Lect 0: Review of Single particle physics

ccf. Ashcroft & Mermin

McGill 2322

§1. Free fermi gas

① density of states $N(\mathcal{E}) = \frac{2}{(2\pi)^3}\int d^3\vec{k}\ \delta\left(\mathcal{E} - \frac{\hbar^2 k^2}{2m}\right)$

$= (2\pi^2)^{-1}\left(\frac{2m}{\hbar^2}\right)^{3/2} \epsilon^{1/2}\, d\epsilon$

Fermi wave vector

$k_F = (3\pi^2 n)^{1/3}$, $\quad n \sim 10^{22}\ cm^{-3} = 10^{28}\ m^{-3}$, $\quad k_F \sim 1\ Å^{-1}$

Fermi velocity

$\frac{\hbar k_F}{m} \sim 10^8\ cm/s = 10^6\ m/s$

Fermi energy $\sim 5\ eV$

density of states at Fermi energy $N(\epsilon) = \frac{1}{\pi^2}\frac{k_F^2}{\hbar v_F}$

② finite temperature and thermodynamic properties.

$$\mu = \mathcal{E}_F\left(1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\epsilon_F}\right)^2 + \cdots\right)$$

$$u = u(T=0)\left[1 + \frac{5}{12}\pi^2\left(\frac{k_B T}{E_F}\right)^2\right]$$

$$C_v = \frac{\pi^2}{3} g(E_F)\, k_B^2\, T = n k_B \frac{\pi^2}{2}\left(\frac{k_B T}{E_F}\right)$$

$$\chi_{\text{spin-suseptibility}} = N(E_F)\,\mu_B^2$$

$$K_{\text{compressibility}} = N(E_F)$$

3 Drude transport

$$\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}, \quad \sigma_0 = \frac{ne^2\tau}{m}$$

$\varepsilon(\omega) = 1 + \frac{i4\pi\sigma(\omega)}{\omega}$, at large frequency $\omega\tau \gg 1$

$$\rightarrow \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \text{ with } \omega_p^2 = \frac{4\pi ne^2}{m} \text{ (plasmon)}$$

Wiedeman-Franz law, thermo-conductivity

$$\kappa = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 T\sigma$$

Matthiessen's law: $1/\tau = 1/\tau_{(1)} + 1/\tau_{(2)}$

§2: Crystal lattices

1 Bravais lattices, Wigner-seitz unit cell,

point group,

Reciprocal lattice (fcc, $\leftrightarrow$ bcc)

First Brillouin zone of fcc, bcc, simple cubic, hcp....

2 Bloch wave,

$$\psi_{kn}(r) = e^{ikr} u_{kn}(r)$$

wannier function $N^{-1/2} \sum_k e^{ik(r-R_i)} u_{k,n}(r)$

Density of state $N(\epsilon) = \frac{2}{(2\pi)^3} \int \frac{dS}{|\nabla_k e|}$,

Von Hove singularity.

Band structures, band gaps,

metal, insulator, semiconductor / semimetal

tight-bind approximation,

plane-wave method calculation of band structure

§3: Electro dynamics in the lattice

$$\begin{cases} \hbar \frac{d\vec{k}}{dt} = e\vec{E} + e\frac{d\vec{r}}{dt}\times\vec{B} \\ \frac{d}{dt}\vec{r} = \frac{\partial \mathcal{E}(k)}{\partial \vec{k}} \end{cases} \qquad (m^*)^{-1}_{ij} = \hbar^2 \frac{\partial^2 \mathcal{E}}{\partial k_i \partial k_j}$$

Cyclotron orbit in k and r-space.

$$\omega_c = \frac{2\pi e B}{\hbar^2}\left(\frac{\partial A}{\partial \mathcal{E}}\right)^{-1},$$ A the area enclosed by the electron orbit

Hall effect: $R_H = 1/ne$

de-Haas-Van Alphen effect $\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\frac{1}{A(\mathcal{E}_F)}$

measuring of Fermi surface, open/closed orbit

§4: semiconductor

concentration of carrier in intrinsic and doped semiconductor.

mobility, diffusion, drift, Einstein relation

$$\mu = \frac{eD}{kT}$$

§: 5 phonons

Quantization of lattice vibration : acoustic / optical phonons.

Debye specific heat / phonon density of states.

anharmonicity : thermal expansion ...

§ magnetism

Pauli paramagnetism, Landau diamagnetism

Curie - Weiss theory for Ferromagnetism