

Quantum Mechanics 2026 HW2

Due 09/23 in Class

September 8, 2026

Problem 1. Harmonic oscillator

Consider a harmonic oscillator with the Hamiltonian $H = p^2/2m + 1/2m\omega^2 x^2$. In last homework we calculated the $(x)_{nn}$. Please reformulate this result in terms of $(p^2)_{nn}$.

1) Calculate the matrix elements x_{nn}^2 . Calculate $\sqrt{(x^2)_{nn}(p^2)_{nn}}$, and find its value at $n = 0$.

2) Calculate the matrix of $x_{nm}(t)$ and $p_{nm}(t)$. Explicitly calculate $[x, p]_{nm}(t)$.

Problem 2. Quantum equation of motion

1) Consider the following the quantum Hamiltonian $H = H_1(p) + H_2(x)$. Prove that the quantum equation of motion of

$$\dot{x} = \frac{\partial H(x, p)}{\partial p}, \quad \dot{p} = -\frac{\partial H(x, p)}{\partial x}, \quad (1)$$

is equivalent to

$$\dot{x} = \frac{1}{i\hbar}[x, H], \quad \dot{p} = \frac{1}{i\hbar}[p, H]. \quad (2)$$

2) Consider the following Hamiltonian,

$$H(x, y, p_x, p_y) = \frac{(p_x - \frac{e}{c}A_x)^2}{2m} + \frac{(p_y - \frac{e}{c}A_y)^2}{2m}, \quad (3)$$

with $A_x = \frac{1}{2}By$, $A_y = -\frac{1}{2}Bx$.

Prove that the quantum equation of motion of

$$\dot{x} = \frac{\partial H}{\partial p_x}, \quad \dot{p}_x = -\frac{\partial H}{\partial x}, \quad (1)$$

$$\dot{y} = \frac{\partial H}{\partial p_y}, \quad \dot{p}_y = -\frac{\partial H}{\partial y}, \quad (4)$$

are equivalent to

$$\dot{x} = \frac{1}{i\hbar}[x, H], \quad \dot{p}_x = \frac{1}{i\hbar}[p_x, H]. \quad (2)$$

$$\dot{y} = \frac{1}{i\hbar}[y, H], \quad \dot{p}_y = \frac{1}{i\hbar}[p_y, H]. \quad (5)$$

Problem 3. De Broglie wave

Please figure out the procedure De Broglie proposed that $p = \hbar k$ by starting from the relation $E = \hbar\omega$.

Problem 4. Schrodinger equation

Please go through the process how Schrodinger proposed his equation by starting with Hamilton-Jacobi equation.

Problem 5. Quantum Potential

Start with the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2 \nabla^2}{2m} \Psi + V\Psi. \quad (6)$$

Express $\Psi(r, t) = e^{iS(r, t)/\hbar}$, and write down the equation that $S(r, t)$ satisfies. Compare it with the Hamilton-Jacobi equation. The difference is called quantum potential proposed by D. Bohm. Show that the term of quantum potential vanishes as $\hbar \rightarrow 0$.