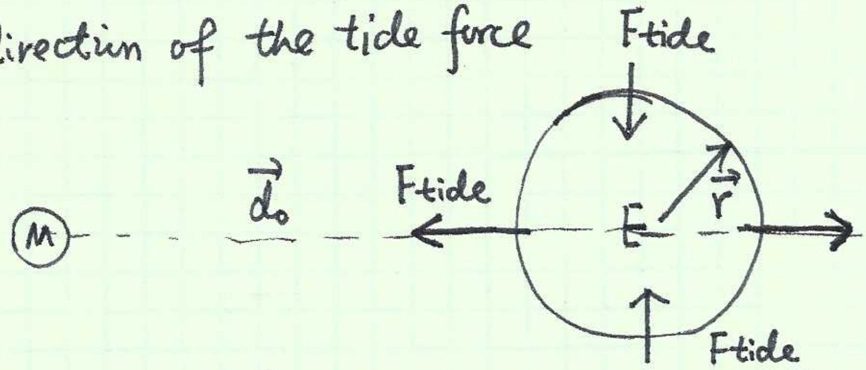
Here the gravity is from the moon!

Comment: If free free in a uniform gravity field, no tide force.

you feel as if completely free, no force.

$$\vec F = m\vec g - m\vec A = m(\vec g - \vec g) = 0.$$

The direction of the tide force



Qudrapolar response ↓ (12 hours period)

$$\vec{F}_{tide} = \frac{G M_{moon} m}{d_0^2} \underbrace{\left[- \hat{r} + 3\hat{d}_0 (\hat{d}_0 \cdot \hat{r}) \right]}_{\text{angular form factor}} \frac{|r|}{d_0}$$

for $\vec{r}$ on the earth. if $\hat{r} \parallel \hat{d}_0 \Rightarrow \vec{F}_{tide} = \frac{G M_m m}{d_0^2} \frac{|r|}{d_0} 2\hat{r}$

$\hat{r} \perp \hat{d}_0 \Rightarrow \vec{F}_{tide} = \frac{G M_m m}{d_0^2} \frac{|r|}{d_0} (-\hat{r})$

§ effective potential for $\vec{F}_{tide}$

We express $\vec{F}_{tide} = - \nabla U_{tide}$

$$\vec{F}_{tide} = - G M_{moon} m \left(\frac{\hat{d}}{d^2} - \frac{\hat{d}_0}{d_0^2} \right) = G M_{moon} m \left[\nabla_r \left(\frac{1}{d} \right) + \nabla_r \left(\frac{\vec{r} \cdot \hat{d}_0}{d_0^2} \right) \right]$$

$$\Rightarrow U_{tide} = - G M_{moon} m \left(\frac{1}{d} + \frac{\vec{r} \cdot \hat{d}_0}{d_0^2} \right)$$

$$\frac{1}{d} = (d_0^2 + r^2 + 2\vec{d}_0 \cdot \vec{r})^{-1/2} = d_0^{-1} \left(1 + \frac{2\hat{d}_0 \cdot \vec{r}}{d_0} + \left(\frac{r}{d_0} \right)^2 \right]^{-1/2}$$

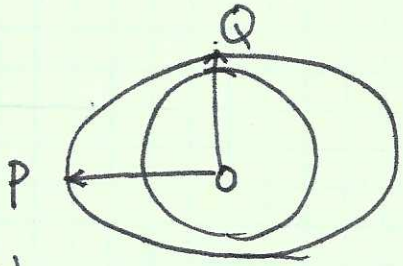
$$= \frac{1}{d_0} \left[1 - \frac{\hat{d}_0 \cdot \vec{r}}{d_0} - \frac{r^2}{2 d_0^2} + \frac{3}{2} \left(\frac{\hat{d}_0 \cdot \vec{r}}{d_0} \right)^2 \right]$$

$$\Rightarrow \boxed{U_{tide} = \text{const} - \frac{G M_{moon} m}{d_0} \left(\frac{r}{d_0} \right)^2 \left[\frac{3(\hat{d}_0 \cdot \hat{r})^2 - 1}{2} \right]}$$

$P_2(\cos\theta)$ Legendre polynomial

This is an additional potential in additional to the gravity of the earth itself.

The hight difference betwee high / low tide



$$(h_P - h_Q)\, mg = \mathcal{U}_{titde}(Q) - \mathcal{U}_{titde}(P)$$

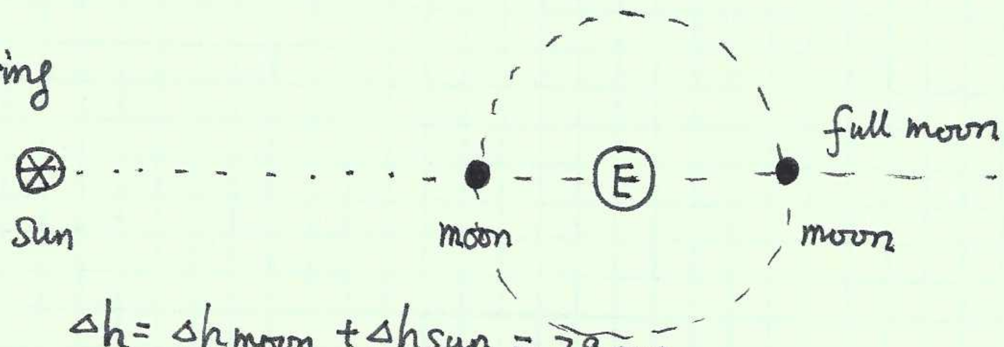
$$mg\,\Delta h \simeq \frac{G M_{moon} m}{d_0} \left(\frac{r_{earth}^2}{d_0^2} \right) \frac{3}{2}$$

$$\Delta h \simeq \frac{3}{2} \frac{G M_{moon}}{d_0} \frac{r_{earth}^2}{d_0^2} \simeq 54\text{ cm} \quad \text{for moon.}$$

The sun has similar effect, $M_{\odot} \gg M_{moon}$, but it's also far away

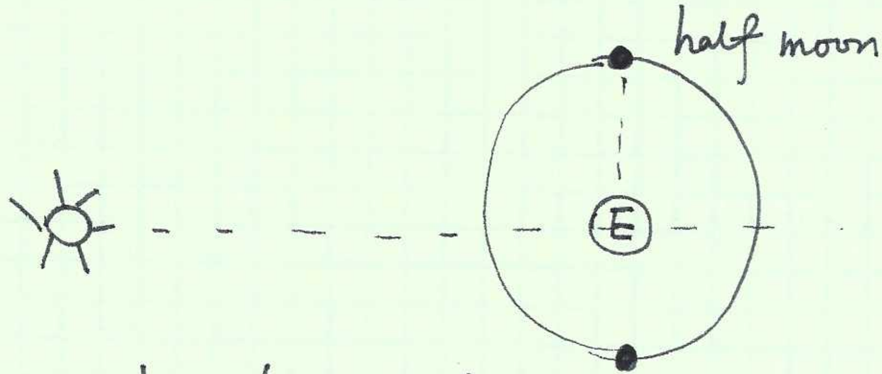
$$\Delta h_{sun} \simeq 25\text{cm}$$

spring



$$\Delta h = \Delta h_{moon} + \Delta h_{sun} = 79\text{ cm}$$

Neap



$$\Delta h = \Delta h_{moon} - \Delta h_{sun} = 29\text{ cm}$$