Quantum Mechanics 2025 HW10

Due 12/16 in Class
December 9, 2025

Problem 1.

A one-dimensional simple harmonic oscillator of angular frequency ω is acted upon by a spatially uniform but time-dependent force (*not* potential)

$$F(t) = \frac{(F_0 \tau/\omega)}{\tau^2 + t^2}, \quad -\infty < t < \infty.$$

At $t = -\infty$, the oscillator is known to be in the ground state. Using the time-dependent perturbation theory to first order, calculate the probability that the oscillator is found in the first excited state at $t = +\infty$.

Bonus question: $F(t)$ is so normalized that the impulse

$$\int F(t)\, dt$$

imparted to the oscillator is always the same—that is, independent of τ; yet for $\tau \gg 1/\omega$, the probability for excitation is essentially negligible. Is this reasonable? [Matrix element of x: $\langle n'|x|n\rangle = (\hbar/2m\omega)^{1/2} \left(\sqrt{n}\delta_{n',n-1} + \sqrt{n+1}\delta_{n',n+1}\right)$.]

Problem 2.

A hydrogen atom in its ground state $[(n, l, m) = (1, 0, 0)]$ is placed between the plates of a capacitor. A time-dependent but spatial uniform electric field (not potential!) is applied as follows:

$$\mathbf{E} = \begin{cases} 0 & \text{for } t < 0, \\ \mathbf{E_0} e^{-t/\tau} & \text{for } t > 0 \quad (\mathbf{E_0} \text{ is the positive } z\text{-direction}) \end{cases}$$

Using first-order time-dependent perturbation theory, compute the probability for the atom to be found at $t \gg \tau$ in each of the three $2p$ states: $(n, l, m) = (2, 1, \pm 1 \text{ or } 0)$. Repeat the problem for the $2s$ state: $(n, l, m) = (2, 0, 0)$. You need not attempt to evaluate radial integrals, but perform all other integrations (with respect to angles and time).

Problem 3.

Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows:

$$V_{11} = V_{22} = 0, \quad V_{12} = \gamma e^{i\omega t}, \quad V_{21} = \gamma e^{-i\omega t} \quad (\gamma \text{ real}).$$

At $t = 0$, it is known that only the lower level is populated—that is, $c_1(0) = 1$, $c_2(0) = 0$.

(a) Find $|c_1(t)|^2$ and $|c_2(t)|^2$ for $t > 0$ by *exactly* solving the coupled differential equation

$$i\hbar \dot{c}_k = \sum_{n=1}^{2} V_{kn}(t) e^{i\omega_{kn} t} c_n, \quad (k = 1, 2).$$

(b) Do the same problem using time-dependent perturbation theory to lowest nonvanishing order. Compare the two approaches for small values of γ. Treat the following two cases separately: (i) ω very different from ω_{21} and (ii) ω close to ω_{21}. Answer for (a): (Rabi's formula)

$$|c_2(t)|^2 = \frac{\gamma^2/\hbar^2}{\gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4} \sin^2 \left[\frac{\gamma^2}{\hbar^2} + \frac{(\omega - \omega_{21})^2}{4} \right]^{1/2} t,$$
$$|c_1(t)|^2 = 1 - |c_2(t)|^2.$$

Problem 4.

An atom is irradiated by right-hand circularly polarized light propagating along the z-direction. The atom transitions from energy level E_{nl} to $E_{n'l'}$. Find the selection rules.

Problem 5.

Assuming spin is neglected, the electronic state in an atom is expressed as

$$\psi_{nlm}(r, \theta, \varphi) = R_n(r) Y_l^m(\theta, \varphi).$$

(a) For an electric dipole spontaneous transition from an initial s-state (with energy E_{nl}, $l = 0$) to a final p-state (with energy $E_{n'l'}$, $l' = 1$), find the branching ratios for the final states with $m = 1, 0, -1$ respectively.

(b) Let the initial state be nlm and final state $n'l'm'$. (1) For a fixed final state $n'l'$, find the branching ratios for transitions to different m' states. (2) Find the branching ratios for transitions to different l' levels (neglecting the differences in the radial wave functions).