Quantum Mechanics 2025 HW9

Due 12/02 in Class

November 26, 2025

Problem 1. 2D Coupled Harmonic Oscillator

Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}\left(x^2 + y^2\right)$$

1) What are the energies of the three lowest-lying states? Is there anydegeneracy?

2) We now apply a perturbation

$$V = \delta m\omega^2 xy$$

where δ is a dimensionless real number much smaller than unity.Find the zeroth-order energy eigenket and the corresponding energy to first order (that is, the unperturbed energy obtained in 1) plus the first-order energy shift) for each of the three lowest-lying states.

3) Solve the $H_0 + V$ problem exactly. Compare with the perturbation results obtained in 2).

(Hint: You may use $\langle n'| x |n\rangle = \sqrt{\hbar/2m\omega}\left(\sqrt{n+1}\delta_{n',n+1} + \sqrt{n}\delta_{n',n-1}\right)$)

Problem 2. Quadratic Perturbation

Consider a harmonic oscillator with $H = H_0 + H'$. $H_0 = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 x^2$, and $H' = \frac{1}{2}\varepsilon m\omega^2 x^2$. Please calculate the ground state energy shift up to the 2nd order, and the ground state wave function up to the 1st order. Please compare your result with the exact solution.

Problem 3. Quadrupole Perturbation

A p-orbital electron characterized by $|n, l = 1, m = \pm 1, 0\rangle$ (ignore spin) is subjected to a potential

$$V = \lambda\left(x^2 - y^2\right) (\lambda = \text{constatnt}).$$

1) Obtain the "correct" zeroth-order energy eigenstates that diagonalize the perturbation. You need not evaluate the energy shifts in detail, but show that the original threefold degeneracy is now completely removed.

2) Because V is invariant under time reversal and because there is no longer any degeneracy, we expect each of the energy eigenstates obtained in 1) to go into itself (up to a phase factor or sign) under time reversal. Check this point explicitly.

Problem 4. Second-order lifting of degeneracy

(This is a tricky problem because the degeneracy between the first and the second state is not removed in first order. See also Gottfried 1966, 397, Problem 1.) This problem is from Schif 1968, 295, Problem 4. A system that has three unperturbed states can be represented by the perturbed Hamiltonian matrix

$$\begin{pmatrix} E_1 & 0 & a \\ 0 & E_1 & b \\ a^* & b^* & E_2 \end{pmatrix}$$

where $E_2 > E_1$. The quantities a and b are to be regarded as perturbations that are of the same order and are small compared with $E_2 - E_1$. Use the second-order nondegenerate perturbation theory to calculate the perturbed eigenvalues. (Is this procedure correct?) Then diagonalize the matrix to find the exact eigenvalues. Finally, use the second-order degenerate perturbation theory. Compare the three results obtained.