Lecture 9: Superconductivity — fundmental properties

— de Gennes' book.

1. Zero resistance below T_c

2. Diamagnetism : London theory

at $T < T_c$, we can divide electrons into "normal fraction" and "superconducting fraction". Newton's second law gives to

$$m\frac{dJ_s}{dt} = me\frac{d}{dt}[n_s\vec{v}] = me\left\{\frac{\partial}{\partial t}[n_s\vec{v}] + \vec{v}\cdot\nabla[n_s\vec{v}]\right\} = n_s e^2 E$$

$$\frac{\partial n_s}{\partial t} + \nabla(n_s\vec{v}) = 0$$

$$\Rightarrow e\frac{\partial}{\partial t}[n_s\vec{v}] - me\vec{v}\cdot\frac{\partial n_s}{\partial t} = n_s e^2 E \Rightarrow \text{for the static state}$$

$$\frac{\partial J_s}{\partial t} = \frac{n_s e^2}{m}E$$

thus $$\nabla\times\frac{\partial}{\partial t}J_s = \frac{n_s e^2}{m}\nabla\times E = \frac{n_s e^2}{m}\left[-\frac{1}{c}\frac{\partial B}{\partial t}\right]$$

$$\Rightarrow \frac{\partial}{\partial t}\left[\nabla\times J_s + \frac{n_s e^2}{mc}B\right] = 0$$

$\nabla\times J_s + \frac{n_s e^2}{mc}B = f(r)$, which should be time-indepenendant.

London assumed that $f(r) = 0 \Rightarrow \boxed{\nabla\times\vec{J}_s = -\frac{n_s e^2}{mc}\vec{B}}$

$$\nabla \times \vec{J}_s = -\frac{n_s e^2}{mc} \nabla \times A \quad \Rightarrow \quad \vec{q} \times \vec{J}_s(q) = -\frac{n_s e^2}{mc} \vec{q} \times \vec{A}(q)$$

* for any finite wave-vector $\vec{q}$, the susceptibility, (Current - current).

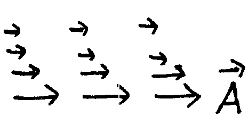
$$\chi_{JJ}^{\perp}(\vec{q}, 0) = -\frac{n_s e^2}{mc},$$

but the longituidnal $\vec{A}_{\parallel}$ is a pure gauge, and should not have

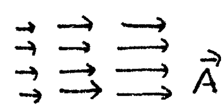
any response $\quad \chi_J^{\parallel}(\vec{q}, 0) = 0$.

We have that in the superconducting state, $\lim_{q \to 0} (\chi_{JJ}^{\perp}(q,0) - \chi_J^{\parallel}(q,0)) \neq 0$.

Thus in the long-wave length limit, the system can still distinguish transverse and longitudinoral.



transverse



longitudinal

* Penetration depth

$$\nabla \times (\nabla \times J_s) = -\frac{n_s e^2}{mc} \nabla \times \vec{B} = -\frac{4\pi n_s e^2}{mc^2} J_s$$

$$\nabla(\nabla \cdot J_s) - \nabla^2 \vec{J}_s = -\nabla^2 \vec{J}_s = -\frac{4\pi n_s e^2}{mc^2} J_s$$

$$\Rightarrow \boxed{\lambda^2 = \frac{4\pi n_s e^2}{mc^2}}$$

* Consider if the superconducting state can be described by a macro-scopic wave-function

$$j(r) = -\frac{ie^*\hbar}{2m^*}\left(\psi^*(r)\left(\nabla - \frac{ie^*A(r)}{\hbar c}\right)\psi(r)\right. + c.c.$$

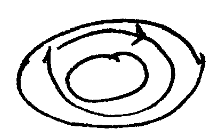
plug in $\psi(r) = \rho^{1/2}(r)\, e^{i\varphi(r)}$ $\Rightarrow \vec{j}_s(r) = \frac{e^*\hbar}{m^*}\rho(r)[\vec{\nabla}\varphi(r) - \frac{e^*}{\hbar c}\vec{A}(r)]$

later on, we will see $e^* = 2e$, $m^* = 2m$. if $\rho(r) = n_s/2$ and const

$\Rightarrow$ London equation.

* flux quantization

Consider we have a multiple-connected geometry.



The flux ~~we~~ trapped insider the hole.

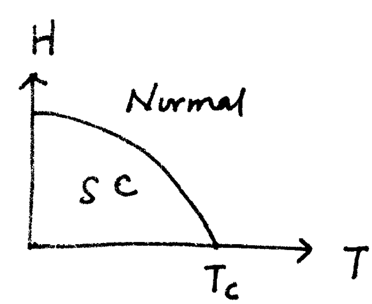
$$\oint \vec{j}_s(r)\,dr = 0 \Rightarrow \int \nabla\varphi(r)\,dr = \frac{e^*}{\hbar c}\int A\,dr$$

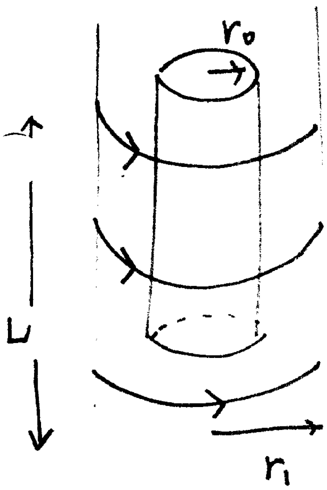
$$\frac{e^*}{\hbar c}\Phi = 2n\pi \Rightarrow \Phi \text{ quantized in unit of } \frac{\hbar 2\pi c}{2e}.$$

$$\Phi_0 = \frac{hc}{2e} = 2\times 10^{-7}\ G\cdot cm^2$$

3. • type I and type II superconductors

• type I: critical field $H_c(T)$





long cylinder : sample in a solenoid

$$H = \frac{4\pi N I}{cL} \qquad N: \text{number of turns}$$

normal state: $F_n = \pi r_0^2 L f_n + \pi r_1^2 L \frac{H^2}{8\pi}$

superconducting state $F_s = \pi r_0^2 L f_s + \pi (r_1^2 - r_0^2) L \cdot \frac{H^2}{8\pi}$

in order to repel the flux, the work did by $\frac{\text{emf}}{\text{induced}}$

$$\int V I \, dt = \int_i^f -\frac{N}{c}\left(\frac{d\phi}{dt}\right) I \, dt = -\frac{N}{c}(\phi_f - \phi_i) I = \frac{N}{c} I \pi r_0^2 H$$

$$= \pi r_0^2 L \frac{H^2}{4\pi}$$

$\Rightarrow$ at $H_c(T)$, we have equilibrium $\Rightarrow$ $f_n = f_s - \frac{H^2}{8\pi} + \frac{H^2}{4\pi}$

$$= f_s + \frac{H^2}{8\pi}$$

another way to derive it is to go through

the Gibbs free energy $G = F - MH$

at H_c we have $G_s(H_c) = G_n(H_c)$

The normal state G doesn't depend on H much $G_n(H_c) \approx F_n(0)$.

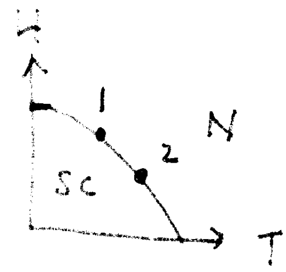
in SC state $dG = -M\,dH = \frac{H\,dH}{4\pi}$ $\left(M = \frac{-H}{4\pi}\right)$

$$G_s(H) - G_s(0) = \int \frac{H\,dH}{4\pi} = \frac{H^2}{8\pi}$$

$$\Rightarrow G_s(H) = f_s(0) + \frac{H^2}{8\pi} \Rightarrow \boxed{f_s(0) + \frac{H^2}{8\pi} = f_n(0)}$$

⑤

$\nearrow H_c(T) \simeq H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$



Latent heat: $dG = -S\,dT - M\,dH$

from $1 \to 2$: in normal state

$-S_n\,dT + M_n\,dH$

from $1 \to 2$ in superconducting state

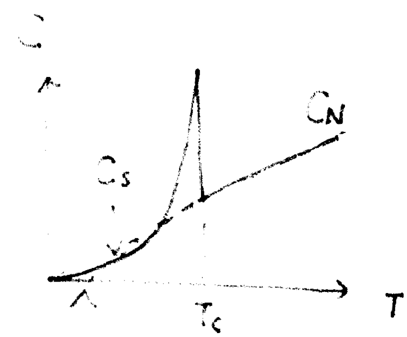
$-S_c\,dT + M_s\,dH$

$$\Rightarrow \frac{dH_c}{dT} = -\frac{S_n - S_s}{M_n - M_s}$$

$$M_n \simeq 0, \quad M_s = -(4\pi)^{-1}H \quad \Rightarrow \boxed{-\frac{dH_c}{dT}\cdot\frac{H_c}{4\pi} = S_n - S_s}$$

$$C_n - C_s = T\frac{d}{dT}(S_n - S_s) = -T\frac{d}{dT}\left(\frac{dH_c}{dT}\cdot\frac{H_c}{4\pi}\right) \quad \text{at zero field}$$

$$\Rightarrow \left[C_n - C_s\right]\Big|_{T=T_c} = -\frac{T}{4\pi}\left(\frac{dH_c}{dT}\right)^2.$$



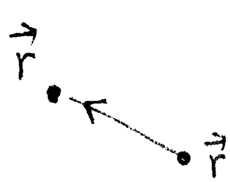
low T:

$C \sim k_B\, e^{-\Delta/k_BT}$

- Pippard non-local form. (Coherence length ξ).

In the Coulumb gauge. $\vec{j}(\vec{r}) = C\cdot\int \frac{(\vec{A}(r)\cdot\vec{R})\vec{R}}{R^4}\, e^{-R/\xi_0}\, d\vec{r}\,'$

where $\vec{R} = \vec{r} - \vec{r}\,'$, C is a constant.



When A is a slowly-varing variable, we must come back to London equation.

set $\vec{A}$ along z-axis $\Rightarrow$ $\vec{j}(\vec{r}) = \vec{A} C \cdot \int \frac{\cos^2\theta}{R^2} e^{-R/\xi} \cdot R^2 dR \sin\theta d\theta \cdot d\varphi$

$$= C \cdot \xi \cdot \frac{2}{3} \cdot 2\pi \vec{A} = -\frac{n_s e^2}{mc}$$

$\Rightarrow$ $C = -\frac{3 n_s e^2}{4\pi m c \xi_0}$, ξ_0 is the correlation length

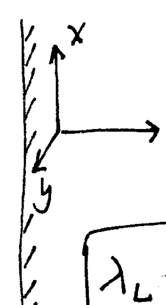
Later on, we can microscopically show $\xi_0 = \frac{\hbar v_F}{\pi \Delta}$, where is the gap in the SC state.

why Pippard proposed this form, is based the Chambers formula in the normal state:

$$\vec{j}(r, \omega) = e^2 \left(\frac{dn}{d\epsilon}\right)_{\epsilon_F} \frac{v_F}{4\pi} \int dr' \frac{\vec{R}\,(\vec{R} \cdot \vec{\mathcal{E}}(r'))}{R^4} e^{-i\frac{\omega R}{v_F}} \cdot e^{-R/l}.$$

- Modification of penetration depth in the Pippard limit ($\xi \gg \lambda_L$).

London penetration depth applies in the case $\lambda \gg \xi$, where $\vec{A}$ is slow-varying over the length-scale of ξ. For type-I superconductor, actually $\lambda_L \ll \xi$, thus $\lambda \neq \lambda_L = \frac{4\pi n_s e^2}{mc}$. ($\vec{A}$ is not slow-varying at the scale of ξ)



Conside a sample of xy-plane, A is only nonzero within a thickness of λ, $\Rightarrow$

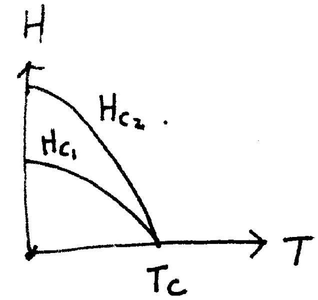
$$\vec{j}(r) \propto -\frac{ne^2}{mc} \frac{\lambda}{\xi_0} \vec{A} \quad (\lambda \ll \xi_0)$$

$\boxed{\lambda_L \ll \lambda \ll \xi}$

self-consistently $\Rightarrow$ $\frac{1}{\lambda^2} = \frac{4\pi n_s e^2}{mc} \frac{\lambda}{\xi_0}$ $\Rightarrow$ $\boxed{\frac{\lambda^3}{\xi_0} = \lambda_L^2 \Rightarrow \frac{\lambda}{\lambda_L} = \left(\frac{\xi_0}{\lambda_L}\right)^{1/3} > 1}$

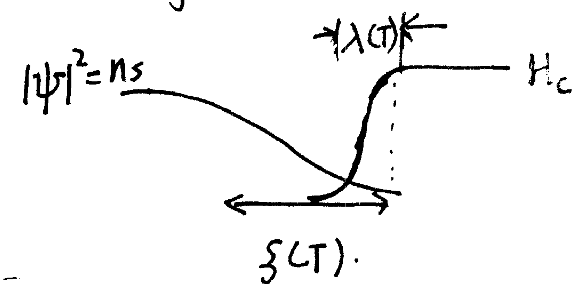
	λ_L	ξ_0	λ_{th}	λ_{exp}
Al	157	16,000	530	490-515
Sn	355	2,300	560	510
Pb	370	830	480	390

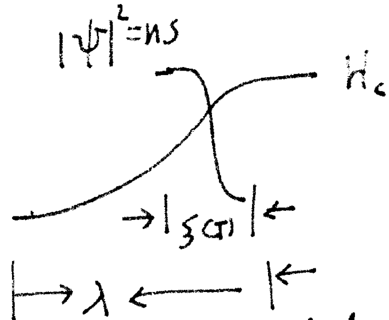
§ type II - superconductor H_{c1} and H_{c2}

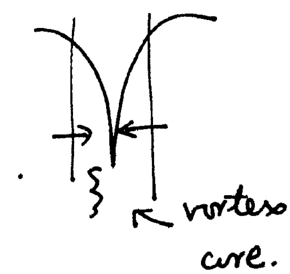


Between H_{c1} and H_{c2}, vortex-state.

Surface energy.







interface - type II

interface - type I

interface: domain between SC/normal face allows magnetic field to enter, thus reduces diamagnetic energy, but it suppresses superconductivity and cost energy.

for type I - superconductor, interface energy > 0, $(\xi \gg \lambda)$.

II ... < 0 $(\xi \ll \lambda)$.

$\longrightarrow$ form vortex lattice with each vortex carries Φ_0, with a normal core with size of ξ.

$$H_{c1} = \frac{\Phi}{\lambda_L^2}, \quad H_{c2} = \frac{\Phi}{\xi^2}.$$