Quantum Mechanics 2025 HW2

Due 09/18 in Class

September 13, 2025

Problem 1. Harmonic osicllator

Consider a harmonic oscillator with the Hamiltonian $H = p^2/2m + 1/2m\omega^2 x^2$. In last homework we calculated the $(\dot{x})_{nn}$. Please reformulate this result in terms of $(p^2)_{nn}$.

1) Calculate the matrix elements x^2_{nn}. Calculate $\sqrt{(x^2)_{nn}(p^2)_{nn}}$, and find its value at $n = 0$.

2) Calculate the matrix of $x_{nm}(t)$ and $p_{nm}(t)$. Explicitly calculate $[x, p]_{nm}(t)$.

Problem 2. Quantum equation of motion

1) Consider the following the quantum Hamiltonian $H = H_1(p) + H_2(x)$. Prove that the quantum equation of motion of

$$\dot{x} = \frac{\partial H(x,p)}{\partial p}, \quad \dot{p} = -\frac{\partial H(x,p)}{\partial x}, \tag{1}$$

is equivalent to

$$\dot{x} = \frac{1}{i\hbar}[x, H], \quad \dot{p} = \frac{1}{i\hbar}[p, H]. \tag{2}$$

2) Consider the following Hamiltonian,

$$H(x, y, p_x, p_y) = \frac{(p_x - \frac{e}{c}A_x)^2}{2m} + \frac{(p_y - \frac{e}{c}A_y)^2}{2m}, \tag{3}$$

with $A_x = \frac{1}{2}By$, $A_y = -\frac{1}{2}Bx$.

Prove that the quantum equation of motion of

$$\begin{aligned} \dot{x} &= \frac{\partial H}{\partial p_x}, \quad \dot{p}_x = -\frac{\partial H}{\partial x}, \\ \dot{y} &= \frac{\partial H}{\partial p_y}, \quad \dot{p}_y = -\frac{\partial H}{\partial y}, \end{aligned} \tag{4}$$

are equivalent to

$$\begin{aligned} \dot{x} &= \frac{1}{i\hbar}[x, H], \quad \dot{p}_x = \frac{1}{i\hbar}[p_x, H]. \\ \dot{y} &= \frac{1}{i\hbar}[y, H], \quad \dot{p}_y = \frac{1}{i\hbar}[p_y, H]. \end{aligned} \tag{5}$$

Problem 3. De Broglie wave

Please figure out the procedure De Broglie proposed that $p = \hbar k$ by starting from the relation $E = \hbar\omega$.

Problem 4. Schrödinger equation

Please go through the process how Schrödinger proposed his equation by starting with Hamilton-Jacobi equation.

Problem 5. Quantum Potential

Start with the Schödinger equation

$$i\hbar\tfrac{\partial}{\partial t}\Psi = -\tfrac{\hbar^2\nabla^2}{2m}\Psi + V\Psi. \tag{6}$$

Express $\Psi(r,t) = e^{iS(r,t)/\hbar}$, and write down the equation that $S(r,t)$ satisfies. Compare it with the Hamilton-Jacobi equation. The difference is call quantum potential proposed by D. Bohm. Show that the term of quantum potential vanishes as $\hbar \to 0$.