Quantum Mechanics 2025 HW11

Due 12/23 in Class
December 12, 2025

Problem 1.

For 1D free space, with the initial condition $G(x, t_a; x_a, t_a) = -i\delta(x - x_a)$, we have

$$G(x_b, t; x_a, t_a) = (-i)\left(\frac{m}{2\pi i t}\right)^{\frac{1}{2}} exp\left[\frac{im(x_b - x_a)^2}{2t}\right].$$

Hint: solve the differential Eq. $i\partial_t G(x, t; x_a, t_a) = -\frac{\hbar^2\partial_x^2}{2m}G(x, t; x_a, t_a)$

Problem 2.

Read lecture notes, and redo the propagator $G(x_b,\ t;\ x_a,\ t_a)$ in free space by using the method of path integral and show that you will arrive at the same results as in problem 1.

Problem 3.

Quantum statistical partition function can be represented by $Z(\beta) = \mathrm{tr}(e^{-\beta H})$. It can be represented in the coordinate representation as $Z(\beta) = \int dx\, G(x, x; \beta)$, where G is the imaginary time propagator defined as $G(x_b, x_a; \beta) = \langle x_b|e^{-\beta H}|x_a\rangle$. Please use the path integral method to represent $Z(\beta)$. (You can read the lecture note)

Problem 4.

Study the Berry phase problem $H = -B\hat{\mathbf{n}} \cdot \mathbf{S}$. Now $\mathbf{S}$ is the spin-1 operators. Please write down the matrices of $\mathbf{S}$ and calculate the Berry connection $\mathbf{A}(\hat{\mathbf{n}})$ for $\hat{\mathbf{n}}$ varying over the unit sphere. Please also calculate the monopole charge $q = \oiint \nabla \times \mathbf{A} \cdot ds$.