HW 3 Phys 211 B

1) Consider a system of spin-1/2 neutral fermions at zero temperature interacting through the Yukwa potential

$$V(r) = \left(+\frac{g}{4\pi r}\right) e^{-r/a}$$

a) Calculate the Landau interaction function $f(pp', \sigma\sigma')$ at the Hartree-Fock level, and get Landau parameters F_0^s F_0^a, F_1^s, F_1^a up to a^4.

b) determine the factors of the specific heat, compressibility and magnetic susceptibility in the Fermi liquid theory compared to the non-interacting Fermi gas with the same bare mass and density.

c) Apply the above result to the screened Coulomb potential in the interacting electron gas by using Thomas-Fermi model.

Consider the low density limit ($k_f \ll k_{FT}$). Compare the magnetic susceptibility and compressibility to the "free-gas" value defined in 2)? Should we take this result seriously?

(from Leggett's HW 3, Phy 490 Fall 2001).

2) Derive the effective mass renormalization.

a) Consider a fermi-liquid system. By summing the Boltzman equation over momentum p, and using the fact that the collision integral must conserve the local particle density, derive the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla_i j_i = 0,$$ where $\vec{j}$ is particle current

defined as $j_i = \frac{1}{V}\sum_{p,\sigma} \frac{\partial \varepsilon_{p\sigma}(r,t)}{\partial p_i} n_{p\sigma}(r,t)$

b) Linearize the expression of j_i, and keep in mind that $\varepsilon_{p\sigma}(r,t)$ also depends on the distribution change $\delta n_{p\sigma}(r,t) = n_{p\sigma}(r,t) - n^0_{p\sigma}$.

Show $$j_i(r,t) = \frac{1+F_1^s/3}{m^*}\frac{1}{V}\sum_{p,\sigma} p\,\delta n_{p\sigma}(r,t)$$

c) In Galilean invariant system, by adiabatical continuity principle. Argue that $j_i(r,t) = \frac{1}{V}\sum_{p\sigma} \frac{p}{m}\delta n_{p\sigma}(r,t)$. Show that $\frac{m^*}{m} = 1 + F_1^s/3$.

d) Can you apply the above result to spin current? Please show

$$\vec{j}_i = \frac{1+F_1^a/3}{m^*}\frac{1}{V}\sum_{p,\sigma} p_i\vec{\sigma}\,\delta n_{p\sigma}(r,t),$$ and we can define the spin effective mass $\frac{1}{m_s^*} = \frac{1+F_1^a/3}{m^*}$, i.e $m_s^* = \frac{1+F_1^s/3}{1+F_1^a/3} m$.

Why m_s^* is not necessarily equal to m?

3. Sound excitations

① First sound or hydrodynamic sound. Prove sound velocity equals

$v^2 = 1/(\chi\rho)$, where χ is the compressibility, ρ is the mass density. Show $v^2 = \frac{n}{m}\left(\frac{\partial\mu}{\partial n}\right)$. In the idea fermi gas, Show $v_{ideal} = \frac{v_F}{\sqrt{3}}$.

and in Fermi liquid $\left(\frac{v_{FL}}{v_{idea}}\right)^2 = \frac{1+F_0^s}{1+\frac{1}{3}F_1^s}$

② in the Class, we have derived the equation for the zero sound

we write $\delta n(p,r,t) = \sum_q \delta n(p)\, e^{i(qr-\omega t)}$ and

$$\delta n(p) = -\frac{\partial n_p^0}{\partial \varepsilon_p} \nu_{\hat p}.$$

and $(S-\cos\theta)\nu_{\hat p} - \cos\theta \sum_{p'} f_{pp'}\left(-\frac{\partial n_{p'}^0}{\partial \varepsilon_p}\right)\nu_{\hat p'} = 0$.

By expanding $\nu_{\hat p} = \sum_{l} Y_{l0}(\hat p)\, u_{l0}$ ($Y_{l0}(\hat p)$ is the spherical harmonic function) (set $\hat q$ along $\hat z$-axis).

Show that

$$\frac{u_{l0}}{2l+1} + \sum_{l'} \Omega_{ll'}(S)\, F_{l'}^s \frac{u_{l'0}}{2l'+1} = 0 , \quad \text{where}$$

$$\Omega_{ll'} = \int_0^{\pi} d\cos\theta \int_0^{2\pi} d\varphi\; Y_{l0}(\cos\theta)\frac{\cos\theta}{\cos\theta - S} Y_{l'0}(\cos\theta)$$

if we truncate at $l=0$, we get $F_0^s \int \frac{d\Omega}{4\pi}\frac{\cos\theta}{\cos\theta - S} = 1$.

and show $S \longrightarrow 1 + 2e^{-2/F_0}$ $(F_0 \to 0)$, and

$$S \longrightarrow \sqrt{F_0/3} \qquad (F_0 \to +\infty)$$

(3) Prove that

$$\Omega_{00} = 1 - \frac{S}{2} \ln\left(\left|\frac{S+1}{S-1}\right|\right)$$

$$\Omega_{01} = \Omega_{10} = S\,\Omega_{00}\,, \quad \Omega_{11} = S^2 \Omega_{00} + 1/3$$

If only $F_0^S \neq 0$, the eigenmode of the zero sound has the configuration of $u_{l0} = (2l+1) \frac{\Omega_{l0}}{\Omega_{00}} u_{00}$ $(l \geq 1)$.

From the symmetry point of view, explain why $\{u_{l0}\}$ $(l=0,1,\cdots)$ can couple together.

If we keep both F_0^S and F_1^S nonzero show that the zero sound frequency satisfies

$$\frac{1}{2} s \ln\left(\left|\frac{S+1}{S-1}\right|\right) - 1 = \frac{1 + \frac{1}{3} F_1}{F_0 + s^2 F_1 + \frac{1}{3} F_0 F_1}$$